Code: CS41 Subject: NUMERICAL & SCIENTIFIC COMPUTING

ALCCS – OLD SCHEME

Time: 3 Hours **FEBRUARY 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE:

- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**
- **All calculations should be up to three places of decimals.**

Q.1 a. Define absolute error, relative error and percentage error.

b. Solve the initial value problem $y' = xy, y(0) = 1$.

c. Explain the cases where Newton's method fails.

d. Fit a line to the following data using principle of least squares:

x	:	1	2	3	4
f(x)	:	-1	1	3	5

e. Explain Euler's method for solving an ordinary differential equation.

f. Evaluate $I = \int_0^1 \frac{1}{1+x}dx$, correct to 3 decimal places by Simpson's rule (h = 0.125).

g. Find a real root of the equation
$f(x): x^3 - x - 1 = 0$
upto 4 – decimal places. **(7 × 4)**

Q.2 a. Find the real root of the equation $x^3 - 2x - 5 = 0$ using Bisection method upto four iterations only. **(9)**

b. Solve by Jacobi's method the following system of linear equations

$$2x_1 - x_2 + x_3 = -1$$
$$x_1 + 2x_2 - x_3 = 6$$
$$x_1 - x_2 + 2x_3 = -3$$

Use the method upto 3-iterations only. **(9)**

Q.3 a. Write the polynomial of lowest degree which satisfies the following sets of numbers using the forward difference interpolation formula. **(8)**

x	0	1	2	3	4	5	6	7
$f(x)$	0	7	26	63	124	215	342	511

b. Show that the LU decomposition method fails to solve the following system of linear equations: **(10)**

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 5 \\ 3 & 2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$$

Q.4 a. Use Gauss-Seidel method to solve the following systems of linear equations:

$$x + y - 2 = 0$$
$$-x + 3y = 2$$
$$x - 3z = -3$$

Initial solution vector is $[0.8, \ 0.8, \ 2.1]^T$ and use the method upto 3 – iteration only. **(9)**

b. Evaluate $\int_1^6 \left[2 + \sin\left(2\sqrt{x}\right)\right] dx$ using Simpson's rule with 5 (intervals). **(9)**

Q.5 a. Solve the initial value problem $\frac{dy}{dx} = 1 + y^2$ where y = 0 when x = 0 using Forth order classical Runge-Kutta method. Also find y(0.2) and y(0.4). **(10)**

b. The population of a town in the decennial census was as given below. Estimate the population for the year 1895. **(8)**

Year(x):	1891	1901	1911	1921	1931
Population (y): (in thousands)	46	66	81	93	101

Q.6 a. Evaluate the integral $I = \int_1^2 \frac{2x\, dx}{1 + x^4}$ using Gauss-Legendre 1-point, 2-point and 3-point quadrature rules. Also compare the result with the exact solution. **(10)**

b. Find a real root of equation $x \log_{10} x = 1.2$ by Newton-Raphson method. Root should be correct to four decimal places. **(8)**

Q.7 a. Find the largest eigen value in modulus & the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

using the power method. **(9)**

b. The following table of values is given:

$x:$	-1	1	2	3	4	5	7
$f(x):$	1	1	16	81	256	625	2401

using the formula $f'(x_1) = \frac{f(x_2) - f(x_0)}{2h}$ and the Richardson extrapolation, find $f'(3)$. **(9)**