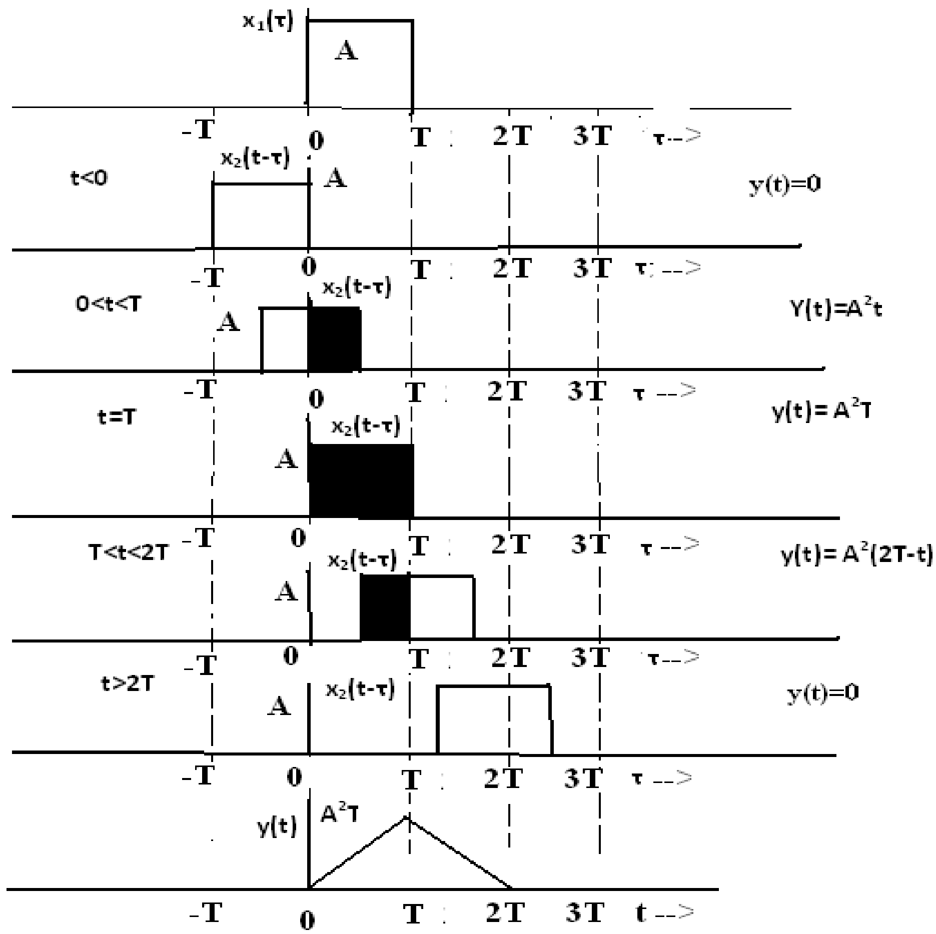
Solution	Marks
Q.2 a. Determine power and energy of the following signals (4) **(i)** $x(t) = Ae^{j\omega_0 t} \quad -\infty < t < \infty$ **(ii) x(t) =sin(ωt)** **Answer:** i) $x(t) = Ae^{jw_0 t} \quad -\infty < t < \infty$ $I = \int_{-T}^{T} \lvert x(t)\rvert^2 dt$ $\lvert x(t)\rvert = \lvert e^{jw_0 t}\rvert = 1$ $= \int_{-T}^{T} \lvert x(t)\rvert^2 dt = A^2 \int_{-T}^{T} 1dt = 2T$ $E = \underset{T\to\infty}{lt}[I] = \infty$ $P = \underset{T\to\infty}{lt}[\frac{I}{2T}] = A^2$ Power is finite , it is a power signal ii) x(t) =sinwt $I = \int_{T} \lvert x(t)\rvert^2 dt$ $= \int_{T} \sin^2 wtdt$ $= \int_{T} \frac{(1-\cos 2wt)}{2} dt = \int_{T} \frac{1}{2} dt - \frac{1}{2}\int_{T} \cos 2wtdt$ $= T/2$ $E = \underset{T\to\infty}{lt}[I] = \infty$ $P = \underset{T\to\infty}{lt}[\frac{I}{T}] = \frac{1}{2}$ Power is finite , it is a power signal **b. Find the convolution of $X_1(t) = A; \quad 0 \le t \le T$ and $X_2(t)= A; \quad 0 \le t \le T$** **Answer:**	 **2** **2**



