**Q.1 a. Verify the proposition $P \vee (P \wedge Q)$ is tautology. (4)**

**Answer:** all end values should be True

**b. "A Boolean algebra is a complemented, distributive lattice" Justify the statement. (4)**

**Answer:**

Complemented Lattice in which every element has a complement, complement are unique. Every complemented distributive lattice has a unique orthocomplementation & is a fact a Boolean algebra.

**c. Explain and compare Isomorphic graph and homorphic graph in detail with suitable examples (4)**

**Answer:**

Isomorphic graph ⟹ 2 Graph are said to be Isomorphic if there exist a 1-1 correspondence f from $V$ to $V'$ such that if $v_1, v_2 \in V$ and $v_1', v_2' \in V'$ and $f(v_1) = v_1'$, $f(v_2) = v_2'$ then the no: of edges between $v_1, v_2$ is the same as the no: of edges between $v_1'$ and $v_2'$. ie $G \cong G'$

Homeomorphic Graph ⟹ 2 graph is said to be Homeomorphic if they can be obtained from the same graph $G$ by adding vertices.

**d. Explain Binary tree and draw two different binary trees with 5 nodes having maximum number of leaves. (4)**

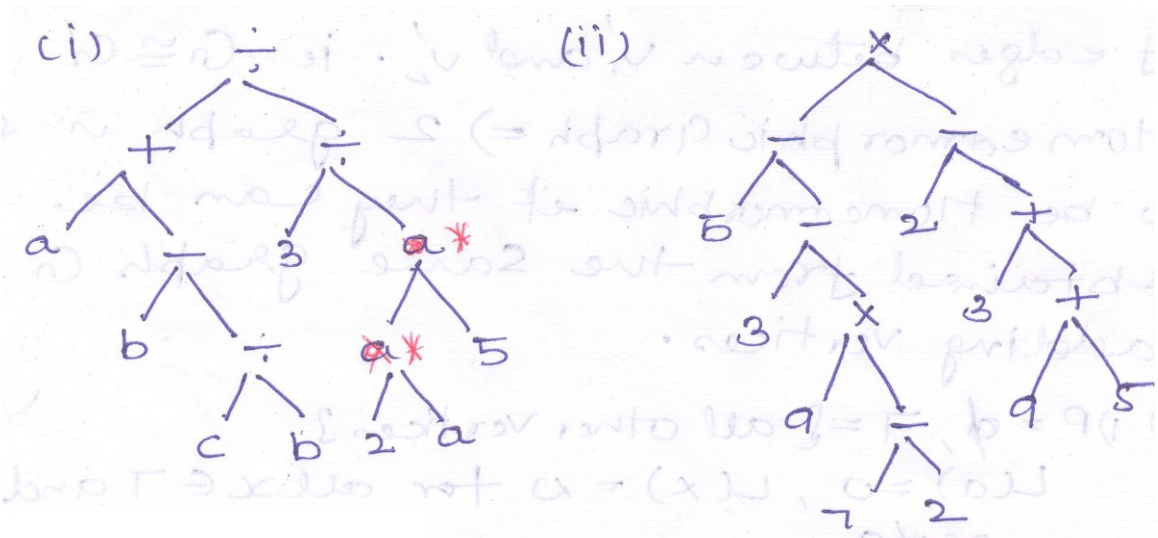
**Answer:**

Binary tree

- no: of vertices is always odd
- Pendant vertices $= (n+1)/2$
- no: of Internal vertices $\frac{n+1}{2} - 1$

**e. Construct the tree of the following algebraic expression. (4)**
**(i) (A + (B – (C + B))) ÷ (3 ÷ (2 * A) * 5)**
**(ii) (5 – (3 – (9 * (7 – 2)))) * (2 – (3 + (9 + 5)))**

**Answer:**




**f. Construct an algorithm to determine if two given trees are identical. (4)**

**Answer:**

2-trees are identical when they have same data & arrangement of Data is also same. To identify if 2 trees are same, we need to traverse Both trees simultaneously and while traversing we need to compare Data & children of trees.

