	Solution			
Q.2 a. Give the comparison between the open loop and closed loop systems.				
OPEN LOOP		CLOSED LOOP		
Utilize a contr obtain the des	oller or control actuator to ired response.	Utilizes feedback to compare the actual output to the desired output response.		
System stability is not a major problem, therefore easier to build		The use of feedback makes the system response relatively insensitive to external disturbances and internal variations in system parameters		
Use open loop only when the inputs are known ahead of time and there is no disturbances		System stability is a major problem because the system tends to overcorrect errors that can cause oscillations or changing amplitude.		
b. With neat Answer:	block diagram explain ar	automatic control system.	04	
Reference input	Error detector Error or actuating signal	Control elements		
	Controller	Feedback path elements		
The general the feedback loop through feedback	lock diagram of an autom as shown in figure. An er back elements, which is a	atic control system is characterized by a cror detector compares a signal obtained function of output response, with the		
reference inp actuating sign to reduce the	al. The control elements original error.	in turn alter the plant in such a manner as		
reference inp actuating sign to reduce the c. C	al. The control elements original error. onsider the circuit (electr	in turn alter the plant in such a manner as <b>'ical) of Figure 1</b>		



#### Fig.1

#### Answer:

(i) This circuit has two storage elements, so these shall be two state variables. We shall identify these as the inductor current  $i_L$  and capacitor voltage  $e_{c}$ ; both these are associated with energy storage.

From the elemental laws of inductor and capacitor the signal flow graph as in Fig. The complete signal flow graph is then constructed by the KCL equation at the node and the KVL equation round the loop. These equations are



(ii) The complete signal flow graph is drawn in Fig. b(iii) From the signal flow graph the two state variable equations can be written as below

$$\frac{di_L}{dt} = \frac{1}{L}e_L = \frac{1}{L}(-e_L - R_1i_L + e) = -\frac{R_1}{L}i_L - \frac{1}{L}e_C + \frac{1}{L}e \qquad \dots (iii)$$
  
and  
$$\frac{de_C}{dt} = \frac{1}{C}i_C = \frac{1}{C}\left(i_C + \frac{e_C}{R_2}\right) = \frac{1}{C}i_L - \frac{1}{R_2C}e_C \qquad \dots (iv)$$
  
Equations (iii) and (iv) can be written in matrix form  
$$\begin{bmatrix} di_L/dt \\ di_C/dt \end{bmatrix} = \begin{bmatrix} -R_1/L & -1/L \\ 1/C & 1/R_2C \end{bmatrix} \begin{bmatrix} i_L \\ e_C \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} e$$
  
These are also known as state space equations.  
(iv) Input  $E(s)$ , output  $E_c(s)$ . From the signal flow graph we have,  
Forward path  $P_1 = \frac{1}{sL} \times \frac{1}{sC} = \frac{1}{s^2CL}; \Delta_1 = 1$ 

2

2

2





Answer:		
1. There are three forward paths		
Path 1: $y_1 - y_2 - y_3 - y_4 - y_5 P_1 = a_{12} a_{23} a_{34} a_{45}$		
Path 2: $y_1 - y_2 - y_4 - y_5 P_2 = a_{12} a_{24} a_{45}$		
Path 3: $y_1 - y_2 - y_5 P_3 = a_{12} a_{25}$		
2. There are four loops		
Loop 1: $y_2 - y_3 - y_2 L_1 = a_{23} a_{32}$		
Loop 2: $y_3 - y_4 - y_3 L_2 = a_{34} a_{43}$		
Loop 3: $y_2 - y_4 - y_3 - y_2 L_3 = a_{24} a_{43} a_{32}$		
Loop 4: $y_4 - y_4 L_4 = a_{44}$	2	
3. Non-touching loops: $y_2 - y_3 - y_2$ and $y_4 - y_4$ Thus the product of the gains of the two non-touching loops: $L_1L_4 = a_{23}a_{32}a_{44}$	1	
4. $\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1L_4 = 1 - (a_{23}a_{32} + a_{34}a_{43} + a_{24}a_{43}a_{32} + a_{44}) + a_{23}a_{32}a_{44}$	1	
5 All the loops are in touch with forward path $P_1$ ,		
Thus $\Delta_1 = 1$ . All the loops are in touch with forward path $P_2$ , thus $\Delta_2 = 1$ .		
Two loops $(y_3 - y_4 - y_3 \text{ and } y_4 - y_4)$ are not touching with forward path $P_3$ .	1	
Thus, $\Delta_3 = 1 - a_{34}a_{43} - a_{44}$ . 5. Thus,		
$\begin{split} M &= \frac{y_5}{y_1} = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta} \\ &= \frac{(a_{12} a_{23} a_{34} a_{45}) + (a_{12} a_{24} a_{45}) + (a_{12} a_{25})(1 - a_{34} a_{43} - a_{44})}{1 - (a_{23} a_{32} + a_{34} a_{43} + a_{24} a_{32} a_{43} + a_{44}) + a_{23} a_{32} a_{44}} \end{split}$		
a. Consider the feedback control system shown in Figure 4. The normal value of process parameter K is 1. Let us evaluate the sensitivity of transfer function $T(s)=C(s)/R(s)$ to variations in parameter K Controller Controlled process $F(s) + 25 \frac{s+1}{s+5} + G(s) = \frac{K}{s(s+1)} + \frac{K}{s(s+1)} + G(s) = $		