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$$y(t) = \begin{cases} A^2 t & 0 \le t \le T \\ A^2(2T - t) & T \le t \le 2T \\ 0 & otherwise \end{cases}$$

c. For the system y(n) = ax[n] + b x(n-1) determine whether the system is

(i) Linear
(ii) Causal
(iii) Stable
(iv) Time-invariant
(v) Memory less

Answer:

For each of the following systems determine whether the system is

a) Linear b) Causal c) Stable d) Time- invariant e) Memory less

i) T[x[n]] = ax[n] + b x(n-1)

a) Linear: Let $x(n) = x_1(n)$ output $y_1(n) = ax_1(n)+bx_1(n-1)$ Let $x(n) = x_2(n)$ output $y_2(n) = ax_2(n)+bx_2(n-1)$ Let $x(n) = x_3(n) = x_1(n) + x_2(n)$ Output $y_3(n) = ax_3(n)+bx_3(n-1)$ $=a[x_1(n) + x_2(n)]+b[x_1(n-1)+x_2(n-1)]$ $y_3(n) = y_1(n) + y_2(n)$ Therefore system is **linear.** b) Causal: Output depends on present and past values of input. Therefore system is **causal.** c) Stable: Let $\lvert x[n]\rvert \le M_x$ output $\lvert y[n]\rvert \le \lvert ax[n]+bx(n-1)\rvert = \lvert a\rvert.\lvert x[n]\rvert + \lvert b\rvert\lvert x[n-1]\rvert$ As 'a' and 'b' are constants of finite value output $\lvert y[n]\rvert$ is bounded and the system is **stable.** d) Time – invariant: Given $T[x[n]] = y[n] = ax[n] + b\, x_1(n-1)$ Let $x[n] = x_1[n]$ Output $y_1[n] = ax1[n] +b\, bx_1(n-1)$ Shift output $y_1[n]$ by 'k' units $y_1[n – k] = ax_1 [n – k] + bx_1(n-k-1)$ Let $x[n] = x_2[n] = x_1[n – k]$ $y_2[n] = ax_2 [n] + b$ $y_2[n] = ax_1[n – k] + b$ $y_2[n] = y_1[n – k]$ Therefore system is **time – invariant.** e) Memory: Output depends only on present and past value of input. Therefore system is	**1x5**

memory system.

Q.3 a. Determine the Fourier's Series representation for signal
$x(t) = \cos(2\pi t) + 4\sin(6\pi t)$

Answer:

Given Signal $x(t) = \cos(2\pi t) + 4\sin(6\pi t)$

Period of cos(2πt) is $T_1 = 1$

Period of sin(6πt) is $T_2 = 1/3$

The Fundamental period x(t) is T = 1sec

Expressing x(t) as

$$x(t) = \frac{1}{2}\left[e^{j(2\pi t)} + e^{-j(2\pi t)}\right] + \frac{4}{2j}\left[e^{j6\pi t} - e - j^{6\pi t}\right]$$

$$= \frac{1}{2}e^{j2\pi t} + \frac{1}{2}e^{-j2\pi t} + \frac{2}{j}e^{j2\pi(3)t} - \frac{2}{j}e^{-j2\pi(3)t}$$

