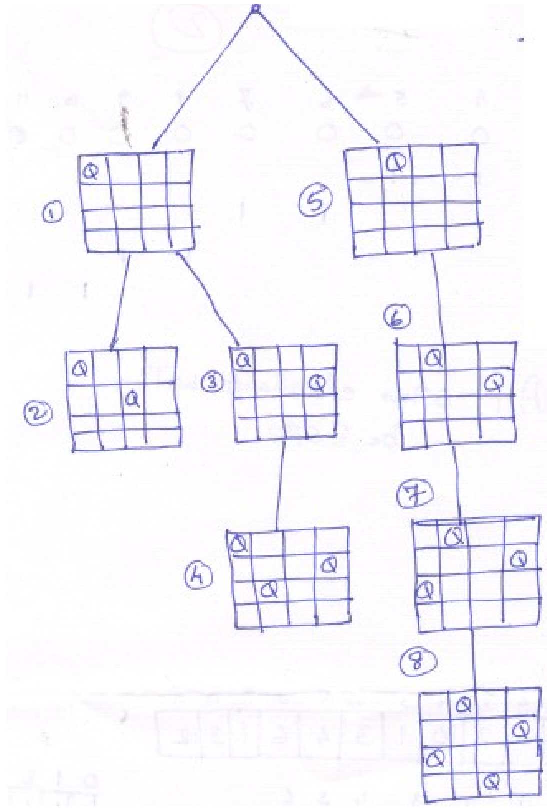
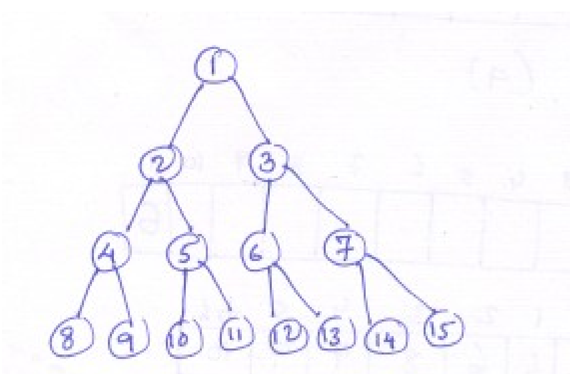
Q.1 a. Find a solution to the 4-Queens problem using back-tracking techniques.

Answer:



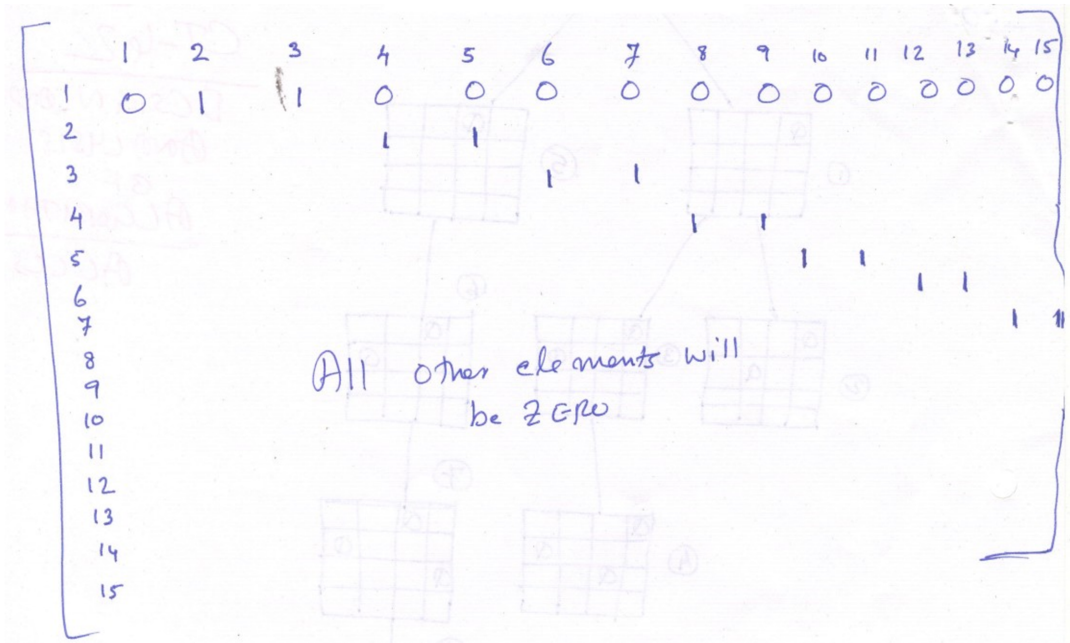
b. Give an Adjacency List representation for a complete binary tree on 15 vertices. Give an equivalent Adjacency Matrix representation. Assume that vertices are numbered from 1 to 15 as in a binary heap.

