Q.2 a. By Using Taylor's series, calculate the value of $f\left(\frac{11}{10}\right)$, Where $f(x) = x^3 + 8x^2 + 15x - 24$ (8)

Answer:

Here $f(x) = x^3 + 8x^2 + 15x - 24$

By Taylor's theorem

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \frac{h^4}{4!} f^{iv}(x) + \cdots \quad \text{— 2 Marks} \quad \text{— (A)}$$

Now To find $f\left(\frac{11}{10}\right)$ i.e. $f\left(1+\frac{1}{10}\right) = f(x+h)$

On comparing, we set

$x = 1,\ h = \frac{1}{10}$ in equation

Equation (A),

$$f\left(\frac{11}{10}\right) = f\left(1+\frac{1}{10}\right) = f(1) + \frac{1}{10} f'(1) + \frac{1}{10^2}\frac{f''(1)}{2!} + \frac{1}{10^3}\frac{f'''(x)}{3!} + \cdots \quad \text{— (B)} \quad \text{2 Marks}$$

Now

2 Marks

$$\left\{\begin{array}{ll} f(x) = x^3 + 8x^2 + 15x - 24 & \Rightarrow f(1) = 0 \\ f'(x) = 3x^2 + 16x + 15 & \Rightarrow f'(1) = 34 \\ f''(x) = 6x + 16 & \Rightarrow f''(1) = 22 \\ f'''(x) = 6 & \rightarrow f'''(1) = 6 \\ f^{iv}(x) = 0 & \Rightarrow f^{iv}(1) = 0 \end{array}\right.$$

Substituting the value of $f(1), f'(1), f''(1), f'''(1)$ --- etc in (B), we get

$$f\left(1+\frac{1}{10}\right) = 0 + \frac{1}{10}(34) + \frac{1}{10^2}\left(\frac{22}{2!}\right) + \frac{1}{10^3}\left(\frac{6}{3!}\right) + \cdots$$

$$f\left(\frac{11}{10}\right) = 3.4 + 0.11 + 0.001$$

$$= \underline{3.511}$$

Hence $f\left(\frac{11}{10}\right) = \underline{3.511}$ — 8 Marks

b. Evaluate $\underset{x \to 0}{lt}\left(\frac{1}{x} - \cot x\right)$ **(8)**

Answer:

Here

$$\lim_{x \to 0}\left(\frac{1}{x} - \cot x\right) = \underset{x \to 0}{lt}\left(\frac{1}{x} - \frac{\cos x}{\sin x}\right) \Rightarrow (\infty - \infty)$$

$$= \underset{x \to 0}{lt} \frac{\sin x - x\cos x}{x \sin x} \Rightarrow \left(\frac{0}{0}\right)$$ — 2 mark

~~Applying L'Hospital rule~~

Try to make std. identity

$$= \underset{x \to 0}{lt} \frac{\sin x - x\cos x}{x^2} \cdot \left(\frac{x}{\sin x}\right)$$

$$= \underset{x \to 0}{lt} \frac{\sin x - x\cos x}{x^2} \cdot \underset{x \to 0}{lt}\left(\frac{x}{\sin x}\right)$$

$$= \underset{x \to 0}{lt} \frac{\sin x - x\cos x}{x^2} = \left(\frac{0}{0}\right)$$ 2 mark

Applying L Hospital rule

$$= \underset{x \to 0}{lt} \frac{\cos x - (\cos x - x\sin x)}{2x}$$

$$= \underset{x \to 0}{lt} \frac{x\sin x}{2x} = \underset{x \to 0}{lt} \frac{\sin x}{2} = 0$$

Hence

$$\underset{x \to 0}{lt}\left(\frac{1}{x} - \cot x\right) = 0$$ — 4 Marks

Q.3 a. Evaluate by using the reduction formula $\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^4\theta\cos 2\theta\, d\theta$ (8)

Answer:

Here $\int_0^{\frac{\pi}{2}} \sin^3\theta \cos^4\theta \cos 2\theta \, d\theta$

$$= \int_0^{\frac{\pi}{2}} \sin^3\theta \cos^4\theta (\cos^2\theta - \sin^2\theta) d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^3\theta \cos^6\theta \, d\theta - \int_0^{\frac{\pi}{2}} \sin^5\theta \cos^4\theta \, d\theta$$

2 Marks

$$= \frac{(6-1)(6-3)(6-5)}{(3+6)(3+4)(3+2)} \int_0^{\pi/2} \sin^3\theta \, d\theta - \frac{(4-1)(4-3)}{(5+4)(5+2)} \int_0^{\frac{\pi}{2}} \sin^5\theta \, d\theta$$

2 Marks

$$= \frac{5}{9}\cdot\frac{3}{7}\cdot\frac{1}{5}\cdot\frac{3-1}{3}\int_0^{\pi/2}\sin\theta\, d\theta - \frac{3}{9}\cdot\frac{1}{7}\cdot\frac{5-3}{3}\cdot\frac{5-1}{5}\int_0^{\pi/2}\sin\theta\, d\theta$$

2 Marks

$$= \frac{30-24}{9\cdot 7\cdot 5\cdot 3}\int_0^{\pi/2}\sin\theta\, d\theta$$

$$= \cancel{\frac{6}{63\times 60}} \quad \frac{\cancel{6}\,2}{63\,\cancel{\times 15}\,5}\int_0^{\pi/2}\sin\theta\, d\theta$$

$$= \frac{2}{315}\,(1)$$

8 Marks

$$= \frac{2}{315} \qquad \because \int_0^{\frac{\pi}{2}} \sin x\, dx = 1$$

Hence

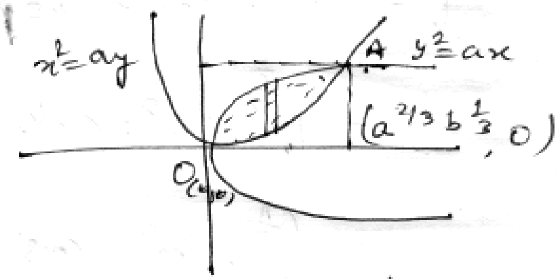
$$\int_0^{\frac{\pi}{2}} \sin^3\theta \cos^4\theta \cos 2\theta \, d\theta = \frac{2}{315} \text{ Ans.}$$

b. Find the common area lie between the parabolas $x^2 = ay$ and $y^2 = bx$ **(8)**

Answer:

Here parabolas are $x^2 = ay$ and $y^2 = bx$

i.e.



The curves are standard parabolas. their common are is the shaded portion in the figure given above i.e. OAO : Here limits are

$y = \frac{x^2}{a}$ to $\sqrt{ax}$ and $x = 0$ to $a^{2/3}b^{1/3}$

i.e. $\iint_A dx\,dy \Rightarrow \int_0^{a^{2/3}b^{1/3}} \int_{\frac{x^2}{a}}^{\sqrt{ax}} dx\,dy$ — 4 Marks

$= \int_0^{a^{2/3}b^{1/3}} \left[y\right]_{\frac{x^2}{a^2}}^{\sqrt{ax}} dx$

$= \int_0^{a^{2/3}b^{1/3}} \left[\sqrt{ax} - \frac{x^2}{a^2}\right] dx$

$= \sqrt{a}\left(\frac{x^{3/2}}{3/2} - \frac{1}{a^2}\frac{x^3}{3}\right)_0^{a^{2/3}b^{1/3}}$

$= \sqrt{a}\left[\frac{2}{3}\left(a^{2/3}b^{1/3}\right)^{3/2} - \frac{1}{3a^2}\left(a^{\frac{2}{3}}b^{\frac{1}{3}}\right)^3 - 0\right]$

$= \sqrt{a}\left[\frac{2}{3}\left(a \cdot b^{\frac{1}{2}}\right) - \frac{1}{3a^2}a^2 b\right]$

