Q.1

a. Show that \((\sim P \land (\sim Q \land R)) \lor (Q \land R) \lor (P \land R) \iff R\)

b. If \(a = b\) then prove that \(ab' + a'b = 0\).

c. Draw the graph \(G\), whose adjacency matrix is given as:

\[
A = \begin{bmatrix}
V_1 & V_2 & V_3 & V_4 & V_5 \\
V_1 & 0 & 1 & 0 & 1 \\
V_2 & 1 & 0 & 0 & 1 \\
V_3 & 0 & 0 & 0 & 1 \\
V_4 & 1 & 1 & 0 & 1 \\
V_5 & 0 & 1 & 1 & 1
\end{bmatrix}
\]

d. Draw the connected regular graphs of degree 0, 1 and 2.

e. What is tree? Define the m-way tree with suitable example.

f. Which of the following graphs are trees:

(i) 

(ii) 

g. State pigeon hole principle. Explain using a suitable example.

Q.2

a. Determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7.
b. Show that \(1^2 - 2^2 + 3^2 + \ldots + (-1)^{n+1}n^2 = \frac{(-1)^{n+1}n^{n+1}}{2}\) \(\text{(9)}\)

Q.3

a. Write each of the following in disjunctive normal form

(i) \((x + y)(x' + y')\)

(ii) \(x'z + xz'\)

(iii) \(x\)

b. Let A = \(\{2,3,4,5\}\). Consider the relation B and C on a set A defined by

\[B = \{(2,2),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5),(5,3)\}\] and

\[C = \{(2,3),(2,5),(3,4),(3,5),(4,2),(4,3),(4,5),(5,2),(5,5)\}\]

then find

(i) \(B \circ C\)

(ii) \(C \circ B\) \(\text{(9)}\)

Q.4

a. Let A = \(\{1,2,3\}\) and consider the two relations

\[R = \{(1,1), (1,2), (1,3), (2,2), (3,3)\}\]

\[S = \{(1,1), (1,2), (2,2), (3,2), (3,3)\}\]

Then find \(R^{-1}, R \cup S\) and \(R \cap S\) by representing the matrices for R and S. \(\text{(9)}\)

b. Show that the operations of meet and join on a lattice are commutative, associattive and idempotent. \(\text{(9)}\)

Q.5

a. “A Boolean algebra is a complemented, distributive lattice” Justify the statement. \(\text{(5)}\)

b. Let \(S = \{a, b, c\}\). Draw the diagram of \((P(S),\subseteq)\). \(\text{(4)}\)

c. Find the shortest path from \(V_1\) to \(V_7\) in the following weighted graph:

![Weighted Graph](image)

Q.6

a. What is planar graph? Check if the following graphs are planar graph. \(\text{(9)}\)
b. Determine the minimal spanning tree for the graph given below using Krushal’s algorithm.

Q.7  a. Describe the language $L = L(M)$ accepted by DFA whose transition graph is shown in the figure:

b. Design a DFA to accept the language $L = \{u : u \text{ has both even number of } 0\text{'s and even number of } 1\text{'s}\}$. Check whether this DFA accepts 110101.