Q.2a. Derive the frequency domain relation between input and output of an ideal continuous to discrete converter. **(8)**

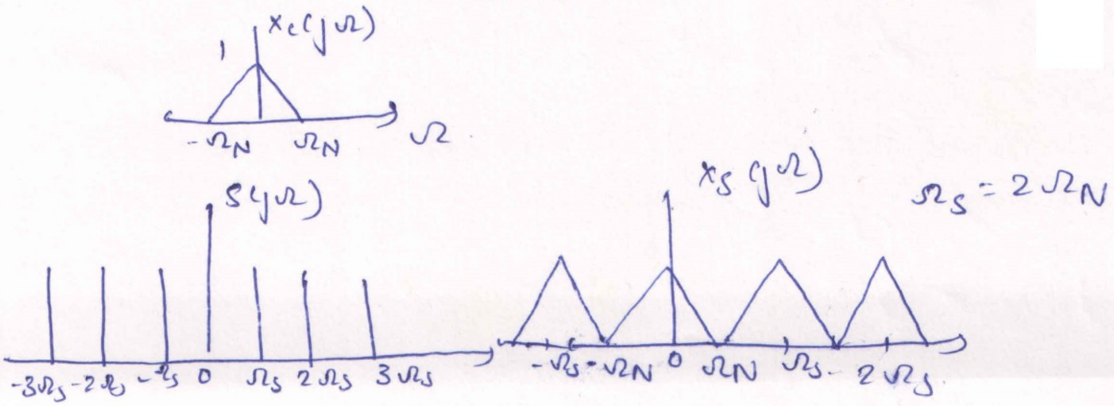
Answer:

(a). $s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$.

$x_s(t) = x_c(t)\, s(t)$

$= x(t) \sum_{n=-\infty}^{\infty} \delta(t-nT)$

$X_s(j\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s))$.



b. With the help of complete calculations show that SNR increases by approximately 6 dB with each bit added to word length. **(8)**

Answer:

Quantization error:

$$e[n] = \hat{x}[n] - x[n]$$

For $(B+1)$ bit quantizer with $\Delta = \frac{X_m}{2^B}$ $X_m \rightarrow$ full scale level of A/D converter

$$-\frac{\Delta}{2} < e[n] \leq \frac{\Delta}{2} \quad \text{if} \quad \left(-X_m - \frac{\Delta}{2}\right) < x[n] \leq \left(X_m - \Delta/2\right)$$

For small, Δ $e[n]$ is assumed to be uniformly distributed from $-\frac{\Delta}{2}$ to $\frac{\Delta}{2}$.

Assuming uncorrelated successive samples with each other and assuming $e[n]$ is uncorrelated with $x[n]$

i.e $e[n]$ is uniformly distributed white noise

Mean; $E[e[n]] = 0$

Variance; $\sigma_e^2 = \int_{-\Delta/2}^{\Delta/2} e^2 \cdot \frac{1}{\Delta}\, de = \frac{\Delta^2}{12}$

$$= \frac{X_m^2 \cdot 2^{-2B}}{12}$$

SNR of $(B+1)$ bit quantizer is

$$SNR = 10 \log \frac{\sigma_x^2}{\sigma_e^2} = 10 \log \frac{12 \cdot 2^{2B} \cdot \sigma_x^2}{X_m^2}$$

$$= 6.02B + 10.8 - 20 \log \frac{X_m}{\sigma_x^2}$$

i.e SNR increases by 6 dB with each bit added to word length

Q.3 a. Find the system function & its ROC for an LTI system with input and output related as $y[n]-\frac{5}{2}y[n-1]+y[n-2]=x[n]$ when **(8)**

(i) system is neither stable nor causal
(ii) system is stable
(iii) system is causal and unstable

Answer:

$$H(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

ROC:

1. for causal ROC $|z|>2$, unstable system.
2. $\frac{1}{2}<|z|<2$, stable
3. $|z|<\frac{1}{2}$, neither stable nor causal

b. What is a 'minimum phase system'? Discuss its three properties. **(8)**

Answer:

