ROLL NO. _____

Code: CT42 Subject: DESIGN AND ANALYSIS OF ALGORITHMS

ALCCS – NEW SCHEME

Time: 3 Hours **FEBRUARY 2013** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE:

- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**

Q.1 a. Express the function $f(n) = 5n^3 + 5n^2 + 10n$ in Θ notation.

b. What is the asymptotic lower bound in determining the complexity of an algorithm?

c. Write two characteristics that distinguishes as dynamic algorithm from greedy algorithm.

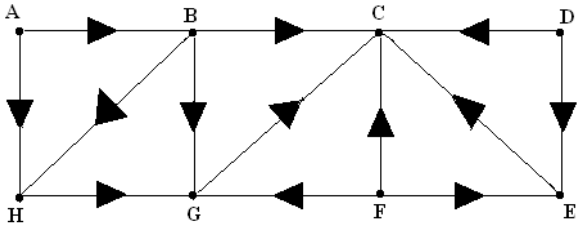
d. Express, in recursive equation form, the time required to search an element from an array of n elements using binary search method.

e. If the Prim's algorithm is used to find minimum cost spanning tree from a weighted connected graph of n nodes, how many edges of the graph will be selected and how many steps will be required to select those edges?

f. A graph has n nodes and 6n edges. When the graph is represented as adjacency matrix then what is the order of that matrix and how many entries in that matrix will be Boolean 1?

g. Define B-Tree. Show all legal B-Trees of minimum degree 2 that represent {1,2,3,4,5}. **(7×4)**

Q.2 a. Write an algorithm to sort a Directed Acyclic Graph (DAG) topologically. Show the ordering of vertices produced by topological sort when it is run on the following DAG. **(12)**



b. Differentiate between Ω and ω notations used to represent complexity. Use growth of function concept to explain your answer. **(6)**

Q.3 a. Prove that $a_n = 2a_{n-1} + 1$; $a_1 = 1$ is of order 2^n. **(8)**

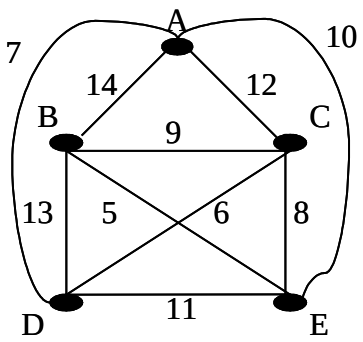
b. Determine the time complexity of longest common subsequence problem? **(10)**

Q.4 a. What are Huffman Codes? What is an optimal Huffman Code for the following set of frequencies, based on the first 8 Fibonacci number:
a:1, b:1, c:2, d:3, e:5, f:8, g:13, h:21
Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers? **(9)**

b. Construct an optimal binary search tree for the following items with probabilities given in the table below. **(9)**

Items	1	2	3	4	5
Probability	0.24	0.22	0.23	0.3	0.01

Q.5 a. In the following graph find all pairs shortest path using Floyd Warshall algorithm. Give the outline of algorithm and write its time complexity.



(10)

b. Divide and conquer paradigm involves three steps at each level of recursion. What are all they? Show that merge-sort algorithm closely follows these steps. Illustrate the operation of merge sort on the array A={3,41,52,26,38,57,9,49} **(8)**

Q.6 a. Write algorithm to find Minimum Spanning Tree (MST) using Krushkal's method and compute its time complexity. Apply the algorithm to determine MST in the graph of Q5(a). **(9)**

b. Draw a Binary Search Tree obtained by entering the months JANUARY to DECEMBER in that order in to an initially empty BST. Convert this BST to a height-balanced one using the definition of AVL trees. **(9)**

Q.7 a. Prove that Hamiltonian cycle problem is polynomial time verifiable. **(9)**

b. Compute the prefix function π for the pattern "abbabbababaab" when the alphabet is $\sum = \{a, b\}$ in the KMP algorithm. **(9)**