ROLL NO. _____

Code: CT22 Subject: DISCRETE MATHEMATICAL STRUCTURES

ALCCS – (NEW SCHEME)

Time: 3 Hours **AUGUST 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE:

- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**

Q.1 a. Show that

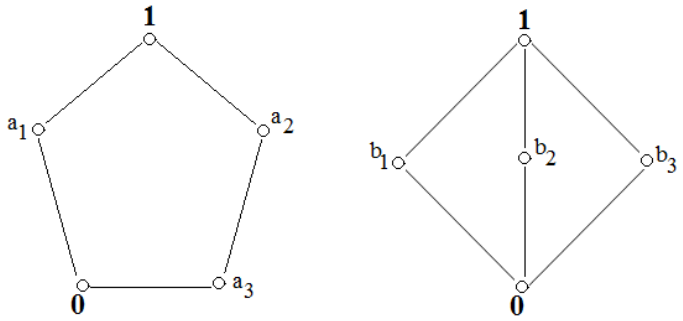
$$\sim(P \wedge Q) \rightarrow (\sim P \vee (\sim P \vee Q)) \Leftrightarrow (\sim P \vee Q)$$

b. Write an equivalent formula for $P \wedge (Q \Leftrightarrow R) \vee (R \Leftrightarrow P)$ which does not contain the biconditional.

c. Let $R = \{\langle 1,2\rangle, \langle 3,4\rangle, \langle 2,2\rangle\}$ and $S = \{\langle 4,2\rangle, \langle 2,5\rangle, \langle 3,1\rangle, \langle 1,3\rangle\}$. Find $R \circ S, S \circ R$, $R \circ (S \circ R)$, $(R \circ S) \circ R$.

d. Show that the maximum number of edges in a simple graph with n vertices is $n(n-1)/2$.

e. Show that the Lattice given by the diagrams below are not distributive.

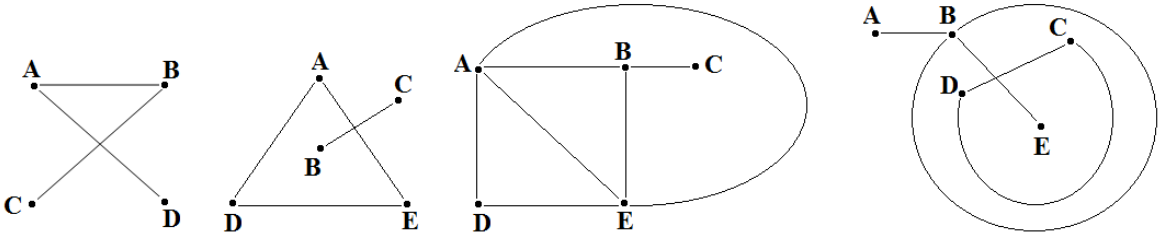


f. Obtain the values of the Boolean forms $x_1 * (x_1' \oplus x_2)$, $x_1 * x_2$ and $x_1 \oplus (x_1 * x_2)$ over the ordered pairs of the two element Boolean algebra.

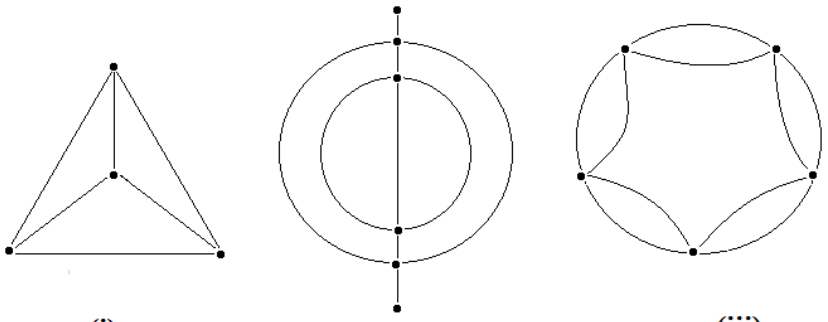
g. Consider the multigraphs G in figures below

(i) Which of them are connected? If a graph is not connected, find its connected components.

