**Q.2 a. From the definition of Laplace transform find the Laplace transform of $x(t) = t$. (4)**

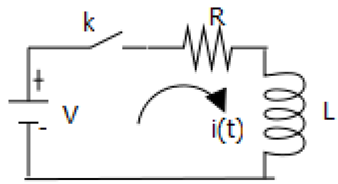
**Answer :**

$$X(s) = \int_0^{\infty} x(t)\, e^{-st}\, dt \qquad (1\,mark)$$

$$L[t] = \int_0^{\infty} t\, e^{-st}\, dt = t \left. \frac{e^{-st}}{-s} \right|_0^{\infty} + \int_0^{\infty} \frac{e^{-st}}{s}\, dt = \left[ t \frac{e^{-st}}{-s} - \frac{e^{-st}}{s^2} \right] = \frac{1}{s^2} \qquad (3\,marks)$$

**b. Using Laplace transform obtain the step response of series RL circuit. (6)**

**Answer :**




**Circuit---(1 mark)**

$$L\frac{di(t)}{dt} + Ri(t) = V\,u(t) \qquad (1\,mark)$$

$$Taking\, LT\, on\, both\;\; sides \quad L\big(sI(s) - t(0+)\big) + R\,I(s) = \frac{V}{s} \qquad (1\,mark)$$

$$I(s) = \frac{V}{L}\frac{1}{s\big(s + R/L\big)} = \frac{A}{s} + \frac{B}{s + R/L} \qquad (1\,mark)$$

$$A = \left. \frac{1}{\big(s + R/L\big)} \right|_{s=} = L/R \quad , \quad B = \left. \frac{1}{s} \right|_{s=-R/L} = -L/R \qquad (1\,mark)$$

$$i(t) = \frac{V}{R}\left[1 - e^{-\frac{R}{L}t}\right] \qquad (1\,mark)$$

**c. Determine the Laplace transform of the signal as shown in Fig.1 (6)**

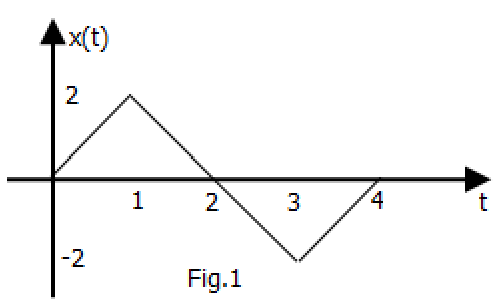
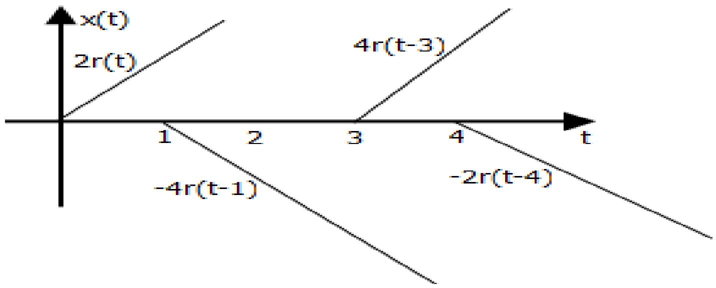



Fig.1

**Answer :**




$x(t) = 2r(t) - 4r(t-1) + 4r(t-3) - 2r(t-4)$

*or* **(4 marks)**

$x(t) = 2tu(t) - 4(t-1)u(t-1) + 4(t-3)u(t-3) - 2(t-4)u(t-4)$

$X(s) = \frac{2}{s^2} - \frac{4}{s^2}e^{-s} + \frac{4}{s^2}e^{-3s} - \frac{2}{s^2}e^{-4s}$ **(2 marks)**

**Q.3 a. Find the current flowing through $R_L = 7.5\Omega$ resistor, using Superposition theorem in the network as shown in Fig.2** **(8)**