**g. Prove that, any connected graph with minimum numbers of edges will form a tree. (4)**

**Answer:**

Yes, The min no: of edges for undirected connected graph is (n-1) edges. To see this, since the graph is connected then there must be unique path from every vertex to every other vertex & removing any edge will make the graph disconnected.

**Q.2 a. Discuss the Demorgan's law. How Tautologies is different from contradictions. Explain with the help of suitable example. (6)**

**Answer:**

Demorgan's Law:⇒

$\overline{A \cup B} = \overline{A} \cap \overline{B}$ and $\overline{A \cap B} = \overline{A} \cup \overline{B}$ } Prove It

Tautologies ⇒ If Truth values is always True

Contradictions ⇒ If Truth values is always false

**b. Prove that p → (q → r) and (p ∧ ¬ r) → ¬q are logically equivalent. (6)**

**Answer:** Construct Truth Table to prove.

**c. Let (S, R) be a poset. Show that (S, $R^{-1}$) is also a poset. (S, $R^{-1}$) is called as dual poset of (S, R). (6)**

**Answer:**

1. Since aRa (Partial order Rel$^n$), $aR^{-1}a$ ($R^{-1}$ is Reflexive
2. Let a, b ∈ S where a ≠ b. If aRb, $bR^{-1}a$
   aRb ⇒ b $\not R$ a ⇒ a $\not R^{-1}$ b
   thus $bR^{-1}a$ ⇒ a $\not R^{-1}$ b ($R^{-1}$ is anti symmetric)
3. If aRb ⇒ $bR^{-1}a$ & bRc ⇒ $cR^{-1}b$; by transitivity of Partial order Rel$^n$, we have aRc. Hence $cR^{-1}a$. Thus $cR^{-1}b$ and $bR^{-1}a$ ⇒ $cR^{-1}a$ ($R^{-1}$ is Transitive)

from 1, 2, 3 $R^{-1}$ is Partial order Rel$^n$.

**Q.3 a. Find DNF of the following identity without using truth table:** **(8)**

$P \wedge (P \to Q)$

$(P \to Q) \wedge (\neg P \wedge Q)$

$(P \wedge \neg(Q \wedge R)) \vee (P \to Q)$

**Answer:**

DNF

1) $P \wedge (P \to Q)$

$\equiv P \wedge (\neg P \vee Q)$

$(P \wedge \neg P) \vee (P \wedge Q)$

2) $P \to Q \equiv \neg P \vee Q$

$\therefore (P \to Q) \wedge (\neg P \wedge Q)$

$\equiv (\neg P \vee Q) \wedge (\neg P \wedge Q)$ Implication law

$\equiv (\neg P \wedge Q) \wedge (\neg P \vee Q)$ Commutative law

$\equiv (\neg P \wedge Q \wedge \neg P) \vee (\neg P \wedge Q \wedge Q)$ distributive law

$\equiv (\neg P \wedge \neg P \wedge Q) \vee (\neg P \wedge Q \wedge Q)$ Associative law

$\equiv (\neg P \wedge Q) \vee (\neg P \wedge Q)$

(c) $(P \wedge \neg(Q \wedge R)) \vee (P \to Q)$

$\equiv (P \wedge \neg(Q \wedge R)) \vee (\neg P \vee Q)$ Implication law

$\equiv (P \wedge (\neg Q \vee \neg R)) \vee (\neg P \vee Q)$ De Morgans law

$\equiv ((P \wedge \neg Q) \vee (P \wedge \neg R)) \vee (\neg P \vee Q)$ distributive

$\equiv (P \wedge \neg Q) \vee (P \wedge \neg R) \vee (\neg P \vee Q)$ Associative

**b. Convert the following SOP expression to POS**

**ABC + AB′C′ + AB′C + ABC′ + A′B′C** **(6)**

**Answer:**

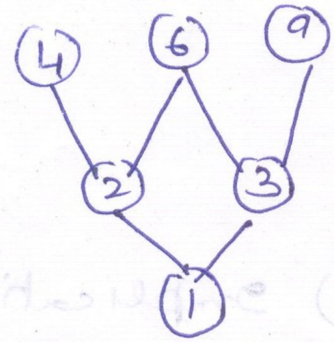
$(A+B+C)(A+\overline{B}+C)(A+\overline{B}+\overline{C})$