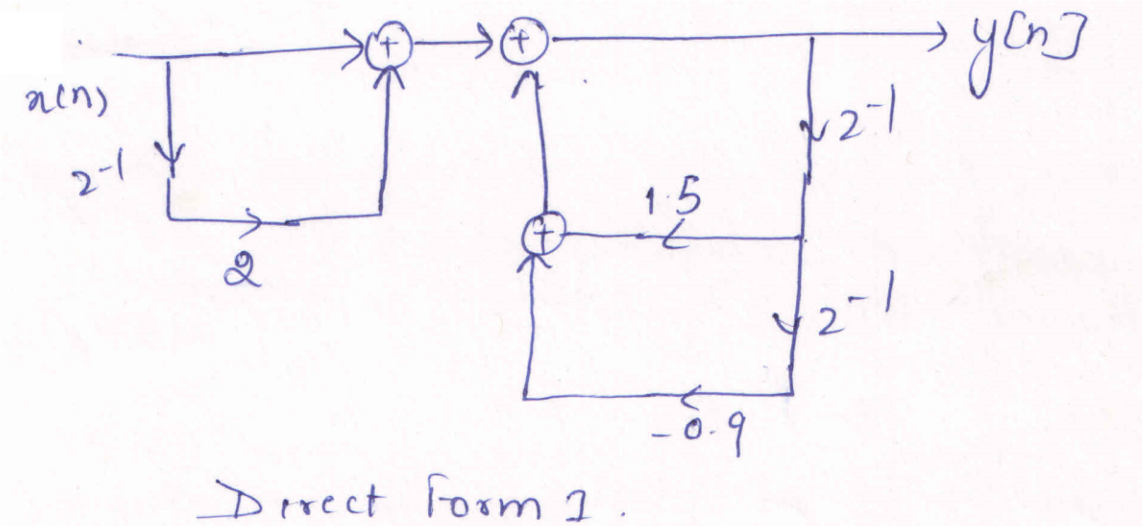
b) if poles of $1/H(z)$ are zeros of $H(z)$ then the system is min. phase system.
It refers to system that are causal and stable
Properties.
- Minimum phase lag
- Minimum group delay property
- Min Energy Delay property.

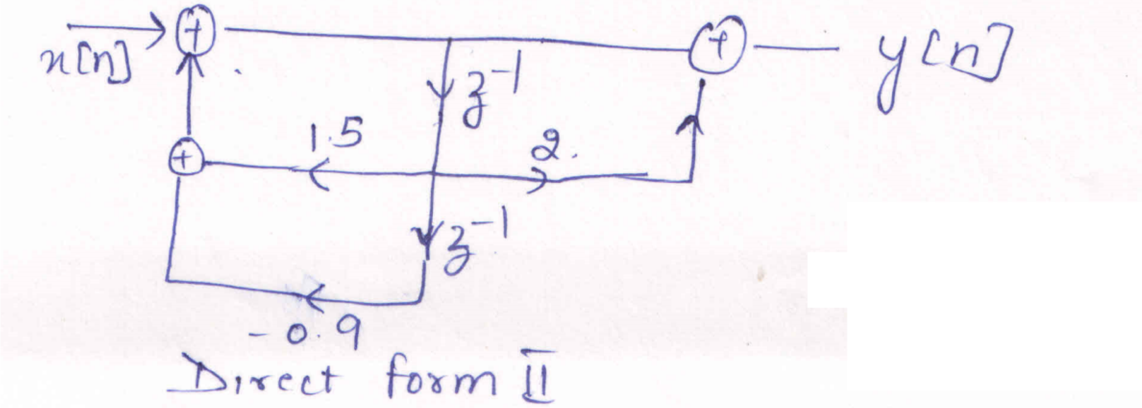
Q.4 a. Consider a LTI system

with $H(z) = \dfrac{1+2z^{-1}}{1-1.5z^{-1}+0.9z^{-2}}$

Implement it using Direct form I & Direct form II. **(8)**

Answer:

