ROLL NO. _____

Code: CT42 Subject: DESIGN AND ANALYSIS OF ALGORITHMS

ALCCS

Time: 3 Hours **FEBRUARY 2014** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE:

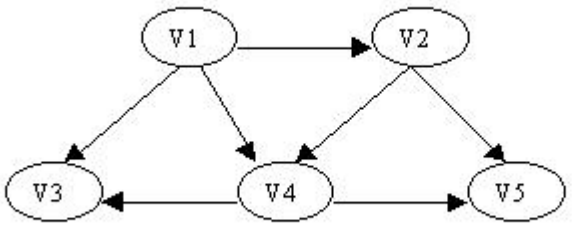
- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**

Q.1 a. Solve the recurrence relation $T(n) = 27\ T(n/3) + \Theta(n^3 \lg n)$

b. Given the following code fragment, what is its Big-O running time?

```
i = n;
while i > 0
  k = k + 2;
 i = i / 2;
```

c. Show the ordering of vertices produced by topological sort in the following graph. What is time complexity of topological sort?



d. Given a sorted array and a value x. Suggest O(n) algorithm to find two values in the array whose sum is equal to x.

e. Suppose that the root of the Red-Black tree is red. If we make it black, does the tree remain a Red Black tree?

f. What are the conditions for a problem to be solved using Dynamic Programming.

g. Explain intractable problem with an example. **(7×4)**

Q.2 a. Give an efficient algorithm that determines whether or not a given directed graph G = (V, E) contains a cycle. Discuss its time complexity. **(9)**

b. Suppose we wish to search a linked list of length *n*, where each element contains a key *k* along with a hash value *h*(*k*). Each key is a long character string. How might we take advantage of the hash values when searching the list for an element with a given key? **(9)**

Q.3 a. What is the difference between the binary-search tree property and the heap property? Can the heap property be used to print out the keys of an n-node tree in sorted order in O(n) time? Explain how or why not. **(9)**

b. Consider a B-tree with degree m. i.e. the number of children c, of any internal node (except the root) is such that $m–1 \leq c \leq 2m–1$. Derive the maximum and minimum number of records in the leaf nodes for such a B-tree with height h ($h \geq 1$). (Assume that the root of a tree is at height 0). **(9)**

Q.4 a. Design a recursive solution to the matrix chain multiplication problem. Find an optimal parenthesization of a matrix chain product whose sequence of dimension is <5, 10, 3, 12, 5, 50, 6> **(12)**

b. What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?
a:1, b:1, c:2, d:3, e:5, f:8, g:13, h:21
Can you generalise your answer to find the optimal code when the frequencies are the first n Fibonacci numbers? **(6)**

Q.5 a. Consider the problem of "Making Change". Coins available are:

- dollars (100 cents)
- quarters (25 cents)
- dimes (10 cents)
- nickels (5 cents)
- pennies (1 cent)

Design an algorithm using greedy approach to make a change of a given amount using the smallest possible number of coins. **(9)**

b. Write a program to merge two arrays in sorted order, so that if an integer is in both the arrays, it gets added into the final array only once. **(9)**

Q.6 a. How can the output of the Floyd-Warshall algorithm be used to detect the presence of a negative-weight cycle? **(6)**

b. Write down the algorithm for counting sort. Illustrate the operation of your algorithm on the array $A = (6,0,2,0,1,3,4,6,1,3,2)$ **(12)**

Q.7 a. Define the set of NP complete languages. Prove that travelling salesman problem is NP complete. **(9)**

b. Write and explain Knuth-Morris-Pratt Algorithm and discuss its computational complexity. **(9)**