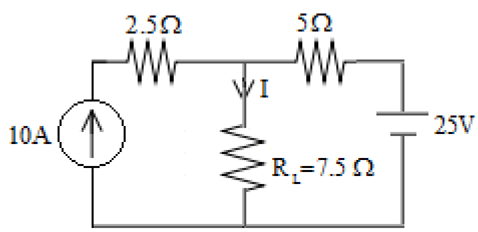
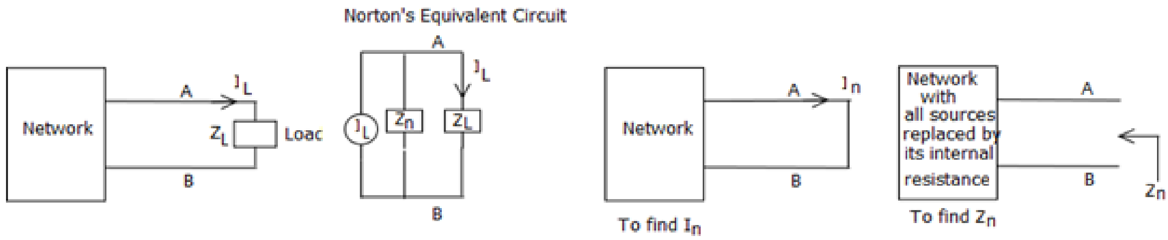



**Fig.2**

**b. State and explain the following theorems**
**(i) Norton's and (ii) Maximum power transfer.** **(8)**

**Answer :**

(i) Norton's theorem: A network across the load branch is converted into a single current source as shown in following figure. Where $I_n$ is called as Norton's current source and $Z_n$ is called as Norton's impedance. The current $I_n$ is found by replacing the load by short circuit and also known as short circuit current and $Z_n$ is the impedance looking from the load into the network replacing all the sources by its internal resistance. **(Explanation -2 marks + diagrams – 2 marks)**




(ii) Maximum power transfer theorem: The condition to transfer maximum power dependent on the type of source, source impedance and the type of load. The following cases are considered.

1. DC source, source impedance is pure resistance and load impedance is pure resistance varying.
2. AC source, complex source impedance and load impedance is pure resistance varying.
3. AC source, complex source impedance and complex load impedance but only resistive part of load impedance is varying.
4. AC source, complex source impedance and complex load impedance but only reactive part of load impedance is varying.
5. AC source, complex source impedance and complex load impedance and both reactive part and resistive part of load is varying. **(4 marks)**

**Q.4 a. State and explain symmetry and reciprocity in two port representation (6)**

**Answer :**

A two port network is said to be symmetric if the ports of the network can be interchanged without changing the port voltages and currents.

If $Y_{11} = Y_{22}$ *or* $Z_{11} = Z_{22}$ ie, the driving point impedances/admittances are identical, then the network is symmetrical.

A two port network is said to be reciprocal if the ratio of response transform to the excitation transform is invariant.

If $Y_{12} = Y_{21}$ *or* $Z_{12} = Z_{21}$ then the network is reciprocal. **(3+3=6 marks)**

**b. For the two port network as shown in Fig.3 find the Y and Z-parameters. (10)**

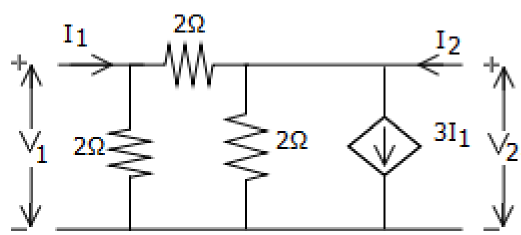
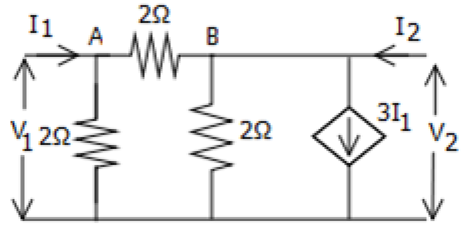



**Fig.3**

**Answer :**




Node equations at node A and B are respectively

$$-I_1+\frac{V_1}{2}+\frac{V_1-V_2}{2}=0 \qquad -(1)$$

$$\frac{V_2-V_1}{2}+\frac{V_2}{2}+3I_1-I_2=0 \qquad -(2)$$

**(2 marks)**

Rearranging the terms in the above equations, we get

$$I_1=V_1-0.5V_2 \qquad -(3)$$

$$I_2=-0.5V_1+V_2+3I_1 \qquad -(4)$$

**(1 mark)**

Substituting (3) in (4), we get $I_2=2.5V_1-0.5V_2$ (5) **(1 mark)**

Comparing equations (3) and (5) with the general expression of a network using Y parameters, we get