$= \frac{2}{3}\left(a^{3/2}b^{\frac{1}{2}} - \frac{a^{\frac{1}{2}}b}{2}\right)$ Ans. — 8 Marks

Q.4 a. State and prove De'Moviere's theorem. (8)

Answer:

Statement: For any rational number n the value or one of the values of

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ — 2 Marks

Proof: Case I. — 2 Marks

Let n be a non-negative integer, then by actual multiplication,

$$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2) = \cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$$

$$= (\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)$$

$$= \cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$$

similarly,

$$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)(\cos\theta_3 + i\sin\theta_3) = \cos(\theta_1+\theta_2+\theta_3) + i\sin(\theta_1+\theta_2+\theta_3)$$

computing in this way, we get

$$(\cos\theta_1 + i\sin\theta_2)(\cos\theta_2 + i\sin\theta_2)\cdots(\cos\theta_n + i\sin\theta_n) = \cos(\theta_1+\theta_2+\cdots+\theta_n) + i\sin(\theta_1+\theta_2+\cdots+\theta_n)$$

Putting $\theta_1 = \theta_2 = \theta_3 = \cdots = \theta_n = \theta$, then

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

Case II: — 2 Marks

Let n be a negative integer i.e.

$n = -m$, where m is +ive Integer

$$(\cos\theta + i\sin\theta)^n = (\cos\theta + i\sin\theta)^{-m}$$

$$= \frac{1}{(\cos\theta + i\sin\theta)^m}$$

$$= \frac{1}{(\cos m\theta + i\sin m\theta)}$$

$$= \frac{\cos m\theta - i\sin m\theta}{(\cos m\theta + i\sin m\theta)(\cos m\theta - i\sin m\theta)}$$

$$= \frac{\cos m\theta - i\sin m\theta}{\cos^2 m\theta + \sin^2 m\theta}$$

$$= \cos m\theta - i\sin m\theta$$

$$= \cos(-m\theta) + i\sin(-m\theta)$$

Put $-m = n$

$$= \cos n\theta + i\sin n\theta$$

Hence, the therom is true for negative integers also.

Case III Let n be a proper fraction $\frac{p}{q}$ where p and q are integers. Let q be a possitive and p may be negative integers.

Now $\left(\cos\frac{\theta}{q} + i\sin\frac{\theta}{q}\right)^q = \cos q\frac{\theta}{q} + i\sin q\frac{\theta}{q} = \cos\theta + i\sin\theta$

Taking the q^{th} root on both sides

$(\cos\theta + i\sin\theta)^{\frac{1}{q}} = \cos\frac{\theta}{q} + i\sin\frac{\theta}{q}$ 2 Marks

Raising p on both side, then

$(\cos\theta + i\sin\theta)^{\frac{p}{q}} = \left(\cos\frac{\theta}{q} + i\sin\frac{\theta}{q}\right)^p \Rightarrow \cos\frac{p\theta}{q} + i\sin\frac{p\theta}{q}$

Hence. $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

It is true for all rational value of n.

b. Separate the real and imaginary part of $\tan(x+iy)$ **(8)**

Answer:

let

$$A+iB = \tan(x+iy)$$

$$= \frac{\sin(x+iy)}{\cos(x+iy)} = \frac{2\sin(x+iy)\cos(x-iy)}{2\cos(x+iy)\cos(x-iy)}$$

$$= \frac{\sin 2x + \sin(2iy)}{\cos 2x + \cos(2iy)}$$

$$= \frac{\sin 2x + i\sinh 2y}{\cos 2x + \cosh 2y} \quad \left(\because \sin i\theta = i\sinh\theta,\ \cos i\theta = \cosh\theta \right)$$

Comparing real and Imaginary parts

$$A = \frac{\sin 2x}{\cos 2x + \cosh 2y};\quad B = \frac{\sinh 2y}{\cos 2x + \cosh 2y}$$

