Q.1  

a. Show that 
\[ \sim (P \land Q) \rightarrow (\sim P \lor (\sim P \lor Q)) \iff (\sim P \lor Q) \]

b. Write an equivalent formula for \( P \land (Q \iff R) \lor (R \iff P) \) which does not contain the biconditional.

c. Let \( R = \{ (1, 2), (3, 4), (2, 2) \} \) and \( S = \{ (4, 2), (2, 5), (3, 1), (1, 3) \} \). Find \( R \circ S, S \circ R, R \circ (S \circ R), (R \circ S) \circ R \).

d. Show that the maximum number of edges in a simple graph with \( n \) vertices is \( n(n - 1)/2 \).

e. Show that the Lattice given by the diagrams below are not distributive.

f. Obtain the values of the Boolean forms 
\[ x_1 \ast (x_1 \oplus x_2), x_1 \ast x_2 \] and \( x_1 \oplus (x_1 \ast x_2) \) over the ordered pairs of the two element Boolean algebra.

g. Consider the multigraphs \( G \) in figures below
   (i) Which of them are connected? If a graph is not connected, find its connected components.
   (ii) Which are cycle free (without cycles)? \( 7 \times 4 \)
Q.2  
   a. Count the number $V$ of vertices, the number $E$ of edges and the number $R$ of regions of each map in figures below and verify Euler’s formula. Also find degree of the outside region. (6)

   ![Maps](image)

   (i) (ii) (iii)

   b. Which of the graphs $G$ in figure below have a Hamiltonian circuit? If not, why not? (6)

   ![Graphs](image)

   c. Convert the infix expression\[ (a + b + c + d) \times (e + f/d) \] into reverse polish expression. (6)

Q.3  
   a. Consider the three trees $T_1, T_2, T_3$. Identify those which represent the same:
   (i) rooted tree
   (ii) ordered rooted tree
   (iii) binary tree (9)

   ![Trees](image)
b. Consider the algebraic expression \( E = (2x + y)(5a - b)^3 \)
   (i) Draw the tree \( T \) which corresponds to expression \( E \).
   (ii) Find the scope of the exponential operator; that is, find the subtree rooted at the exponential operator.
   (iii) Find the preorder of \( T \).

Q.4  

a. For a 3-ary tree with \( n \) internal nodes, prove that \( t = 2(n-1) + 3 \) where \( t \) is terminal nodes.

b. Suppose the following list of letters is inserted into an empty binary search tree
   (i) Find the final tree \( T \).
   (ii) Find the inorder traversal of \( T \).

Q.5  

a. Describe the word \( w \) in the language \( L \) accepted by automation \( M \) in Figure below:

b. Let \( A = \{0, 1\} \), construct an finite automation \( M \) such that \( L(M) \) will consist of
   (i) words without substring ‘000’.
   (ii) words which starts and end with same double letter.

Q.6  

a. Let \( A = \{1, 2, 3, 4, 5\} \) be ordered by
   Hasse diagram (\( H \)). Insert the correct symbol, \( <, >, \) or \( \parallel \) (not comparable),
   between each pair of elements:
   (i) \( 1 \ldots 5 \)
   (ii) \( 2 \ldots 3 \)
   (iii) \( 4 \ldots 1 \)
   (iv) \( 3 \ldots 4 \)

b. Consider the ordered set \( A \) in the above Hasse diagram
   (i) Find all minimal and maximal elements of \( A \).
   (ii) Does \( A \) have a first element or a last element?

Q.7  

Among integers 1 to 1000
   (i) How many of them are not divisible by 3 nor by 5 nor by 7?
   (ii) How many are not divisible by 5 or 7 but divisible by 3?