ROLL NO. ___________________

Code: CS41 Subject: NUMERICAL & SCIENTIFIC COMPUTING

ALCCS – OLD SCHEME

Time: 3 Hours **AUGUST 2012** Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE:

- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**
- **All calculations should be up to three places of decimals.**

Q.1 a. What are the different types of errors in numerical computations?

b. Discuss convergence of Iteration method.

c. Find the relative error in calculation of $\frac{7.342}{0.241}$. Numbers are correct to three decimal places. Determine the smallest interval in which true result lies.

d. Write the algorithm for Regula falsi method.

e. Using Newton's divided difference formula, find a polynomial function satisfying the following data:

x:	-4	-1	0	2	5
f(x):	1245	33	5	9	1335

Hence find f(1).

f. Use the two point Gauss – Legendre quadrature formula to evaluate $\int_{5}^{12} \frac{1}{x} dx$.

g. Use trapezoidal rule to evaluate $\int_{0}^{1} x^3 dx$ considering five sub-intervals. **(7×4)**

Q.2 a. Use Regula-Falsi method to find the real root of the equation $x^3 - x^2 - 2 = 0$ correct to three decimal places. **(9)**

b. Use Newton-Raphson method to find all roots of the equation $\cos x - x^2 = x$ correct to four decimal places. **(9)**

Q.3 a. Solve the equations

$$x + y = z = 6$$
$$3x + (3 + \in)y + 4z = 20$$
$$2x + y + 3z = 13$$

using Gauss elimination method, where ϵ is small such that $1 \pm \in^2 \approx 1$. **(9)**

b. Apply Gauss-Seidel iteration method to solve the following equations: **(9)**

$$10x - 2y - z - u = 3$$
$$-2x + 10y - z - u = 15$$
$$-x - y + 10z - 2u = 27$$
$$-x - y - 2z + 10u = -9$$

Q.4 a. Using the Jacobi method, find all the eigenvalues and the corresponding eigenvectors of the matrix:-

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$ **(9)**

b. Given the matrix A = I + L + U where

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

L and U are strictly lower and upper triangular matrices respectively. Decide whether
(i) Jacobi
(ii) Gauss-Seidel
methods converge to the solution of Ax = b. **(9)**

Q.5 a. Using Lagrange's interpolation formula, find y(10) from the following table:

x	5	6	9	11
y	12	13	14	16

(9)

b. Apply Hermite's interpolation formula to find a cubic polynomial which meets the following specifications: **(9)**

x_i	y_i	y_i'
0	0	0
1	1	1

Q.6 a. The speed, v meters per second, of a car, t seconds after it starts, is shown in the following table:

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.60	10.08	18.90	21.60	18.54	10.26	5.40	4.50	5.40	9.00

Using Simpson's rule, find the distance travelled by the car in 2 minutes. **(9)**

b. A differentiation rule of the form

$$f(x_0) = \alpha_0 f_0 + \alpha_1 f_1 + \alpha_2 f_2$$

Where $x_k = x_0 + kh$ is given. Find the values of $\alpha_{0,}\ \alpha_1$ and α_2 so that the rule is exact for $f \in P_2$. Find the error term. **(9)**

Q.7 a. Solve the equation $\frac{dy}{dx} = 1 - y$ with the initial condition x = 0, y = 0 using Euler's algorithm and tabulate the solutions at x = 0.1, 0.2, 0.3. **(9)**

b. Use Taylor's series method to solve

$$\frac{dy}{dx} = x + y\ ;\ y(1) = 0$$

numerically up to x = 1.2 with h = 0.1. Compare the final result with the value of explicit solution. **(9)**