Q.1
a. Let \( f(n) \) and \( g(n) \) be asymptotically non-negative functions. Using the basic definition of \( \Theta \)-notation, prove that \( \max(f(n), g(n)) = \Theta(f(n) + g(n)) \)

b. Define: “topological sort” of a directed acyclic graph. What is the time complexity of topological sort?

c. Explain briefly how ‘Divide and Conquer’ approach is used in Quick Sort.

d. Define Linear and Quadratic Probing.

e. What properties need to be satisfied by a Binary Search Tree to be a Red-Black tree?

f. How B+ trees are different from B trees?

g. If any NP-complete problem belongs to class P, then is \( P = NP \)?

Q.2
a. Consider the following recurrence
\[
T(n) = T(n/3) + T(2n/3) + n
\]
Obtain asymptotic bound using recursion tree method.

b. What is the basic idea behind Rabin-Karp algorithm? What is expected running time of this algorithm?

c. Write a brief note on NP-completeness and the classes-P, NP and NPC.

Q.3
a. Write pseudocode for the basic depth-first-search algorithm. Perform depth-first-search on the following directed graph \( G \).
b. Sort the following list using quick sort algorithm:
\(<50, 40, 20, 60, 80, 100, 45, 70, 105, 30, 90, 75>\)
Also discuss worst and best case of quick sort algorithm. (8)

Q.4 a. Define Max-heap. Write Max-Heapify algorithm that maintain max-heap property. (9)

b. Write an algorithm that perform left rotation over a node X in a Red-Black tree. What
time does this procedure take in worst case? (9)

Q.5 a. Create an AVL search tree from the given set of values:

b. Show that the total running time of merge-sort is \(O(n \log n)\). (4)

c. Write Kruskal’s algorithm to find a minimum spanning tree of a Graph. (6)

Q.6 a. Given the characters S <a, b, c, d, e, f> with the following probability P=<29, 25, 20,
12, 05, 09>. Build a binary tree using greedy Huffman algorithm. (8)

b. Given two sequences of characters,
P=<MLNOM>
Q=<MNOM>
Obtain the longest common subsequence. (10)

Q.7 a. Discuss Knuth Morries Pratt (KMP) algorithm. Compute whether the pattern
P=10100111 is present in the string T=1001010100111 or not. (10)

b. Given the four matrices P_{5\times4}, Q_{4\times6}, R_{6\times2}, T_{2\times7}, find the optimal sequence for the
computation of multiplication operation. (8)