Answer:

Answer:  

$$T(s) = \frac{C(s)}{R(s)} = \frac{25K}{s^2 + 5s + 25k} = \frac{1}{1 + \frac{s^2 + 5s}{25K}}$$
Therefore  $S_R^T = \frac{\partial T}{\partial K} \times \frac{K}{T} = \frac{\sigma(s+5)}{s^2 + 5s + 25K}$ 
Since The Normal Value of K is 1, we have
$$S_k^T = \frac{\partial T}{\partial K} \times \frac{K}{T} = \frac{s(s+5)}{s^2 + 5s + 25K}$$
Since The Normal Value of K is 1, we have
$$S_k^T = \frac{s(s+5)}{s^2 + 5s + 25K}$$
1
$$S_k^T may be evaluated at various values of frequency. At a particular frequency  $\omega=5$ , the magnitude of sensitivity is approximately  $|S_k^T| = 1.41$ .
$$T(s) = \frac{25K}{s^2 + (1 + 4K)s + 25K} = \frac{25K}{s(s+1) + K(4s + 25)}$$

$$S_k^T = \frac{s(s+1)}{s(s+1) + K(4s + 25)} = \frac{s(s+5)}{s^2 + 5s + 25K} \text{ for K=1}$$
The magnitude of sensitivity at  $\omega=5$  is  $|S_k^T| = 1$ .
$$\frac{R(s)}{1 + 25} + \frac{25}{s^2 + 5s + 25K} \text{ for K=1}$$
A two-loop configuration for T(s)
$$\frac{R(s)}{s(s+1) + \alpha} = \frac{\alpha}{s(s+1) + \alpha}$$
Fig.5
Answer:
$$T(s) = \frac{\alpha}{s(s+1) + \alpha}$$
1
$$1$$$$

# CONTROL ENGINEERING JUN 2015

$S^T = \frac{\partial In(\alpha)}{\partial I(\alpha)} = 1$	
To find $S_{\alpha} = \partial ln(\alpha)$	
$\frac{\partial In[s(s+1)+\alpha]}{\alpha} = \frac{\alpha}{\alpha}$	
$\partial In(\alpha)$ $s(s+1)+\alpha$	-
Then $S_{\alpha}^{T} = 1 - \frac{\alpha}{s(s+1) + \alpha} = \frac{s(s+1)}{s(s+1) + \alpha}$	2
To evaluate $S_{\alpha}^{T}$ , at s =j $\omega$ =j0.1and j1	1
s(s+1) = j0.1(j0.1+1) = -0.01 + j0.1	
$S_2^T(j0.1) = \frac{-0.01 + j0.1}{(2 - 0.01) + j0.2} = 0.05 \angle 93^0$	1
$S_2^T(j2) = \frac{-1+j1}{(2-1)+j1} = 1 \angle 90^0$	2
In terms of absolute values	
$ S_2^T(j0.1)  = 0.05,  S_2^T(j2)  = 1$	1
It is observed that sensitivity increases with frequency.	
Q.5 a. A unity feedback control system is characterized by an open-loop	
transfer function. $G(s)H(s) = \frac{K}{s(s+10)}$	
Determine the system gain K, so that the system will have a damping ratio of 0.5. For this value of K, find the rise time, peak time, settling time and peak overshoot. Assume that the system is subjected to a step of 1V.	
Answer:	
The characteristic equation for the present problem is	
1+G(s)H(s)=0.	
$1 + \frac{K}{s(s+10)} = 0$	
$s^2 + 10s + K = 0.$	1
For the reference system shown in figure, the characteristic equation is given by	
$1 + \frac{\omega_n^2}{s(s + 2\zeta\omega_n)} = 0$	
$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0.$	1
Compare equation we get	
$2\zeta\omega_n s = 10$ and $\omega_n^2 = K$	