$I_1=Y_{11}V_1+Y_{12}V_2$ *and* $I_2=Y_{21}V_1-Y_{22}V_2$ **(1 mark)**

$Y_{11}=1,\quad Y_{12}=-0.5,\quad Y_{21}=2.5,$ *and* $Y_{22}=-0.5$ **(1 mark)**

The relation between Y and Z –parameters is given by

$Z_{11}=\frac{Y_{22}}{\Delta Y},\; Z_{12}=\frac{-Y_{12}}{\Delta Y},\; Z_{21}=\frac{-Y_{21}}{\Delta Y},\; Z_{22}=\frac{Y_{11}}{\Delta Y}$ *and* $\Delta Y=Y_{11}Y_{22}-Y_{12}Y_{21}=0.75$ (2 *marks*)

$Z_{11}=-0.67\Omega,\; Z_{12}=0.67\Omega,\; Z_{21}=-3.33\Omega,\; Z_{22}=1.33\Omega$ (2 marks)

**Q.5 a. An RLC series circuit has $R=5\Omega,\ L=100mH\ and\ C=150\mu F$ and is connected to a 230V, variable frequency supply.**
**Find**
**(i) resonant frequency (ii) impedance at resonance**
**(iii) voltage drop across inductor and capacitor at resonance**
**(iv) Q-factor (v) band width. (8)**

**Answer :**

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times10^{-3}\times150\times10^{-6}}} = 41.1\,Hz \qquad (2\,marks)$$

$$impedance\ at\ resonance,\ Z = R = 5\Omega \qquad (1\,mark)$$

$$At\ resonce,\ drop\ across\ inductor = drop\ across\ capacitor\ \ \therefore |V_L| = \omega_0 L \times \frac{V}{R} = 2\pi\times41.1\times100\times10^{-3}\times230/5$$

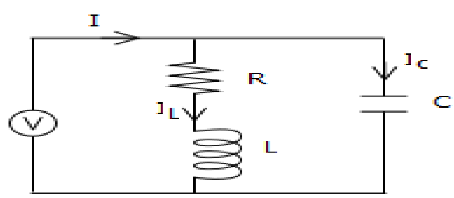
$$= 1187.7V \qquad (2\,marks)$$

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi\times41.1\times100\times10^{-3}}{5} = 5.16 \qquad (1\,mark)$$

$$Band\ width = \frac{f_0}{Q} = \frac{41.1}{5.16} = 7.96\,Hz \qquad (2\,marks)$$

**b. Derive the expressions for resonant frequency and impedance at resonance for parallel resonant circuit. (8)**

**Answer :**




**circuit--- (1 mark)**

$$Y = Y_1 + Yc = \frac{1}{R + j\omega L} + j\omega C = \frac{R}{R^2 + \omega^2 L^2} - j\left[\frac{\omega L}{R^2 + \omega^2 L^2} - \omega C\right] \qquad (2\,marks)$$

$$at\ resonance\ reactive\ comonent\ is\ zero \therefore\ \frac{\omega_0 L}{R^2 + \omega^2{}_0 L^2} - \omega_0 C = 0 \qquad (1\,mark)$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \qquad (1\,mark)$$

At resonance admittance is

$$\frac{R}{R^2 + \omega_0{}^2 L^2} = Y_0 \qquad and \quad R^2 + \omega_0{}^2 L^2 = \frac{L}{C} \qquad (2\,marks)$$

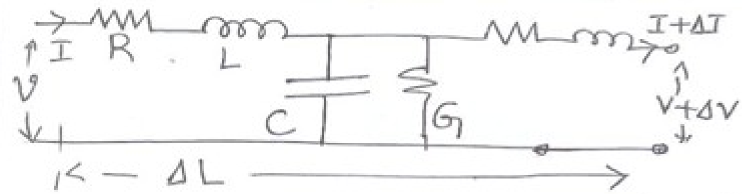
$$Z_0 = \frac{L}{CR} \qquad (1\,mark)$$

**Q.6 a. Draw the lumped element equivalent of transmission line and explain the series and shunt components of the line. Also define secondary constants of the line. (8)**