(ii) Which are cycle free (without cycles)? **(7 × 4)**

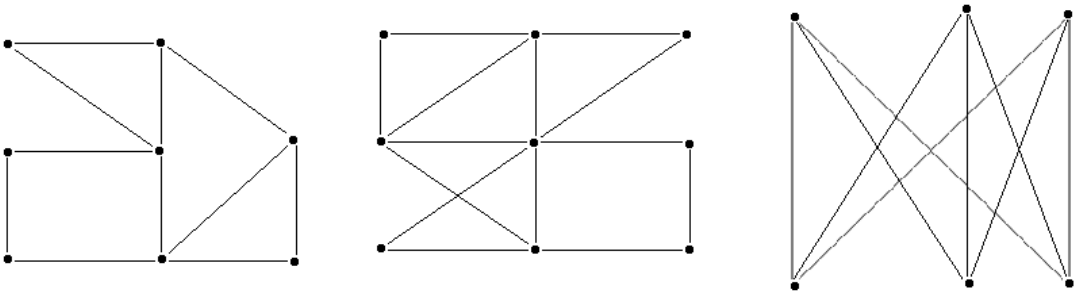


Q.2 a. Count the number V of vertices, the number E of edges and the number R of regions of each map in figures below and verify Euler's formula. Also find degree of the outside region. **(6)**



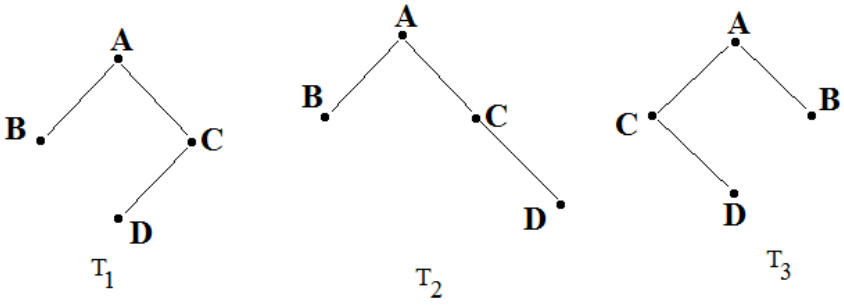
(i) (ii) (iii)

b. Which of the graphs G in figure below have a Hamiltonian circuit? If not, why not? **(6)**



c. Convert the infix expression
$(a+b\uparrow c\uparrow d)*(e+f/d)$ into reverse polish expression. **(6)**

Q.3 a. Consider the three trees T_1, T_2, T_3. Identify those which represent the same:
(i) rooted tree (ii) ordered rooted tree
(iii) binary tree **(9)**



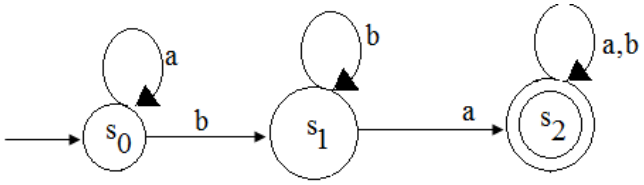
T_1 T_2 T_3

b. Consider the algebraic expression $E = (2x + y)(5a - b)^3$
(i) Draw the tree T which corresponds to expression E.
(ii) Find the scope of the exponential operator; that is, find the subtree rooted at the exponential operator.
(iii) Find the preorder of T. **(9)**

Q.4 a. For a 3-ary tree with n internal nodes, prove that $t = 2(n-1)+3$ where t is terminal nodes. **(8)**

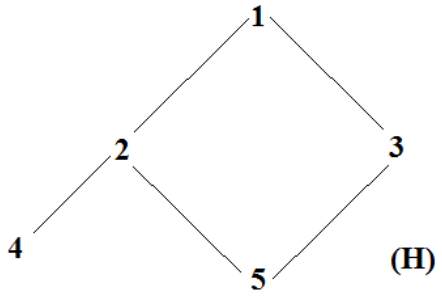
b. Suppose the following list of letters is inserted into an empty binary search tree
J, R, P, G, W, E, M, H, P, A, F, Q
(i) Find the final tree T.
(ii) Find the inorder traversal of T. **(10)**

Q.5 a. Describe the word w in the language L accepted by automation M in Figure below: **(9)**



b. Let $A = \{0,1\}$, construct an finite automation M such that L(M) will consist of
(i) words without substring '000'.
(ii) words which starts and end with same double letter. **(9)**

Q.6 a. Let $A = \{1,2,3,4,5\}$ be ordered by Hasse diagram (H). Insert the correct symbol, <, >, or $\|$ (not comparable), between each pair of elements:
(i) 1 ____5
(ii) 2 ____3
(iii) 4 ____1
(iv) 3 ____4 **(9)**



(H)

b. Consider the ordered set A in the above Hasse diagram
(i) Find all minimal and maximal elements of A.
(ii) Does A have a first element or a last element? **(9)**

Q.7 Among integers 1 to 1000
(i) How many of them are not divisible by 3 nor by 5 nor by 7? **(9)**
(ii) How many are not divisible by 5 or 7 but divisible by 3? **(9)**