ROLL NO. ___________________

Code: CT22 **Subject: DISCRETE MATHEMATICAL STRUCTURES**

ALCCS – NEW SCHEME

Time: 3 Hours **AUGUST 2013** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE:

- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**

Q.1 a. Use equivalences to show that $(A \vee B) \rightarrow A \equiv B \rightarrow A$

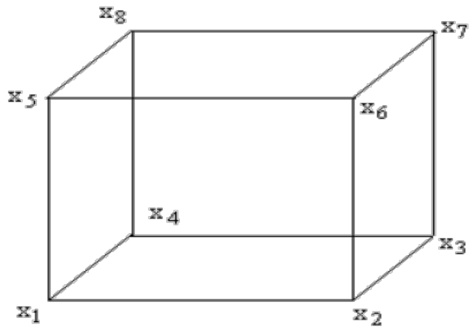
b. There are 37 people at a house party. Prove that at least four of them must have birthdays in the same month.

c. What do you mean by the term contradiction? Check if the following statement is a contradiction?
"If the sky is cloudy then it will rain and it will not rain"

d. What do you mean by a planar graph? Is $K_{3,3}$ a planar graph?

e. In a class , 8 students play football and hockey,7 students do not play football or hockey, 13 students play hockey and 19 students play football. How many students are there in the class?

f. Define Hamilton path. Determine if the following graph has a Hamilton circuit.



g. If $G = (\{S\}, \{0,1\}, \{S \rightarrow 0S1, S \rightarrow \epsilon\}, S)$, find L(G). **(7 × 4)**

Q.2 a. Find DNF and CNF of the following identity without using truth table:
$(A \rightarrow (B \vee C)) \rightarrow (A \wedge D)$ **(6)**

b. State DeMorgan's laws. Prove it using the truth table. **(5)**

c. Check the validity of the following argument. If valid, construct a formal proof, if not explain why.

"If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it is always the case that wages for such persons are not equal therefore the labour market is not perfect." **(7)**

Q.3 a. Calculate the number of distinct natural numbers not exceeding 1000 which are multiples of 10, 15, 35 or 55. **(10)**

b. Show that if R_1 and R_2 are equivalence relations on A, then $R_1 \cap R_2$ is an equivalence relation. **(8)**

Q.4 a. Draw the Hasse diagram for the poset $(\wp(A), \subset)$ where $A = \{1,2,3,4\}$ and $\wp(A)$ is the power set of A. **(6)**

b. If $[L, \wedge, \vee]$ is a complemented and distributive lattice, then prove that the complement $\bar{a}$ of any element $a \in L$ is unique. **(6)**

c. Prove that if $(A,\leq)$ has a least element, then $(A,\leq)$ has a unique least element. **(6)**

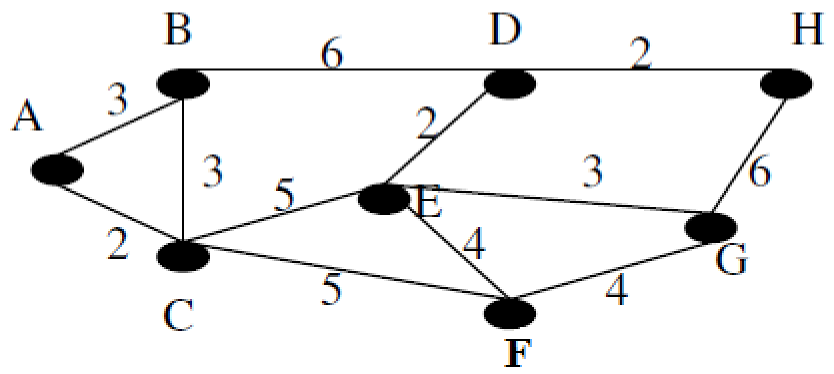
Q.5 a. State and prove Euler's formula for a connected planar graph G = (V, E). Also prove that if $|V| > 2$, then $|E| \leq 3|V| - 6$. **(9)**

b. Prove that a simple graph is connected if and only if it has a spanning tree. **(5)**

c. State Kuratowski's Theorem. For what purpose this theorm is used? Show by an example, how this theorem is used. Give an example of a graph that you prove to be non-planar using this theorem. **(4)**

Q.6 a. What is a Binary Search Tree (BST)? Make a BST for the following sequence of numbers.
45,32,90,34,68,72,15,24,30,66,11,50,10
Traverse the BST so created in Postorder. **(2+5+2)**

b. What is the difference between a spanning tree and a minimum spanning tree. Apply Prim's algorithm on the following graph to find minimum spanning tree. **(9)**



Q.7 a. Let L be a language over {0, 1} such that each string starts with a 0 and ends with a minimum of two subsequent 1's. Construct:
(i) the regular expression to specify L.
(ii) a finite state automata M, such that M (L) = L.
(iii) a regular grammar G, such that G(L) = L. **(9)**

b. Determine the values of the following prefix notation:
+, -, -, 3, 2, -, 2, 3, /, 6, –, 4, 2 **(4)**

c. Find the state table for the NFA with the state diagram given below. **(5)**