Answer:



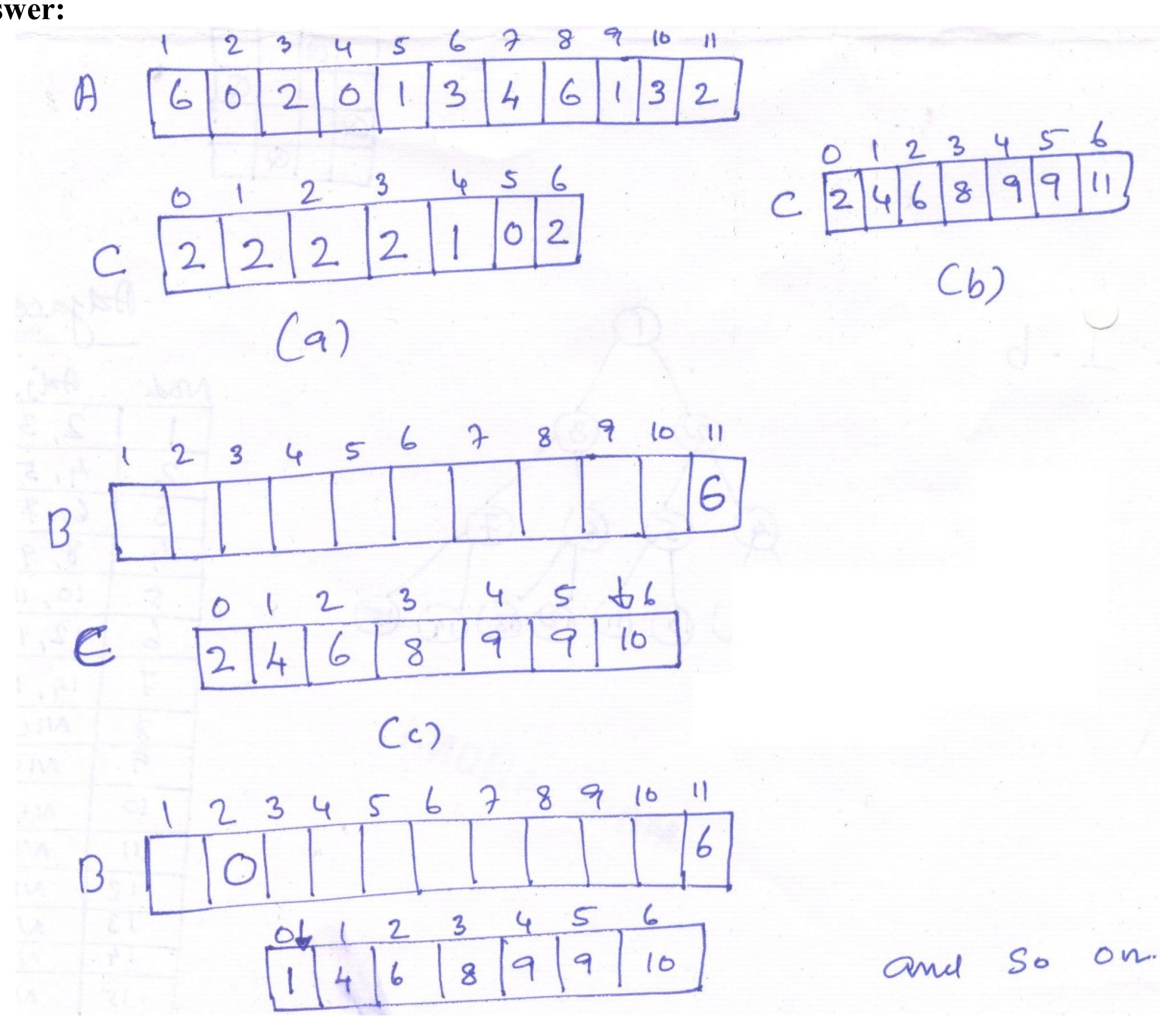
Adjacency List

Node	Adj. Nodes
1	2, 3
2	4, 5
3	6, 7
4	8, 9
5	10, 11
6	12, 13
7	14, 15
8	NIL
9	NIL
10	NIL
11	NIL
12	NIL
13	NIL
14	NIL
15	NIL