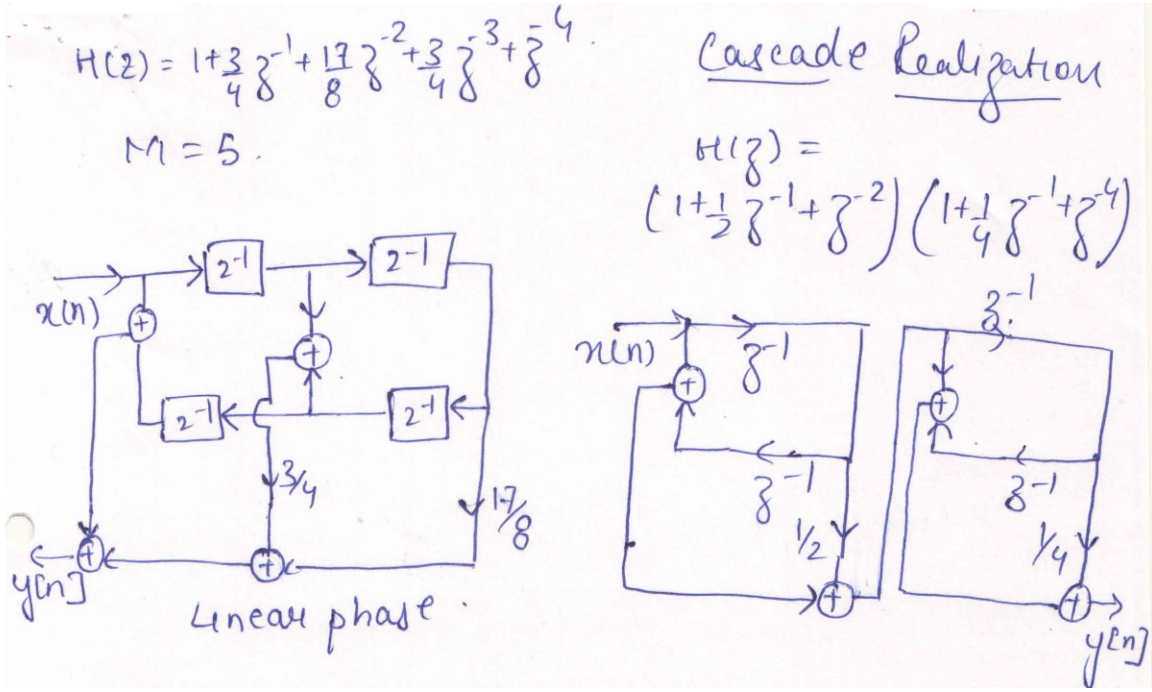




b. Determine FIR linear phase and cascade realization of the system function which is expressed as

$$H(z) = \left(1+\frac{1}{2}z^{-1}+z^{-2}\right)\left(1+\frac{1}{4}z^{-1}+z^{-2}\right) \quad \textbf{(8)}$$

Answer:



Q.5 a. Explain the mapping of s-plane to z-plane using Bilinear Transformation. Can we obtain a discrete time Low Pass filter with Linear phase characteristics by applying bilinear transformation to a continuous time low pass filter with linear phase characteristics? Justify. **(8)**

Answer:

$H_a(s) = \frac{b}{s+a}$.

$H(s) = \frac{Y(s)}{X(s)}$.

$a Y(s) + a Y(s) = b X(s)$.

$y(nT) - y(nT-T) + \frac{aT}{2} y(nT)$

$+ \frac{aT}{2} y(nT-T) = \frac{bT}{2} x(nT) + \frac{bT}{2} x(nT-T)$.

$$H(z) = \frac{b}{\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + a}$$

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

No, we cannot apply BT for obtaining a Linear phase LP discrete filter from a continuous time filter due to warping in phase response of filter.

Ideal Linear phase factor, $e^{-s\alpha}$

Substituting, $s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ (in bilinear transformation) and evaluating on unit circle, phase angle is

$-\left(\frac{2\alpha}{T}\right)\tan(\omega/2)$. Using small angle approximation

$\tan(\omega/2) \approx \frac{\omega}{2}$ ∴ phase angle is $-\frac{2\alpha}{T}\cdot\frac{\omega}{2} = -\frac{\omega\alpha}{T}$

b. Explain the process of windowing using illustrations. Obtain frequency domain characteristics of rectangular window function. **(8)**

Answer:

(b) Simplest method of FIR filter design is called window method.