Fourier Series representation is

$$x(t) = \sum_{k=-\infty}^{\infty} X(k)e^{jk\omega_0 t}$$

Comparing with above equation

$$X(k) = \begin{cases} -\frac{2}{j} & k = -3 \\ \frac{1}{2} & k = -1 \\ \frac{1}{2} & k = 1 \\ \frac{2}{j} & k = +3 \\ 0 & otherwise \end{cases}$$

4

b. State and prove the following properties of continuous time and periodic signals
(i) Time shifting
(ii) Time Reversal

Answer:

Time shifting

When a time shift is applied to a periodic signal x(t), the period T of the signal is preserved. The Fourier series coefficients b_k of the resulting signal $y(t)=x(t-t_0)$ may be expressed as **1**

$$b_k = \frac{1}{T}\int_T x(t-t_0)e^{-jk\omega_0 t}dt$$

Letting $\tau = t - t_0$ in the integral, and noting that the new variable τ will also range over an interval of duration T, we obtain

$$\frac{1}{T}\int_T x(\tau)e^{-jk\omega_0(\tau+t_0)}d\tau = e^{-jk\omega_0 t_0(\tau+)}\frac{1}{T}\int_T x(\tau)e^{-jk\omega_0\tau}d\tau$$

3

$$= e^{-jk\omega_0 t_0}a_k = e^{-jk(2\pi/T)t_0}a_k$$

Where a_k is the k^{th} Fourier series coefficient of x(t). that is , if

$$x(t)\overset{FS}{\leftrightarrow}a_k,$$

Then

$$x(t-t_0)\overset{FS}{\leftrightarrow}e^{-jk\omega_0 t_0}a_k = e^{-jk(2\pi/T)t_0}a_k.$$

One consequence of this property is that, when a periodic signal is shifted in time, the magnitudes of its Fourier series coefficients remain unaltered. That is $|b_k|=|a_k|$.

Time Reversal

The period T of a periodic signal x(t) also remains unchanged when the signal undergoes time reversal. To determine the Fourier series coefficients of y(t)=x(-t). **1**

$$x(-t) = \sum_{k=-\infty}^{\infty} a_k e^{-jk2\pi t/T}$$

Let k=-m

$$y(t) = x(-t) = \sum_{m=-\infty}^{\infty} a_{-m}e^{-jm2\pi t/T}$$

$b_k = a_{-k}$. $\quad x(t)\overset{FS}{\leftrightarrow}a_k,$ **3**

Then

$x(-t) \overset{FS}{\leftrightarrow} a_{-k}$.

c. Find the Fourier Series representation of the signal x(t) shown in fig.1(4)

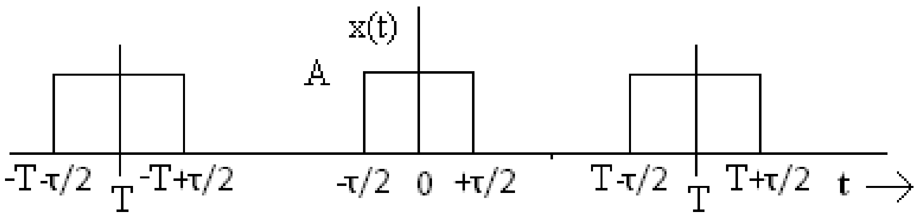
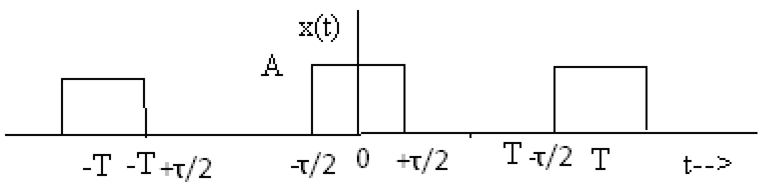


Fig.1

Answer:



$$x(t) = \begin{cases} A & -\tau/2 \le t \le \tau/2 \\ 0 & -\tau/2 \le t \le T-\tau/2 \end{cases}$$