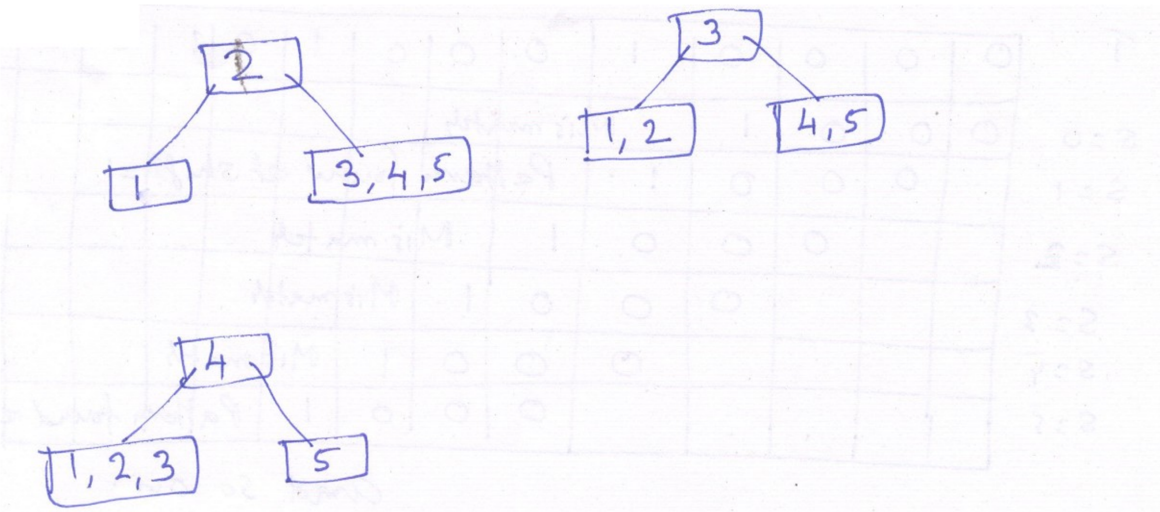
c. Illustrate the operation of Counting Sort on the array A=(6,0,2,0,13,4,6,1,3,2)

Answer:



d. Show all legal B-trees of minimum degree 2 that represent (1,2,3,4,5)

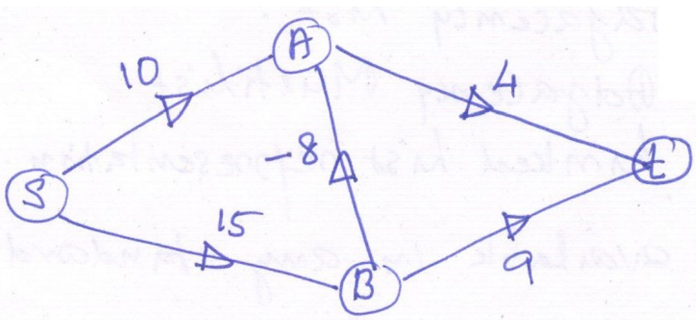
Answer:



Note:- Value 1 and 5 can never be at the root node as they can not be the middle value of any sequence taken from the given set of values.

e. Give a simple example of a directed graph with negative weight edges for which Dijkstra's algorithm produces in correct answers.

Answer:



In the above diagram, Dijkstra's Algorithm will produce 14 as answer for the shortest path from S to t where as it should be 11

f. State Max Flow – Min Cut Theorem.

Answer: Refer page 723 of Text Book-I

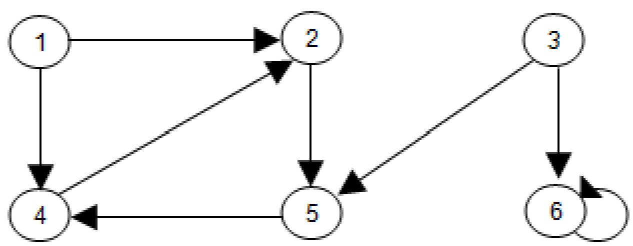
g. Show the comparisons the naive string matcher makes for the pattern P=0001 in the text T = 000010001010001 **(7×4)**

Answer:

T	0	0	0	0	1	0	0	0	1	0	1	0	0	0	1
s=0	0	0	0	1	Mismatch										
s=1		0	0	0	1	Pattern found at shift=1									
s=2			0	0	0	1	Mismatch								
s=3				0	0	0	1	Mismatch							
s=4					0	0	0	1	Mismatch						
s=5						0	0	0	1	Pattern found at s=5					

and so on.