Commonly used windows.

rectangular $[w_R[n]] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise.} \end{cases}$

triangular window $w[n] = \begin{cases} 2n/M, & 0 \le n \le \frac{M}{2} \\ 2 - 2n/M, & \frac{M}{2} < n \le M \\ 0, & \text{otherwise.} \end{cases}$

Hanning window

$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M) & 0 \le n \le M \\ 0, & \text{otherwise.} \end{cases}$$

Hamming

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise.} \end{cases}$$

$$W_R(\omega) = e^{-j\omega\left(\frac{M-1}{2}\right)} \frac{\sin\left(\frac{\omega M}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}.$$

Q.6 a. Find DFT of the sequence which is expressed as: **(8)**

$$x[n] = \begin{cases} \frac{1}{5}, & -1 \le n \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Answer:

$$\therefore X(K) = \frac{1}{5}\left[1 + 2\cos\left(\frac{2\pi K}{3}\right)\right].$$

$$k = 0, 1, \cdots N-1.$$

$$X(K) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}.$$

b. Sketch the linear and circular convolution of the two finite length sequences
$x_1[n] = \{1,2,3,4,5,6\}$ and $x_2[n] = \{0,0,1\}$ **(8)**

Answer:

Linear convolution

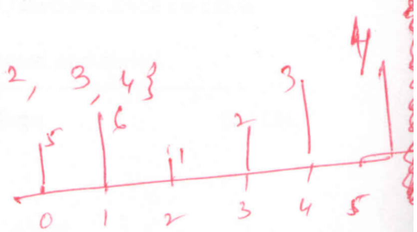
$x_1(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5, 6\}$

$x_2[n] = \{\underset{\uparrow}{0}, 0, 1\}$

$x_1[n] * x_2(n) = \{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5, 6\}$

Circular convolution

$x_1(n) \circledast x_2[n] = \{\underset{\uparrow}{5}, 6, 1, 2, 3, 4\}$



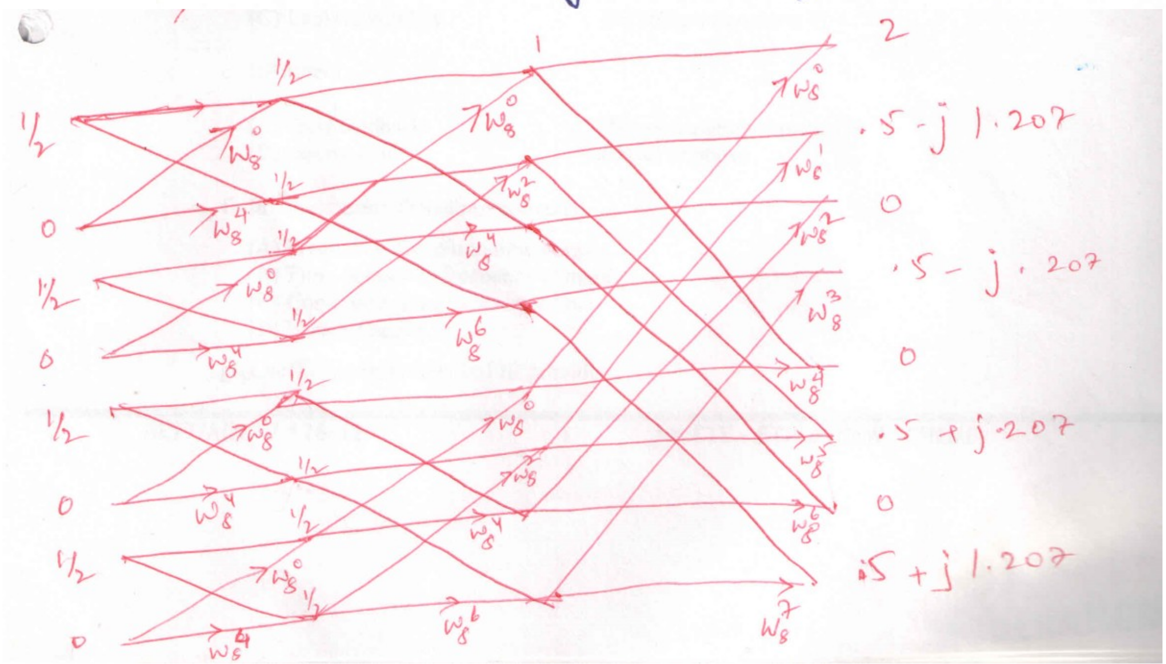
Q.7 a. Determine 8-point DFT of

$$x[n] = \left\{\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0,0,0\right\}$$