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jkw_0 t}$$

$$X[K] = \frac{1}{T}\int_T x(t)e^{jkw_0 t}dt$$

$$X[K] = \frac{1}{T}\int_{-\tau/2}^{\tau/2} Ae^{jkw_0 t}dt$$

$$X[K] = \frac{2A}{Tkw_0}\sin[kw_0\tau/2]$$

or

$$X[K] = \frac{A\tau}{T}\sin c[k\tau/T]$$

4

Q.4 a. State and prove the following properties of continuous signal Fourier Transform:
(i) Time shifting property
(ii) Frequency differentiation property 4

Answer:

If $x(t) \xleftrightarrow{FT} X(j\omega)$

then

$x(t-t_0) \xleftrightarrow{FT} X(j\omega)e^{-j\omega t_0}$	
Shift in time domain will result in multiplying by an exponential in frequency domain	1
Proof. $F\{x(t-t_0)\} = \int_{-\infty}^{\infty} x(t-t_0)e^{-j\omega t}dt$	
Let $t - t_0 = \tau$	
$t = \tau + t_0$ and $dt = d\tau$	
$= \int_{-\infty}^{\infty} x(\tau)e^{-j\omega(\tau+t_0)}d\tau$	
$= \int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}d\tau \;\; e^{-j\omega t_0}$	4
$= X(j\omega)e^{-j\omega t_0}$	
ii). Frequency differentiation Property:	
Statement:	
If $x(t) \xleftrightarrow{FT} X(j\omega)$	1
Then	
$-jtx(t) \xleftrightarrow{FT} \frac{d}{dw}X(jw)$	
Differentiating signal in frequency domain is same as multiplying by t in time domain	
Proof.	
$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$	
Differentiating with respect to w	
$\frac{dX(jw)}{dw} = \int_{-\infty}^{\infty} x(t)e^{-j(w)t}(-jt)\,dt$	
$\frac{dX(jw)}{dw} = \int_{-\infty}^{\infty} (-jt)x(t)e^{-j(w)t}\,dt$	

Therefore $-jtx(t) \overset{FT}{\longleftrightarrow} \frac{d}{dw} X(jw)$

b. Find the FT of the signal x(t) as shown in fig.2 **4**

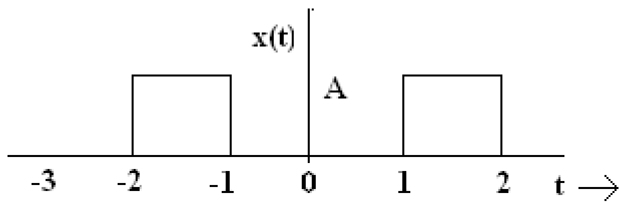
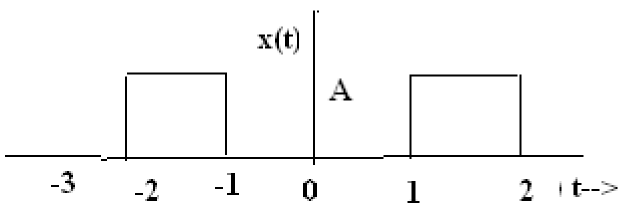


Fig.2

Answer:



Let $x_1(t)$=A rect(t)↔A sinc(f)=$X_1(jw)$ Here A=A and T=1

Given signal x(t) can be expressed as

X(t)= $x_1(t-1.5)$+ $x_1(t+1.5)$

Therefore the Fourier transform is

FT{x(t)}= $X_1(jw)$ [$e^{-j(1.5t)}$ +$e^{j(1.5)t}$] **2**

X(jw)= A sinc(f) [$e^{-j(1.5t)}$ +$e^{j(1.5)t}$]

X(jw)= 2A sinc(f) cos(1.5t)

Note: Alternative answers are also possible.