Since $\zeta = 0.5$	
We get $2\times 0.5 \approx -10$	
we get, $2 \times 0.5 \omega_n = 10$	
$\omega_n = 10 rad / sec$	
Hence, K=10 <sup>2</sup> =100	2
From the formulae derived in the last section, we get	_
• Rise time $t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega_n \sqrt{1-\zeta^2}} = 0.242$ seconds	1
• Reak time $t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = 0.363$ seconds	1
• Settling time $t_s = \frac{4}{\zeta \omega_n} = 0.8$ seconds	1
• Peak overshoot $M_P = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.163 = 16.3\%$	1
function consisting of two poles Two zeros and variable gain K. The zeros are located at $-2$ and $-1$ : and the poles at $-0.1$ and $+1$ . Using Routh stablity criterion, determine the range of values of K for which the closed-loop system has 0, 1 and 2 poles in the right-half of s-plane.	
Answer:	
$G(s) = \frac{K(s+1)(s+2)}{(s+0.1)(s-1)}$	
The characteristic quaion of the system is given as	
1+G(s) = 0	
(s+0.1)(s-1)+K(s+1)(S+2) = 0	1
$(1+K)s^2+(3K-0.9)s+(2K-0.1)=0$	
Applying Routh criteria $S^2 (1+K) = (2K-0.1)$	
$S^2$ (3K-0.9) 0	
S <sup>0</sup> (2K-0.1) 0	
i) No poles in right half s-plane (system stable) K+1>0 or K>-1	1
3K-0.9>0 or K>0.3	
2K-0.1>0 or K>0.05	2

Hence, K>0.3	
ii) 1pole in right half s-plane (=one change of sign in first column terms) -1 <k<0.05< td=""><td>2</td></k<0.05<>	2
iii) 2 pole in right half s-plane (+two changes in sign in first column terms)	
0.05 <k<0.3.< td=""><td>2</td></k<0.3.<>	2
Q.6 a. Briefly discuss rules followed to plot the root locus.	08
Answer: RULES FOR ROOT LOCUS PLOTTING	
$1 + KG(s)H(s) = 1 + F(s) = 1 + \frac{K\prod_{i=1}^{m} (s + z_i)}{\prod_{j=1}^{n} (s + p_j)} = 0$	
; $m \le n$ ; $K \ge 0$	
<b>1.</b> The root locus plot consists of <b>n</b> root loci (branches) as $K$ varies from 0 to $\square$ . The loci are symmetric with respect to real axis.	
<b>2.</b> As <i>K</i> increases from 0 to $\mathbb{Z}$ , each root locus starts from an openloop pole with <i>K</i> = 0 and ends on an open-loop zero or on $\mathbb{Z}$ with <i>K</i> = $\mathbb{Z}$ . The number of root loci ending at $\mathbb{Z}$ equals the number of open-loop poles minus zeros.	
<b>3.</b> The $(n - m)$ root loci which tend to $\mathbb{Z}$ do so along straight line asymptotes radiating out from a single point $s = -\mathbb{Z}_a$ on the real axis (called the centroid) where	
$-\sigma_a = \frac{\sum (real \ part of \ open - loop \ poles) - \sum (real \ part \ of \ open - loop \ zero}{n-m}$	
These (n - m) asymptotes have angles	
$\phi_a = \frac{(2q+1)180^0}{n-m}  ;  q = 0, 1, \cdots, (n-m-1)$	
<b>4.</b> A point on the real axis lies on the locus if the number of open-loop poles plus zeros on the real axis to the right of this point is odd. By use of this fact, the real axis can be divided into segments on-locus and not-on-locus; the dividing points being the real open-loop poles and zeros.	