Q.5 a. Show that the vectors $\vec{A}, \vec{B}, \vec{C}$, if **(8)**

$\vec{A} = 5\vec{i} + 6\vec{j} + 7\vec{k}$, $\vec{B} = 7\vec{i} - 8\vec{j} + 9\vec{k}$, $\vec{C} = 3\vec{i} + 20\vec{j} + 5\vec{k}$ **are coplanar**

Answer:

Here $\vec{A} = 5\vec{p} + 6\vec{q} + 7\vec{k}$

$\vec{B} = 7\vec{p} - 8\vec{q} + 9\vec{k}$

and $\vec{C} = 3\vec{p} + 20\vec{q} + 5\vec{k}$

For coplanar vector $\vec{A}, \vec{B}, \vec{C}$, then

$\vec{A}\cdot(\vec{B}\times\vec{C}) = 0$ — 2 Marks

$$\therefore \vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} 5 & 6 & 7 \\ 7 & -8 & 9 \\ 3 & 20 & 5 \end{vmatrix}$$

$$5[-40-180] - 6[35-27] + 7[140+24]$$

$$= -5\times 220 - 6\times 8 + 7\times 24$$

$$= -1100 - 48 + 1148$$

$$= 0$$ 8 Marks

Hence $\vec{A}, \vec{B}$ and $\vec{C}$ are coplanar.

b. Prove that $\vec{i}\times(\vec{p}\times\vec{i})+\vec{j}\times(\vec{p}\times\vec{j})+\vec{k}\times(\vec{p}\times\vec{k})=2\vec{p}$ **where** $\vec{p}=p_1\vec{i}+p_2\vec{j}+p_3\vec{k}$ **(8)**

Answer:

Here $\vec{p} = p_1 i + p_2 j + p_3 k$, then

L.H.S $\quad i \times (\vec{p} \times \vec{i}) + j \times (\vec{p} \times j) + \vec{k} \times (\vec{p} \times \vec{k})$

$i \times \left[(\vec{p_1} i + \vec{p_2} j + \vec{p_3} k) \times i\right] + j \times \left[(\vec{p_1} i + \vec{p_2} j + \vec{p_3} k) \times j\right]$

2 mark $\quad + k \times \left[(\vec{p_1} i + \vec{p_2} j + \vec{p_3} \vec{k}) \times k\right]$

$= i \times \left[p_1 (i \times i) + p_2 (j \times i) + p_3 (k \times i)\right] + j \times$

$\left[\vec{p_1}(i \times j) + \vec{p_2}(j \times j) + \vec{p_3}(k \times j)\right] + k \times$

$\left[\vec{p_1}(i \times k) + \vec{p_2}(j \times k) + \vec{p_3}(k \times k)\right]$

$= i \times \left[0 - p_2 k + p_3 j\right] + j \times \left[p_1 k + 0 - \vec{p_3} i\right] +$

2 mark $\quad k \times \left[-p_1 j + 0 + \vec{p_2} i\right]$

$= -p_2 (i \times k) + p_3 (i \times j) + p_1 (j \times k) - p_3 (j \times i) +$

$(-p_1)(k \times j) + p_2 (k \times i)$

$= \underline{p_2 j} + p_3 k + \underline{p_1 i} + p_3 k + \underline{p_1 i} + \underline{p_2 j}$

2 mark $\quad = 2 p_1 i + 2 p_2 j + 2 p_3 k = 2(p_1 i + p_2 j + p_3 k)$

$= 2\vec{p}$ Ans.

Q.6 a. Solve $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9=\frac{e^{-3x}}{x^3}$ **(8)**

Answer:

Here $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=\frac{e^{-3x}}{x^3}$

Let $\frac{d}{dx}=D$, then

$$(D^2+6D+9)y=\frac{e^{-3x}}{x^3}$$

A.E $m^2+6m+9=0$

or $(m+3)^2=0 \Rightarrow m=-3,-3$ 2 Marks

$$C.F=(c_1+xc_2)e^{-3x}$$ 3 Marks

P.I $\frac{1}{D^2+6D+9}\left(\frac{e^{-3x}}{x^3}\right)=e^{-3x}\frac{1}{(D-3)^2+6(D-3)+9}\cdot\frac{1}{x^3}$