**c. Define Boolean algebra and prove that the power set of any given set forms a Boolean algebra.** **(4)**

**Answer:** explain Boolean algebra

**Q.4 a. Let A = {1, 2, 3, 4, 6, 9} and relation R defined on A be "a divides b". Draw Hasse diagram for this relation. (6)**

**Answer:**




**b. Explain the term equivalence class and partition and let say R be an equivalence relation on set A = {6, 7, 8, 9, 10} defined by R = {(6,6), (7,7), (8,8), (9,9), (10,10), (6,7), (7,6), (8,9), (9,8), (9,10), (10,9), (8,10), (10,8)}. Find the equivalence classes of R and hence find the partition of A corresponding to R (8)**

**Answer:**

Equivalence class and partition:-

A = {6, 7, 8, 9, 10}

[6] = 6, 7 = [7]

[8] = 8, 9, 10 = [9] = [10]

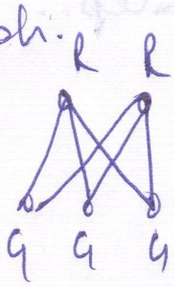
Partition = {(6, 7) (8, 9, 10)}

**c. Explain the term chromatic number and prove that chromatic number of complete bipartite graph $k_{m,n}$ where m and n are positive integers is two (4)**

**Answer:**

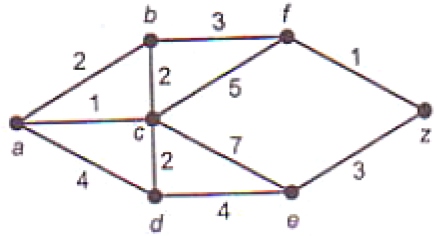
Chromatic No: = min no: of colour Required for a Graph.

$K_{m,n}$ =




only 2 colour Req

**Q.5** **a. Explain Dijkstra's algorithm and find the shortest path between the vertex *a* and vertex *z* using Dijkstra's algorithm for the following graph.** **(4)**



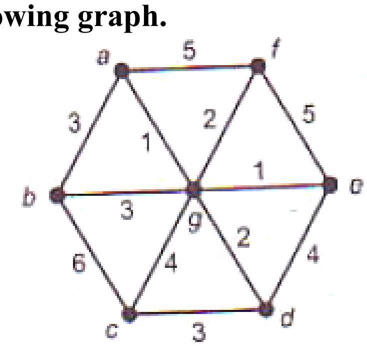

**Answer:**

1) $P = \phi$, $T = \{$all other vertices$\}$
$L(a) = 0$, $L(x) = \infty$ for all $x \in T$ and $x \neq a$

2) find the vertex $v$ which has the smallest Label. This Label will be called the permanent Label of $v$. Now set
$P = P \cup \{v\}$ and $T = T - \{v\}$
If $v = z$ then $L(z)$ is the Length of Shortest Path.

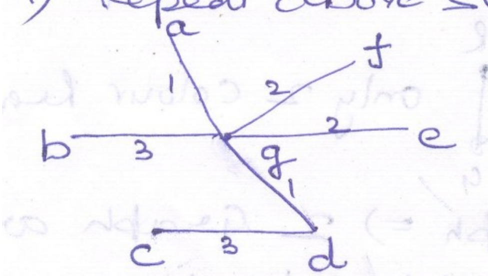
$\Rightarrow$ ans is $a, b, f, z = \underline{6}$

**b. Explain Kruskal's algorithm and find the minimum spanning tree using Kruskal's algorithm for the following graph.** **(8)**