Q.2 a. What are the different ways of representing a graph in the memory of a computer? Represent the following graph using any three ways. **(9)**



Answer:

The most popular ways of representing a graph in the memory of a computer are:

(1) Adjacency Matrix

(2) Incidence Matrix

(3) Adjacency List.

(4) Adjacency Multilist

(5) Linked list representation.

Definitions are available in the Text book

Adjacency Matrix

	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

Incidence matrix

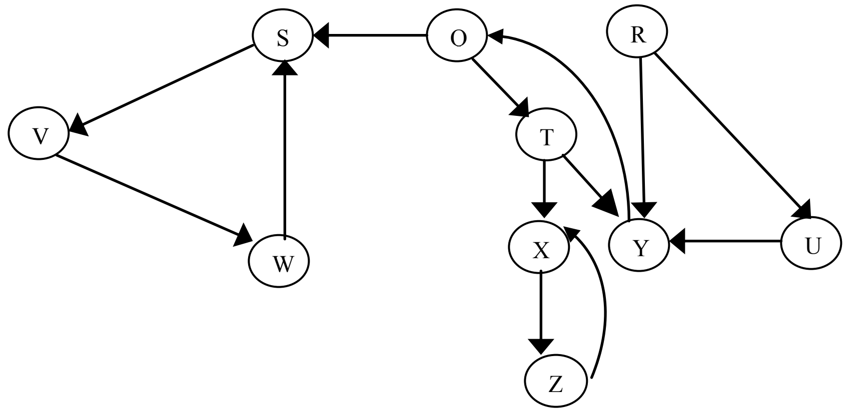
	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
1	−1	−1	0	0	0	0	0	0
2	0	1	−1		0	1	0	0
3	0	0	0	−1	−1	0	0	0
4	1	0	0	0	0	−1	1	0
5	0	0	1	1	0	0	−1	0
6	0	0	0	0	1	0	0	1

(Note :- Self loops and Parallel edges are to be managed differently. Here e_8 is originating from node 6, but not falling anywhere. Therefore it will be treated as a self loop)

Adjacency list

node	Adj. nodes
1	2, 4
2	5
3	5, 6
4	2
5	4
6	6

b. Write an algorithm for Depth First Search on a directed graph. Show how your algorithm will work on the following graph. Your algorithm should consider the vertices in alphabetical order and assume that each adjacency list is ordered alphabetically. **(9)**



Answer:

Depth First Search algorithm available in any standard book
(Page: 604 of Text Book1)