2 Marks

$$=e^{-3x}\left[\frac{1/x^3}{D^2-6D+9+6D-18+9}\right]$$

$$=e^{-3x}\frac{1}{D^2}\left(\frac{1}{x^3}\right)=\frac{e^{-3x}}{D^2}\left(x^{-3}\right)$$

2 Marks

$$=e^{-3x}\frac{1}{D}\left(\frac{x^{-2}}{-2}\right)$$

$$=e^{-3x}\left[\frac{x^{-1}}{(-2)(-1)}\right]=\frac{e^{-3x}}{2x}$$

Hence the solution is

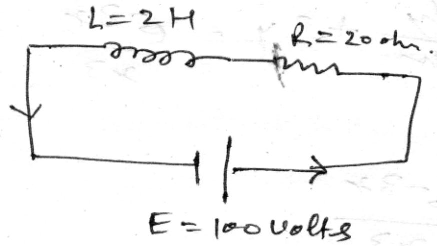
$$y=C.F+P.I$$

$$(c_1+xc_2)e^{-3x}+\frac{e^{-3x}}{2x}$$ 8 Marks

b. An inductance of 2 henries and a resistance of 20 ohms are connected in series with e.m.f E Volts. If the current is zero when t = 0, find the current at the end of 0.01 sec, if E = 100 volts. (8)

Answer:

Here the circuit contains with $L = 2H$, $R = 20$ ohms, $E = 100$ volt,



The differential equation of the circuit is

$$L\frac{di}{dt} + Ri = E$$

2 Marks

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \cong \frac{di}{dt} + Pi = Q$$

$$P = \frac{R}{L}, \quad Q = \frac{E}{L}$$

$$\therefore I.F. = e^{\int \frac{R}{L} dt} = e^{\frac{Rt}{L}}$$ — 2 Marks

Its solution is

$$i e^{\frac{Rt}{L}} = \frac{E}{L}\int e^{\frac{Rt}{L}} dt$$

$$\Rightarrow i e^{\frac{Rt}{L}} = \frac{E}{L}\cdot\frac{L}{R} e^{\frac{Rt}{L}} + C = \frac{E}{R} e^{\frac{R}{L}t} + C$$

when $t = 0$, $i = 0$ then

$$0 = \frac{E}{R} + C \Rightarrow C = -\frac{E}{R}, \text{ then}$$

$$i e^{\frac{R}{L}t} = \frac{E}{R} e^{\frac{R}{L}t} - \frac{E}{R} \Rightarrow i = \frac{E}{R} - \frac{E}{R} e^{-\frac{R}{L}t}$$

or $$i = \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$ — 6 Marks

On putting the values of E, R and L in above equation

$$i = \frac{100}{20}\left[1 - e^{-\frac{20}{2}t}\right] = 5(1 - e^{-10t})$$

$$= 5[1 - e^{-10\times 0.01}] = 5[1 - e^{-0.1}]$$

$$= 5\left[1 - \frac{1}{e^{0.1}}\right] \quad \text{at } t = 0.01 \text{ sec}$$

$$\therefore i = 0.475 \text{ (Approximate)}$$

8 Marks

Q.7 Examine the following series: (16)

(i) $\sum \sqrt{(n^4+1)} - \sqrt{(n^4-1)}$

(ii) $\sum \frac{(n+1)^n}{n^{n+1}} x^2$

Answer: (i)

Let $u_n = \sqrt{n^4+1} - \sqrt{n^4-1}$

$$= \frac{\{\sqrt{n^4+1} - \sqrt{n^4-1}\} \times \sqrt{n^4+1} + \sqrt{n^4-1}}{\sqrt{n^4+1} + \sqrt{n^4-1}}$$

$$= \frac{(n^4+1) - (n^4-1)}{\sqrt{n^4+1} + \sqrt{n^4-1}}$$

2 Marks

$$= \frac{2}{\sqrt{n^4+1} + \sqrt{n^4-1}}$$

$$= \frac{2}{n^2\left[\sqrt{1+\frac{1}{n^4}} + \sqrt{1-\frac{1}{n^4}}\right]}$$

Let $v_n = \frac{1}{n^2}$ 2 Mark

Applying comparision Test i.e.