**Answer:**

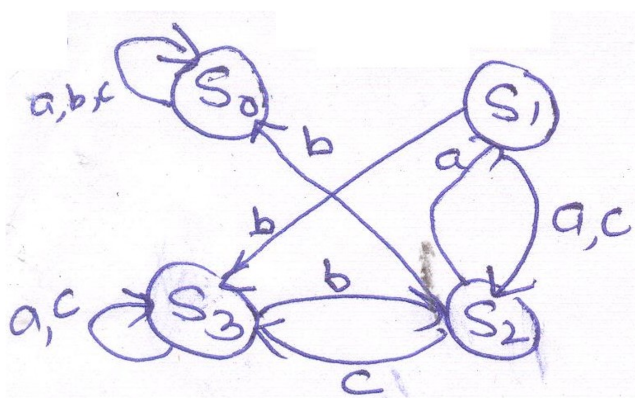
algo:-
1) get 1st edge by joining the vertices topping
2) get the 2nd edge by joining the vertices second in the list
3) Consider the 3rd (next) edge in the list. It should not form a cycle.
4) Repeat above step...




c. **Draw the state transition diagram for the finite set of states, S = {$s_0$, $s_1$, $s_2$, $s_3$), a finite set of input alphabets, I = {a, b, c} and the transition function is given in table,** **(6)**

| | **Input** | | |
|---|---|---|---|
| **State** | **a** | **b** | **c** |
| $s_0$ | $s_0$ | $s_0$ | $s_0$ |
| $s_1$ | $s_2$ | $s_3$ | $s_2$ |
| $s_2$ | $s_1$ | $s_0$ | $s_3$ |
| $s_3$ | $s_3$ | $s_2$ | $s_3$ |

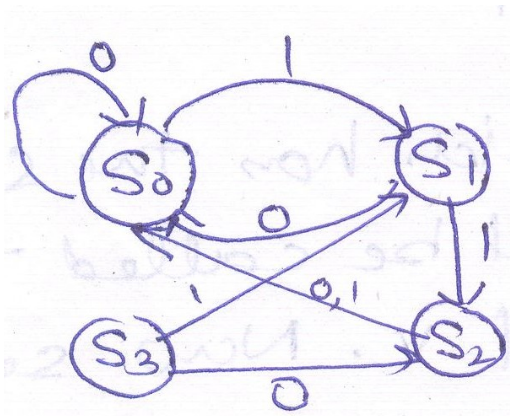
**Answer:**




**Q.6** a. **Draw the diagraph of the machine whose state transition table is shown. Remember to label the edges with the appropriate inputs. Finite set machine, M = (S, I, F, $s_0$, F) where finite set of states, S ={$s_0$, $s_1$, $s_2$, $s_3$}, finite set of input alphabets, I ={0, 1} and transition function is given in the table,** **(8)**

| State | Input 0 | Input 1 |
|---|---|---|
| $s_0$ | $s_0$ | $s_1$ |
| $s_1$ | $s_0$ | $s_2$ |
| $s_2$ | $s_0$ | $s_0$ |
| $s_3$ | $s_2$ | $s_1$ |

**Answer:**




**b. Write short notes on the following:** **(8)**

**(i) Pigeon hole principle**
**(ii) Planer graph**
**(iii) Equivalence relation**
**(iv) Complete bipartite graph**

**Answer:**

Short notes :- Pigeonhole:-
• if n items are to put in m containers. and n>m
Then at least one container must contain more then one item.
• Planer graph:- a graph that can be drawn without crossing edge.
• Equivalence Relⁿ:- ie which is Reflexsive, Symmetric, Transtive
• Complete Bipartite:- 2 set of vertex are there in a Graph. Each vertex of each set is connected to vertex of other set. But not connected with in a set.

**c. Find the number of ways in which we can put 6 letters in 10 envelops.** **(2)**

**Answer:** $10^6$

**Q.7 a. Find a nonempty set and a relation on the set that satisfy each of the following combination of properties. Simultaneously draw a diagraph of each relation.**
**(i) Reflexive and symmetric but not transitive** **(4)**
**(ii) Reflexive and but not anti-symmetric** **(4)**

**Answer:**

**b. Let *f* and *g* be the functions from the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. Determine the compositions of *f* and *g* and of *g* and *f*.** **(8)**

**Answer:**

**c. How many elements are there in the power set of set $A = \{\phi, \{\phi\}\}$?** **(2)**

**Answer:** **2**