Ready Queue:-

Q R S T U V W X Y Z

WAITING STACK

① ⑥ ② ④ ③ ⑤ ⑦ ⑧ ⑨ ⑩
Q, S, T, X, Y, Z, V, W, R, U

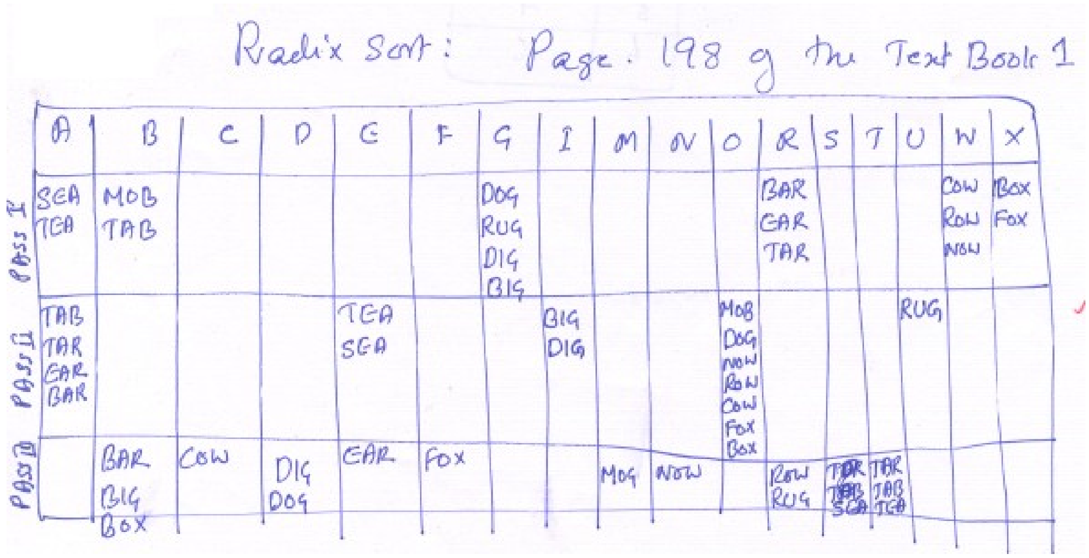
PROCESSED NODES

Q, T, Y, X, Z, S, V, W, R, U

Q.3 a. Write down the algorithm for Radix Sort. Illustrate your algorithm on the following list of English words: **(9)**

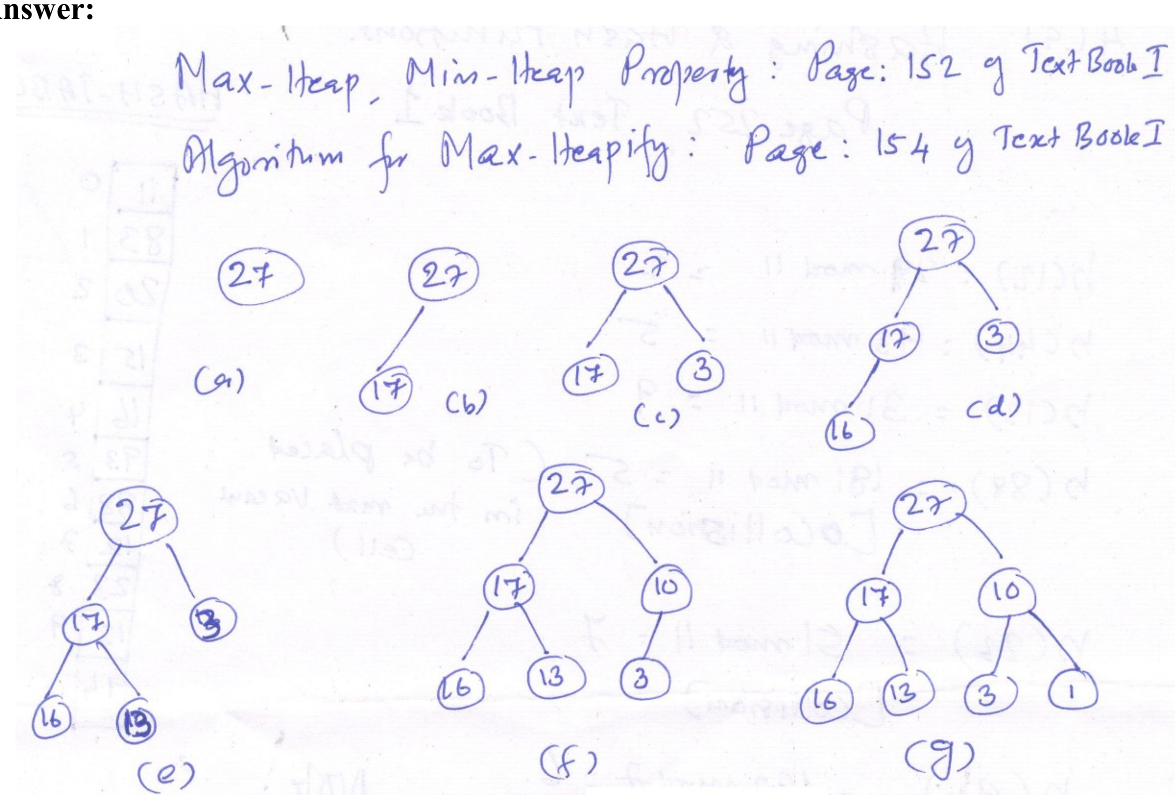
COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX

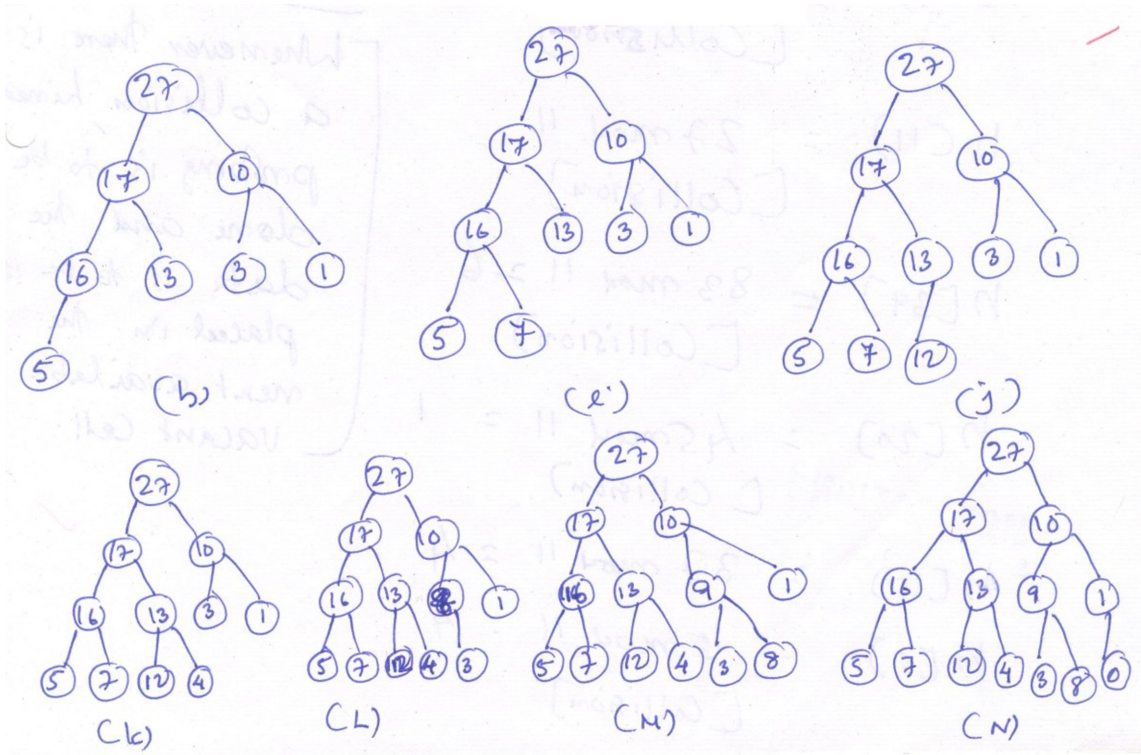
Answer:



b. Define Max-Heap and Min-Heap property. Write an algorithm for creating a max-Heap when the input is in the form of an array of integer numbers. Illustrate the operation of your algorithm on the array A= (27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0) **(9)**

Answer:





Q.4 a. What do you mean by hashing? Explain any five popular hash functions. Draw the 11-item hash table resulting from hashing the keys 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, 5 using the has function h(i) = (2i+5) mod 11 **(9)**

Answer:

Hashing & Hash Functions:

Page 252, Text Book 1

HASH-TABLE

0	11
1	83
2	20
3	15
4	16
5	93
6	88
7	12
8	23
9	13
10	94

h(12) : 29 mod 11 = 7

h(44) : 93 mod 11 = 5

h(13) = 31 mod 11 = 9

h(88) = 181 mod 11 = 5 [Collision] (To be placed in the next vacant cell)

h(23) = 51 mod 11 = 7 [Collision]

h(94) = 193 mod 7 = 6
[Collision]

h(11) = 27 mod 11
[Collision]

h[39] = 83 mod 11 = 6
[Collision]

h[20] = 45 mod 11 = 1
[Collision]

h[16] = 37 mod 11 = 4

h[5] = 15 mod 11 = 4
[Collision]

Note:
Whenever there is a collision linear probing is to be done and the data is to be placed in the next available vacant cell.

b. Taking a suitable example explain how disjoint sets are represented using linked lists. Show the data structure that results and the answers returned by the Find_Set operations in the following programme. Use the linked-lit representation with the weighted union heuristics. Assume that if the sets containing X_i and X_j have the same size, then Union (X_i, X_j) appends X_j's list on to X_i's list. **(9)**

```
for i = 1 to 16
        Make –Set (Xi)
for i = 1 to 15 by 2
        Union (Xi, Xi+1)
for i = 1 to 13 by 4
        Union (Xi, Xi+2)
Union (X1, X5)
Union (X11, X13)
Union (X1, X10)
Find-Set (X2)
Find_Set (X9)
```