$$\frac{u_n}{v_n} = \frac{2}{\sqrt{1+\frac{1}{n^4}} + \sqrt{1-\frac{1}{n^4}}}$$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{2}{\sqrt{1+\frac{1}{n^4}} + \sqrt{1-\frac{1}{n^4}}}$$

$$= \frac{2}{2} = 1,$$ which is finite and non-zero therefore by comparision Test both series are convergent or divergent togeter

2 Mark

Since $U_n = \frac{1}{n^2} = \frac{1}{n^p}$

Here $p = 2 > 1$, therefore

U_n is convergent.

Hence ΣU_n is also convergent.

(ii)

Here $U_n = \frac{(n+1)^n}{n^{n+1}} \cdot x^n$

Since power is involved in U_n, the cauchy roots test is applicable

$\therefore (U_n)^{\frac{1}{n}} = \frac{n+1}{n} \cdot \frac{1}{(n)^{\frac{1}{n}}} \cdot x$ — 2 mark

$\therefore \lim_{n\to\infty} (U_n)^{\frac{1}{n}} = \left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right) \cdot \lim_{n\to\infty} \frac{1}{n^{\frac{1}{n}}}\right] x$

$= (1+0)x = x \quad \left(\because \lim_{n\to\infty} \frac{1}{n^{\frac{1}{n}}} = \frac{1}{1} = 1\right)$ — 2 mark

By Cauchy root test, we conclude that

(i) If $x < 1$, the series is convergent

(ii) If $x > 1$, the series is divergent

(iii) If $x = 1$, test fails to decide the nature of series

2 mark

When $x = 1$, then

$U_n = \frac{(n+1)^n}{n^{n+1}} = \frac{1}{n}\left(1+\frac{1}{n}\right)^n$

Let $V_n = \frac{1}{n}$

Applying comparison Test

$$\frac{u_n}{v_n} = \frac{1}{n}\left(1+\frac{1}{n}\right)^n \times \frac{n}{1} = \left(1+\frac{1}{n}\right)^n$$

$$\therefore \lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

which is finite and non zero, therefore by comparison Test, Σu_n and Σv_n converge or diverge together

since $v_n = \frac{1}{n} = \frac{1}{n^p}$, Here $p=1$

$\therefore$ Σv_n is divergent, therefore Σu_n is also divergent.

Hence Σu_n is convergent if $x<1$ and Σu_n is divergent if $x \geq 1$.

Q.8 Find the Laplace Transform of f(t) , where (16)

(i) $f(t) = \begin{cases} \frac{t}{a}, & \text{where} \quad 0 < t < a \\ 1, & \text{where} \quad a < t > \infty \end{cases}$

(ii) $f(t) = \frac{e^{-t}\sin t}{t}$

Answer: (i)

Here $f(t) = \begin{cases} \frac{t}{a}, & 0 < t < a \\ 1, & a < t < \infty \end{cases}$

By the definition of Laplace transform

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt \text{ — 2 Mark}$$

$$= \int_0^a e^{-st} \frac{t}{a}\,dt + \int_a^\infty e^{-st} \cdot 1 \cdot dt$$

2 Mark

$$= \left(\frac{t}{a} \frac{e^{-st}}{-s}\right)_0^a - \int_0^a \frac{1}{a} \frac{e^{-st}}{-s}\,dt + \left(\frac{e^{-st}}{-s}\right)_a^\infty$$

$$= -\frac{e^{-as}}{s} - 0 + \frac{1}{sa}\left(\frac{e^{-st}}{-s}\right)_0^a - \frac{1}{s}(0 - e^{-as})$$

$$= -\frac{e^{-as}}{s} + \frac{1}{sa}\left(\frac{e^{-as}}{-s} + \frac{1}{s}\right) + \frac{e^{-as}}{s}$$

$$= \frac{1}{as^2}(1 - e^{-as}) \text{ — 2 Mark}$$

(ii)