**5.** The intersections (if any) of root loci with the imaginary axis can be determined by use of Routh criterion.

**6.** The angle of departure  $\mathbb{Z}_p$  of a locus from a complex open-loop pole is given by  $\mathbb{Z}_p = 180_0 + \mathbb{Z}$ , where  $\mathbb{Z}$  is the net angle contribution at this pole of all other openhoop poles and zeros.

**7.** The angle of arrival  $\mathbb{Z}_z$  of a locus at a complex zero is given by  $\mathbb{Z}_z = 180_0 - \mathbb{Z}_z$ , where  $\mathbb{Z}$  is the net angle contribution at this zero of all other open-loop poles and zeros.

**8.** Points at which multiple roots of the characteristic equation occur (breakaway points of root loci)

are the solutions of 
$$\frac{dK}{ds} = 0$$
 where  $K = -\frac{\prod_{j=1}^{n} |(s+p_j)|}{\prod_{i=1}^{m} |(s+z_i)|}$ 

# b. A unity feedback control system has an open-loop transfer function of

$$G(s) = \frac{K(s+4/3)}{s^2(s+12)}$$

Plot the root locus. Find the value of K for which all the roots are equal. What is the value of these?

#### Answer:

Poles: s=0, 0, -12 Zeros: s= -4/3  $-\sigma_A = \frac{-12+4/3}{3-1} = -16/3$   $\phi_A = \frac{\pm 180^0(2q+1)}{3-1} = \pm 90^0$   $K = \frac{s^2 + 24s}{(s+4/3)} + \frac{s^2(s+12)}{(s+4/3)} = 0$ or  $-3s(s+8)(s+4/3) + s^2(s+12) = 0$  or  $s(s+4)^2 = 0$  s = 0, s = -4, -4Breakaway point is at s=-4 Equal roots (3Nos) are at s=-4



# b. Explain in detail Nyquist stability criteria. Answer: NYQUIST STABILITY TEST The Cauchy criterion (from complex analysis) states that when taking a closed contour in the complex plane, and mapping it through a complex function G(s), the number of times, N, that the plot of G(s) encircles the origin is equal to the number of zeros, Z, of G(s) enclosed by the frequency contour minus the number of poles, *P*, of G(s) enclosed by the frequency contour. N = Z - PEncirclements of the origin are counted as positive if they are in the same direction as the original closed contour or negative if they are in the opposite direction. When studying feedback control, we are not as interested in G(s)H(s)as in the closed-loop transfer function G(s)H(s)/[1+G(s)H(s)]If 1+G(s)H(s) encircles the origin, then G(s)H(s) will enclose the point -1. Since we are interested in the closed-loop stability, we want to know if there are any closed-loop poles (zeros of 1+G(s)H(s)) in the right-half plane. 3 The Nyquist Stability Criterion usually written as Z = P + N, where > Z is the number of right hand plane poles for the closed loop system (or zeros of 1+G(s)H(s)> *P* is the number of open-loop poles (in the RH side of the s-plane) of G(s)H(s)(or poles of 1+G(s)H(s)), and > *N* is the number of clockwise encirclements of (-1,0) "A feedback control system is stable if and only if the number of counterclockwise encirclements of the critical point (-1,0) by the *GH* polar plot is equal to the number of poles of *GH* with positive real parts." (Nyquist Stability 3 Criterion Definition) Q.8 a. A unity negative feedback system has an open-loop transfer function of $G(s) = \frac{K}{(s+4)}$ Consider a cascade compensator $G_c(s) = \frac{s+\alpha}{s}$ Select the values of K and $\alpha$ to achieve a.