Answer:

Linked list representation of Disjoint sets in Page : 565 of Text Book – 1

for i = 1 to 16
Make-set (X_i)

Answ: $\{x_1\}$ $\{x_2\}$ $\{x_3\}$ .... $\{x_{16}\}$

for i = 1 to 15 by 2
Union (X_i, X_{i+1})

Answ: $\{x_1, x_2\}$ $\{x_3, x_4\}$ $\{x_5, x_6\}$ .... $\{x_{15}, x_{16}\}$

for i = 1 to 13 by 4
Union (X_i, X_{i+2})

Answ: $\{x_1, x_2, x_3, x_4\}$ $\{x_5, x_6, x_7, x_8\}$ .... $\{x_{13}, x_{14}, x_{15}, x_{16}\}$

Union (X_1, X_5)

Answ: $\{x_1, x_2, x_3 \cdots x_8\}$

Union (X_{11}, X_{13})

Answ: $\{x_9, x_{10}, x_{11} \cdots x_{16}\}$

Union (X_1, X_{10})

Answ : $\{x_1\ x_2\ x_3 \cdots\cdots x_{16}\}$

Find Set (X_2) and Find Set (X_9) will give the same answer, which is the representative of the set.

Q.5 a. What do you understand by "Closest Pair of Points Problem"? How can you compute closet pair of points in (i) O (n^2) time (ii) O (n log n) time. **(6)**

Answer:

Given n points in space, find a pair of points with the smallest distance between them. This problem can be solved in two ways:

(1) Brute-Force Algorithm

One can compute the distance between all the $n(n-1)/2$ pairs of points. Then pick up the pair with the smallest distance. This will take $O(n^2)$ time.

(2) Recursive Divide & Conquer Approach

(i) Sort the points according to their x-coordinate

(ii) Split the set of points in to two equal sized subsets by a vertical line $x = x_{mid}$

(iii) Solve the problem recursively in the left and right subsets. This yields the left side and right side minimum distances d_{Lmin} and d_{Rmin} respectively.

(iv) Find the minimal distance d_{LRmin} among

The set of pairs of points in which one point lies on the left of the dividing vertical and the second point on lies to the right.

(v) The final answer is the minimum among d_{Lmin}, d_{Rmin} and d_{CRmin}.

b. Suppose that a graph G has a minimum spanning tree already computed. How quickly can we update the minimum spanning tree if we add a new vertex and incident edges of G. **(6)**

Answer:

Add the new edges one by one to the minimum spanning tree already computed. Whenever a cycle is generated break the cycle by removing the largest edge from the cycle.

c. Give an algorithm to compute the second best minimum spanning tree of a graph G.**(6)**

Answer:

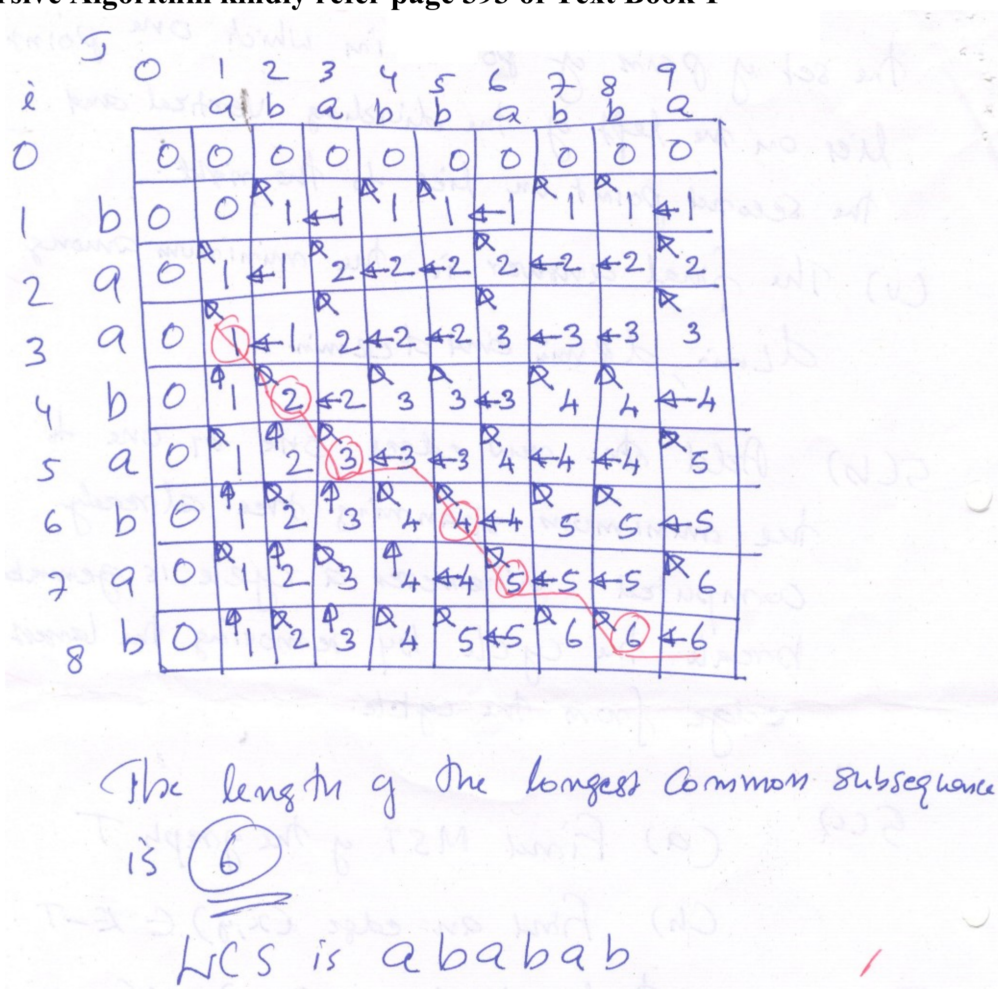
(a) Find MST of the graph T

(b) Find an edge $(x,y) \in E-T$ that minimizes $w(x,y) - w(max[x,y])$

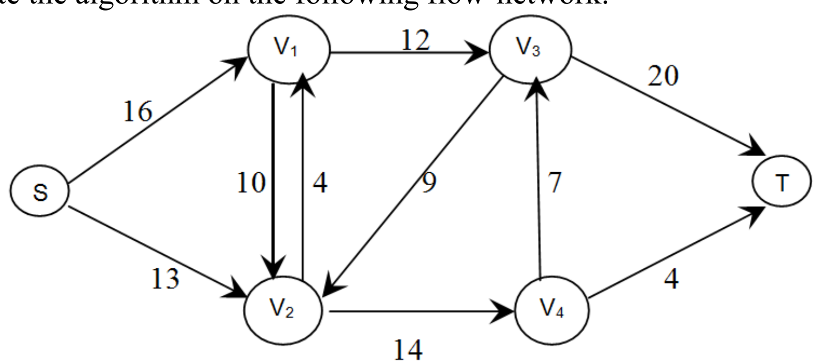
(c) Output $T' = T - max[x,y] + (x,y)$

Q.6 a. Write down the recursive formula that generates the optimal sub-structure of Longest Common Subsequence (LCS) problem. Determine an LCS of (b, a, a, b, a, b, a, b) and (a, b, a, b, b, a, b, b, a) **(9)**

Answer: For Recursive Algorithm kindly refer page 393 of Text Book-I



b. Write down Ford-Fulkerson method to find the maximum flow in a flow-network. Execute the algorithm on the following flow-network. **(9)**



Answer: Refer pages 724 & 726 of Text Book-I

Q.7 a. Write down Knuith-Morris-Pratt(KMP) algorithm for string matching. Compute the prefix function for the pattern: ababbabbabbababbabb **(12)**

Answer: **For KMP Algorithm kindly refer page 1005 of Text Book-I**

Prefix Function

	a	b	a	b	b	a	b	b	a	b	b	a	b	a	b	b	a	b	b
k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Π(k)	0	0	1	2	0	1	2	0	1	2	0	1	2	3	4	5	6	7	8

b. Prove that (i) if any NP-complete problem is polynomial-time solvable, then P=NP (ii) if any problem in NP is not polynomial time solvable, then no NP-complete problem is polynomial time solvable. **(6)**

Answer: **Refer page 1069 of Text Book-I**

<u>TEXT BOOKS</u>

I. Introduction to algorithms – Cormen and others, Prentice Hall of India, second edition, 2002

II. Algorithms – Johnsonbaugh and Schaefer , Pearson Education prentice Hall