Here $f(t) = \dfrac{e^{-t}\sin t}{t}$

we know that $L\{\sin t\} = \dfrac{1}{s^2+1} = \bar{f}(s)$ (say) — 2 mark

and $L\{e^{-t}\sin t\} = \dfrac{1}{(s+1)^2+1} = \bar{f}(s+1) = \bar{F}(s)$

By the property of division by t i.e. — 2 mark

$$L\left\{\frac{e^{-t}\sin t}{t}\right\} = \int_s^\infty \bar{f}(s)\,ds \text{ — 2 mark}$$

$$= \int_s^\infty \frac{ds}{(s+1)^2+1}$$

$$= \left[\tan^{-1}(s+1)\right]_s^\infty$$

$$= \tan^{-1}\infty - \tan^{-1}(s+1)$$

$$= \frac{\pi}{2} - \tan^{-1}(s+1)$$

$$= \cot^{-1}(s+1) \qquad \left\{ \because \frac{\pi}{2} - \tan^{-1}x = \cot^{-1}x \right.$$

Hence

$$L\left\{\frac{e^{-t}\sin t}{t}\right\} = \underline{\cot^{-1}(s+1)} \text{ — 8 Mark}$$

Q.9 a. Find the Inverse Laplace Transform of $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$. **(8)**

Answer:

$$\frac{2s^2-6s+5}{s^3-6s^2+11s-6} = \frac{2s^2-6s+5}{(s-1)(s-2)(s-3)} = \frac{A}{(s-1)} + \frac{B}{(s-2)} + \frac{C}{(s-3)}$$

$$\Rightarrow 2s^2-6s+5 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$$

Putting $s = 1, 2, 3$ successively, then we get

2 Marks

$$A = \frac{1}{2}, \quad B = -1, \quad \text{and } C = \frac{5}{2}$$

$$\therefore L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\} = \frac{1}{2}L^{-1}\left\{\frac{1}{s-1}\right\} - L^{-1}\left\{\frac{1}{s-2}\right\} + \frac{5}{2}L^{-1}\left\{\frac{1}{s-3}\right\}$$

2 Marks for each

$$= \frac{1}{2}e^{t} - e^{2t} + \frac{5}{2}e^{3t}$$

$$= \frac{(e^{t}+5e^{3t})}{2} - e^{2t}$$

b. Apply convolution theorem, find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$ **(8)**

Answer:

Here

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$$

Let $\bar{f}(s) = \frac{s}{s^2+a^2}$ and $\bar{g}(s) = \frac{s}{s^2+a^2}$

and $\bar{f}(t) = \cos at \quad g(t) = \cos at$

we know that the convolution theorem is given by

$$L^{-1}\{\bar{f}(s) * \bar{g}(s)\} = \int_0^t \cos au \cos a(t-u)\,du$$

2 Marks

$$\therefore L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\} = \frac{1}{2}\int_0^t 2\cos au \cos(at-au)\,du$$

$$= \frac{1}{2}\int_0^t \cos at + \cos a(t-2u)\,du$$

$$= \frac{1}{2}\left[u \cdot \cos at + \frac{\sin a(t-2u)}{-2a}\right]_0^t$$

$$= \frac{1}{2}\left[t\cos at - \frac{1}{2a}\sin a(t-2t) - 0 + \frac{1}{2a}\sin at\right]$$

$$= \frac{1}{2}\left[t\cos at + \frac{\sin at}{2a} + \frac{\sin at}{2a}\right]$$

$$= \frac{t\cos at}{2} + \frac{\sin at}{2a}.$$

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\} = \frac{t\cos at}{2} + \frac{\sin at}{2a}.$$

8 Marks

TEXT BOOK

I. Engineering Mathematics – Babu Ram, Pearson Education Limited, 2012