**Answer :**

6 (a)

Marks




(2)

R → Series resistance per unit length is atributed for the losses, when current flows through conductor. (1)

L → Series inductance per unit length, which is atributed for the magnetic field when current flows through conductor. (1)

C → Shunt capacitance per unit length, which is present when two conductors are seperated by dielectric. (1)

G → G is the conductance per unit length which is attributed for the losses in dielectric. (1)

R, L, G and C are called primary constants. (1)

characteristic impedance $Z_0$ and propagation constant $\gamma$ are called secondary constants. (1)

8

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad \text{and} \quad \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

b. A generator of 1V, 1000Hz, supplies power to a 100Km open wire line terminated in Zo have the following parameters R=10Ω/Km, G=0.8μ mhos/Km, L=0.004 H/Km and C=0.008 μF/Km.
Calculate

(i) Characteristic impedance (ii) Propagation constant
(iii) Attenuation constant (iv) Phase constant
(v) Phase velocity (vi) Wave length of the line (8)

**Answer :**

(b)

$\omega = 2\pi f = 2\pi \times 1000 = 6.28 \times 10^3$ rad/sec

$R + j\omega L = 10 + j25.12 = 27.04 \angle 68.29^\circ$

$G + j\omega C = (0.8 + j50.24) \times 10^{-6}$ ℧ $= 50.25 \times 10^{-6} \angle 89.09^\circ$ ℧

(i) $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \frac{27.04 \angle 68.29}{50.25 \times 10^{-6} \angle 89.09} = 733.56 \angle -10.4^\circ \ \Omega$ (2)

(ii) $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = 0.0369 \angle 78.69 = 0.00724 + j0.0362$ (2)

(iii) $\alpha = 0.00724$ NP/Km (1)

iv $\beta = 0.0362$ rad/Km (1)

(v) $v_p = \frac{\omega}{\beta} = \frac{6.28 \times 10^3}{0.0362} = 173.486 \times 10^3$ Km/s (1)

(v) $\lambda = 2\pi/\beta = 2\pi/0.0362 = 173.65$ KM. (1)

8

**Q.7 a. Define the reflection coefficient and voltage standing wave ratio. Derive the expression for input impedance in terms of reflection coefficient of the line. (8)**

**Answer :**

7(a) Reflection coefficient :- is defined as ratio of reflected signal to incident signal. → (1)

VSWR : is defined as ratio of magnitude of Maximum voltage to minimum voltage, $S = \frac{|V_{max}|}{|V_{min}|}$ → (1)

Derivation: We know $V(z) = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\gamma L} + \Gamma e^{-\gamma L}\right]$ — (1)

$$I(z) = \frac{I_R(Z_R+Z_0)}{2Z_0}\left[e^{\gamma L} - \Gamma e^{-\gamma L}\right]$$ — (1)

$$Z(z) = \frac{V(z)}{I(z)} = \frac{I_R Z_R (Z_R+Z_0)/2Z_R\left[e^{\gamma L} + \Gamma e^{-\gamma L}\right]}{I_R(Z_R+Z_0)/2Z_0\left\{e^{\gamma L} - \Gamma e^{-\gamma L}\right]}$$ — (1)

$$Z_Z = Z_0\left[\frac{e^{\gamma L} + \Gamma e^{-\gamma L}}{e^{\gamma L} - \Gamma e^{-\gamma L}}\right]$$ — (1)

$$\text{or } Z_Z = Z_0 \frac{\cancel{e^{\gamma L}}}{\cancel{e^{\gamma L}}}\left[\frac{1 + \Gamma e^{-2\gamma L}}{1 - \Gamma e^{-\gamma L}}\right]$$ — (2)

8

where $\Gamma$ is reflection coefficient.