Use Radix -2 decimation in time FFT algorithm. **(8)**

Answer:

$X(K) = \{2, 0.5-j1.207, 0, 0.5-j0.207, 0, 0.5+j.207, 0, 0.5+j1.21]$.



b. Write short note on The Goertzel Algorithm & Bit reversal. **(8)**

Answer:

(b) Goertzel algorithm is an example of how the periodicity of W_N^{kn} can be used to reduce computation.

$$W_N^{-kn} = e^{j(2\pi/N)Nk} = e^{j2\pi k} = 1.$$

Bit reversal: if (n_2, n_1, n_0) is binary rep. of index of sequence $x[n]$ then $x[n_2\, n_1\, n_0]$ is stored in array position $X_0[n_0\, n_1\, n_2]$.

Q.8 a. Show that Fourier Transform of windowed signal consists of Fourier Transform of window replicated at frequencies $\pm\omega_0$ and $\pm\omega_1$ and scaled by complex amplitudes of individual complex exponentials that make up the signal. **(8)**

Answer:

$$V(e^{j\omega}) = \frac{A_0}{2} e^{j\theta_0} W(e^{j(\omega-\omega_0)}) + \frac{A_0}{2} e^{-j\theta_0} W(e^{j(\omega+\omega_0)}) + \frac{A_1}{2} e^{j\theta_1} W(e^{j(\omega-\omega_1)}) + \frac{A_1}{2} e^{-j\theta_1} W(e^{j(\omega+\omega_1)}).$$

b. Prove that power density spectrum is Fourier Transform of auto correlation function. **(8)**

Answer:

$$P_{xx}(f) = \int_{-T_0}^{T_0} R_{xx}(\tau) e^{-j2\pi f\tau} d\tau$$

$$\stackrel{F}{=} \{R_{xx}(\tau)\}$$

$$R_{xx}(\tau) = \frac{1}{2T_0} \int_{-T_0}^{T_0} x^*(t) \cdot x(t+\tau) \cdot dt$$

$$P_{xx}(f) = \frac{1}{2T_0} \left[\left| \int_{-T_0}^{T_0} x(t) e^{-j2\pi f t} dt \right|^2 \right]$$

statistical power density is.

$$\phi_{xx}(f) = \lim_{T_0 \to \infty} E\left[\frac{1}{2T_0} \left| \int_{-T_0}^{T_0} x(t) e^{-j2\pi f t} dt \right|^2 \right]$$

Q.9 a. Explain Hilbert Transform relationships. **(8)**

Answer:

$$X_I(e^{j\omega}) = \frac{-1}{2\pi} \int_{-\pi}^{\pi} X_R(e^{j\theta}) \cot\left(\frac{\omega-\theta}{2}\right) d\theta$$

$$X_R(e^{j\omega}) = x[0] + \frac{1}{2\pi} \int_{-\pi}^{\pi} X_I(e^{j\theta}) \cot\left(\frac{\omega-\theta}{2}\right) d\theta$$

b. Show that when complex cepstrum of a sequence is causal, both poles & zeros of its z-transform lie inside the unit circle. **(8)**

Answer:

(b). A causal complex cepstrum $\hat{x}[n]$ is equivalent to minimum phase condition. $\hat{x}[n]$ is inverse fourier Transform $\hat{X}(e^{j\omega})$

$\hat{X}(e^{j\omega}) = \log|X(e^{j\omega})| + j\arg[X(e^{j\omega})]$

$\therefore$ $X(z)$ have all its poles & zeros inside unit circle.

TEXT BOOK

I. Discrete-Time Signal Processing (1999), Oppenheim, A. V., and Schafer, R. W., with J. II R. Buck, Second Edition, Pearson Education, Low Price Edition