(i) Peak overshoot of about 20%

(ii) Setting time (2%basis)  $\approx 1$  sec

For the values of K and  $\alpha$  found in part (a) calculate the unit ramp input steady-state error

#### Answer:

In this simple example we can proceed analytically. Compensated forward path transfer function is

$$G_c(s)G(s) = \frac{K(s+\alpha)}{s(s+4)} \qquad \dots (i)$$

Its characteristic equation is

$$1 + G_c(s)G(s) = 0$$
 or  $s(s+4) + K(s+\alpha) = 0$ 

or

$$s^2 + (4+K)s + K\alpha = 0$$

Required :

 $M_p = e^{-\pi\xi/\sqrt{(4-\xi^2)}} = 0.2$ 

Which yields

Settling time

# $t_s = \frac{4}{\xi \omega_n} = 1$

 $\xi = 0.456$ 

Which gives

From the characteristic equation

 $K\alpha = \omega_n^2 = (8.77)^2 = 76.9$ 

 $\omega_n = \frac{4}{0.456} = 8.77 rad / sec$ 

$$K + 4 = 2\xi\omega_n = 8$$

Transfer function of pre-filter

$$G_p(s) = \frac{1}{s + 19.23}$$

1

1

1

2

2

1

1

1

06

b)  $R(s) = \frac{1}{s^2}$ , unit ramp

$$e_{ss} = \lim_{s \to 0} \frac{1}{sG_c(s)G(s)} = \lim_{s \to 0} \frac{(s+4)}{K(s+\alpha)} = \frac{4}{K\alpha}$$

$$e_{ss} = \frac{4}{4 \times 19.23} = 0.05$$

### b. Give the comparisons between phase lag and phase lead compensator.

Answer:

Phase lead compensator	Phase Lag Compensator
Bandwidth increases High frequency gain increases	Bandwidth decreases
Dynamic response becomes faster	Dynamic response slows down
Susceptable to high frequency noise	High frequency noise suppressed
<ul> <li>No significant decreases in steady- state error</li> </ul>	Steady-state error reduced
• Application: When fast dynamics response is required	Application; When low steady-starequired.
<ul> <li>Cannot be applied when phase angle of uncompensated system is decreasing rapidly near crossover frequency</li> </ul>	Cannot be applied if uncompensa phase angle in low frequency re sufficient to provide requisite pha







1

2

1

1

1

 $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ 

The state transition matrix  $\Phi(t)$  is given by

$$e^{At} = I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \dots$$

Substituting values of A and collecting terms, we get

$$e^{At} = \begin{bmatrix} 1+t+0.5t^2+\dots & 0\\ t+t^2+\dots & 1+t+0.5t^2+\dots \end{bmatrix}$$
$$= \begin{bmatrix} \Phi_{11} & \Phi_{12}\\ \Phi_{21} & \Phi_{22} \end{bmatrix}$$

The terms  $\Phi_{11}$  and  $\Phi_{22}$  are easily recognized as series expansion of  $e^t$ . To recognize  $\Phi_{_{21}}$  , more terms of the infinite series should be evaluated. In fact  $\Phi_{_{21}}$ is te<sup>t</sup>.

The computation of  $e^{At}$  by expanding it into a power series in t and then adding the corresponding elements in the matrix terms of the infinite series, is practical only for simple cases.

The state transition matrix is

$$\Phi(t) = \begin{bmatrix} e^t & o \\ te^t & e^t \end{bmatrix}$$

The time response of the system is given by

$$x(t) = \Phi(t) \left[ x_0 = \int_0^t \Phi(-\tau) B u \ d\tau \right]$$

Now with u=1,

$$\Phi(-\tau)Bu = \begin{bmatrix} e^{-\tau} & o \\ \tau e^{-\tau} & e^{-\tau} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} e^{-\tau} \\ e^{-\tau}(1-\tau) \end{bmatrix}$$

$$\int_0^t \Phi(-\tau) Bud\tau = \begin{bmatrix} \int_0^t e^{-\tau} \\ \int_0^t e^{-\tau} (1-\tau) d\tau \end{bmatrix} = \begin{bmatrix} 1-e^{-t} \\ te^{-1} \end{bmatrix}$$



## **TEXT BOOK**

1. Control Systems Engineering, ), I.J. Nagrath and M. Gopal, Fifth Edition (2007 New Age International Pvt. Ltd