**b. A lossless transmission line with Characteristic impedance 60Ω is 400meters long. It is terminated with load $Z_L = (40 + j80)\Omega$ and operated at frequency 1MHz. The velocity of the wave is $2.4\times10^8$ m/sec. Find: (8)**

**(i) Reflection coefficient (ii) VSWR**

**(iii) Input impedance**

**Answer :**

(b) (i) Given $Z_0 = 60\Omega$; $L = 400m$; $Z_L = 40+j80$; $f = 1MHz$.
$v_p = 2.4\times10^8$ m/sec.

(i) Reflection coefficient $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$\Gamma = \frac{40+j80-60}{40+j80+6} = 0.644\angle 65.38^\circ$$ — (2)

(ii) $S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.644}{1-0.644} = 4.618$ → (1)

(iii) $Z_Z = Z_0\left[\frac{1+\Gamma e^{-2\gamma L}}{1-\Gamma e^{2\gamma L}}\right]$ or $|\Gamma|e^{j\phi} = \Gamma$
$e^{-2\gamma L} = e^{-j2\beta L}$ $\because (\alpha = 0)$

$$Z_Z = Z_0\left[\frac{1 + |\Gamma|\angle\phi - 2\beta L}{1 - |\Gamma|\angle\phi - 2\beta L}\right]$$ — (1)

$\beta = \frac{2\pi f}{v_p} = 0.02618$ rad/m $= 26.18$ rad/KM — 1

$$Z_Z = 60\left[\frac{1+(0.644)\angle 65.38^\circ - 2(26.18\times 180^\circ/\pi \times 0.4)}{1-(0.644)\angle 65.38 - 2(26.18\times 180/\pi \times 0.4)}\right]$$

$$Z_Z = 60\left[\frac{1+0.644\angle -1134.62^\circ}{1-0.644\angle -1134.62^\circ}\right] = 60\left[\frac{1.4698\angle -20.93}{0.8178\angle 40^\circ}\right]$$ 3

$$\boxed{Z_Z = 107.84\angle -60.93^\circ\ \Omega}$$

8

**Q.8 a. Mention the properties of smith chart.** **(4)**

**Answer :**

8(a) Properties:

(i) Smith chart consists of resistance and reactance circles.

(ii) Complex impedance is shown by single point on chart which is superimposition of real and imaginary circle.

(iii) VSWR and reflection coefficients can be easily determined.

(iv) Vmax and Vmin locations can be found from load end or vice versa easily.

(v) Open circuit and short circuit positions can be easily obtained on chart.

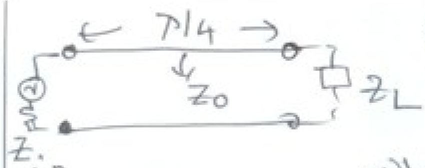
(vi) To design stub matching, smith chart can be used with less complex calculations.

4

**b. Explain the quarter wave impedance matching circuit.** **(4)**

**Answer :**

(b)




Quarter wave line is used to match source impedance to load impedance when characteristic impedance of quarter wave line $Z_0 = \sqrt{Z_{in} \cdot Z_L}$.

4 marks

**c. A load impedance of $Z_L = (60 - j\,80)\Omega$ is required to be matched to a $50\Omega$ line by using a short circuited Stub of length 'L' located at distance 'd' from the load. The wave length of operation is 1meter. Use smith chart to find L and d. (8)**

**Answer :**

8 (c) $Z_L = (60 - j80)\,\Omega$ ; $Z_0 = 50\,\Omega$.

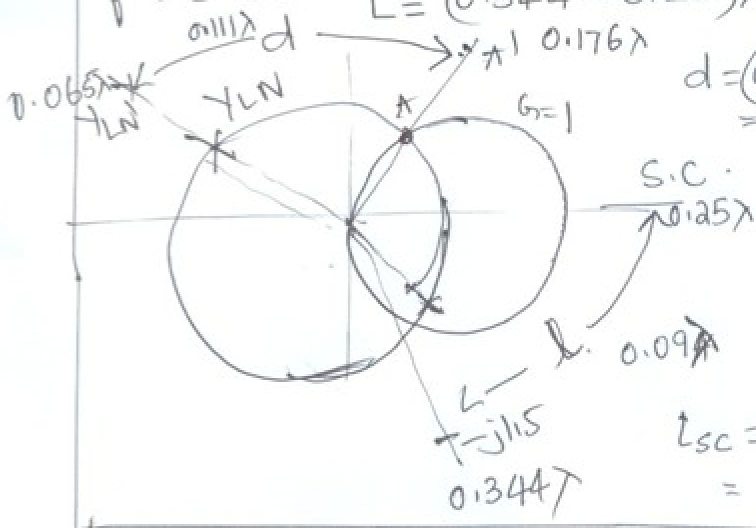
$Z_{LN} = \frac{60 - j80}{50} = 1.2 - j1.6$

* mark 1.2 − j1.6 on chart, take diametric opposite of this point as load $Y_{LN}$.
* Draw constant S circle with radius $OY_{LN}$.
* This circle intersects G=1 circle at two points say A and B.
* This distance between $OY_{LN}Y_{LN}'$ and $OAA'$ is location 'd' of the stub.

$d = (0.176 - 0.065)\lambda = 0.111\lambda = 0.111$ mts. $= 11.1$ cms.

* At A reactance is +j1.5; mark 0 − j1.5. The distance from S.C to (−j1.5) is L → length of the stub.

$L = (0.344 - 0.25)\lambda = 0.094\ m = 9.4\ cm$.



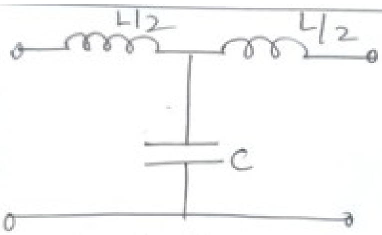

$d = (0.176 - 0.065)\lambda = 0.111\lambda$

$l_{sc} = (0.344 - 0.25)\lambda = 0.094\ m$

**Q.9 a. With neat circuit diagram and graphs explain the variation of attenuation and phase constant with respect to frequency in a constant K, T-Section low pass filter. (8)**

**Answer :**

9 (a)




$Z_1 = j\omega L$

$Z_2 = \frac{1}{j\omega C}$

$Z_1 Z_2 = \frac{L}{C} = R_0^2$

$\sinh \frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \frac{j\omega\sqrt{LC}}{2}$

$\sinh \frac{\gamma}{2} = j\, f/f_c$

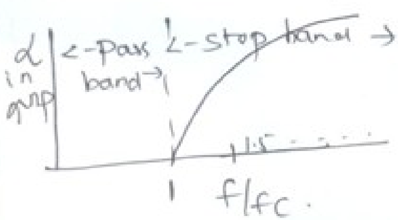
f is in the pass band, $(f/f_c) < 1$

$\alpha = 0$ ; $\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$

$\alpha = 0$; $\beta = 2\sin^{-1}(f/f_c)$

f in the stop band ; $f/f_c > 1$ :

$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$ ; $\alpha = 2\cosh^{-1}(f/f_c)$ & $\beta = \pi$ (4)





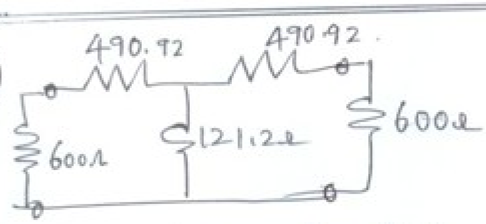
(4)

8

b. **Design a T-pad attenuator to give an attenuation of 20dB and to work in line of 600Ω impedance.** **(8)**

**Answer :**

(b)




(2)

$D = 20\,dB$ ; $R_0 = 600$ (2)

$N = \text{Antilog}\frac{D}{20} = 10$ (2)

$R_1 = 600\left(\frac{10-1}{10+1}\right) = 490.9\,\Omega$ (2)

$R_2 = 690\left(\frac{2\times 10}{100-1}\right) = 121.2\,\Omega$

8

## TEXT BOOK

1. **Transmission Lines and Networks; Umesh Sinha, 8th Edition; Reprint 2004, Satya Prakashan, Incorporating Tech India Publications, New Delhi**