Q.5 a. Find a Fourier transformer $x[n] = a^{|n|}$, $|a| < 1$

Answer: **4**

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} a^{|n|} e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} a^n e^{-j\omega n} + \sum_{n=-\infty}^{-1} a^{-n} e^{j\omega n}$$

$$X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}} + \frac{ae^{j\omega}}{1-ae^{-j\omega}}$$

$$= \frac{1-a^2}{1-2a\cos\omega + a^2}$$

6

b. A casual LTI is described by the difference equation

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

(i) Find the system function $H(z) = Y(z)/X(z)$ for this system. Plot the poles and zeros of $H(z)$ and indicate region of convergence.

(ii) Find the unit sample response of system.

Answer:

$$h[n] = a^n u[n]$$

$$x[n] = \beta^n u[n]$$

$$H(e^{j\omega}) = \frac{1}{1-\alpha e^{-j\omega}}$$

and

6

$$X(e^{j\omega}) = \frac{1}{1-\beta e^{-j\omega}}$$

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) = \frac{1}{(1-\alpha e^{-j\omega})(1-\beta e^{-j\omega})}$$

$$A = \frac{\alpha}{\alpha-\beta}, \quad B = -\frac{B}{\alpha-\beta}$$

$$y[n] = \frac{1}{\alpha-\beta}\left[\alpha^{n+1}u[n] - \beta^{n+1}u[n]\right]$$

c. For the system equation y(n) - 4 y(n-1), = x(n) find

(i) The transfer function and

(ii) Impulse response

Answer:

For the system equation y(n) - 4 y(n-1), = x(n) find the transfer function and impulse response.

Solution:

Given y

Taking Fourier transform

$$Y(e^{j\Omega})[1-4e^{-j\Omega}] = X(e^{j\Omega})$$
$$H(e^{j\Omega}) = \frac{1}{[1-4e^{-j\Omega}]}$$
$$h(n) = (4)^n u(n)$$

4

Q.6 a. Find the frequency response of an LTI system having impulse response
h(t)=(1+t)e^{-2t} u(t)

Answer:

Given h(t)=(1-t)e^{-2t} u(t)

$$h(t) = e^{-2t}u(t) - te^{-2t}u(t)$$
$$H(jw) = \int_{-\infty}^{\infty} h(t)e^{-jwt}dt$$

$$e^{-at}u(t) \leftrightarrow \frac{1}{a+jw}$$
$$te^{-at}u(t) \leftrightarrow \frac{1}{(a+jw)^2}$$

3

$$F\{e^{-2t}u(t) - te^{-2t}u(t)\} = \frac{1}{2+jw} - \frac{1}{(2+jw)^2}$$
$$H(jw) = \frac{1+jw}{(2+jw)^2}$$

b. State and prove sampling theorem for Low pass signal.

3

Answer:

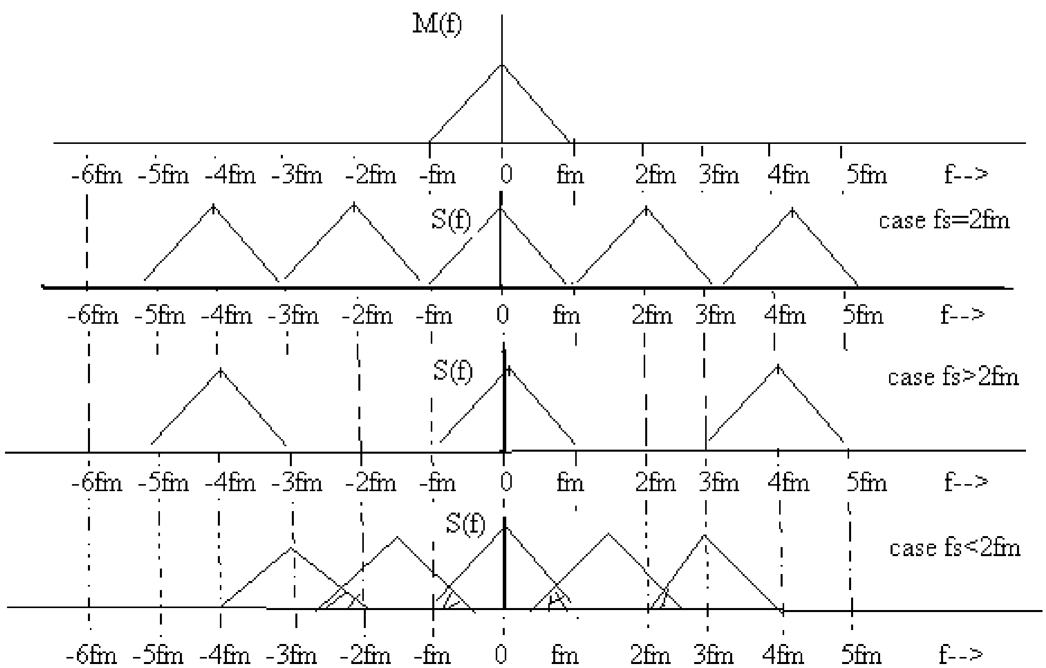
Sampling theorem.

Statement: Let m(t) is a message signal band limited to f_mHz, if this signal is sampled at a rate $f_s \geq 2f_m$ then we can reconstruct the message signals from the sampled value with minimum distortion.

i.e $f_s \geq 2f_m$

2

where fs is sampling frequency

and fm is maximum message frequency Let m(t)=message signal $m(t) \leftrightarrow M(f)$ $\delta_T(t) = \sum_n \delta(t - nT)$ is periodic delta function with Fourier series $\delta_T(f) = \frac{1}{T}\delta(f - nf_s)$ Sampled signal S(t)=m(t)∂_T(t) Multiplication in time domain is same as convolution in frequency domain $\therefore S(f) = M(f) * \delta_T(f)$ $= M(f) * \left[\frac{1}{T}\sum \delta_T(f - nf_s)\right]$ Convolving any function with delta function yield the same function $\therefore S(f) = \frac{1}{T}\sum_n M(f - nf_s)$ Spectrum of sampled signal is periodic with period fs.  	4

c. Determine the Nyquist rate for the following signals
(i) x(t) = cos(600πt)+ sin(800πt)
(ii) x(t) = cos(600πt) cos(800πt)

Answer:
Determine the Nyquist rate for the following signals

i) x(t)=cos(600πt)+ sin(800πt)

f_1=300 Hz and

f_2=400 Hz

$f_{Nyq}=2fm_{(max)}$=2x400=800Hz

ii) x(t)= cos(600πt) cos(800πt)

=0.5[cos(1400 πt)+cos(200 πtt)]

f_1=700 Hz and

f_2=100 Hz

$f_{Nyq}=2fm_{(max)}$=2x700=1400Hz

Q.7 a. Find the Laplace transform of the following signals.
(i) $X(t)=te^{-2t}u(t)$
(ii) $X(t) =e^{-3t}\cos(5t)\, u(t)$

Answer: **2**

i) $X(t)=te^{-2t}u(t)$

$$L[e^{-2t}u(t)] = \frac{1}{s+2}$$

using differentiation property

$$L[(-t)^n x(t)] = \frac{d^n X(S)}{dS^n}$$

for $n=1$ **2**

$$L[(t)x(t)] = -\frac{dX(S)}{dS}$$

$$L[(t)e^{-2t}u(t)] = -\frac{d\left[\frac{1}{s+2}\right]}{dS}$$

$$= \frac{1}{(s+2)^2}$$

ii.) X(t) = e^{-3t} cos(5t) u(t)

$$L[\cos(5t)u(t)] = \frac{s}{s^2+5^2}$$

using frequency shifting property

$$L[e^{-3t}\cos(5t)u(t)] = \frac{(s+3)}{(s+3)^2+25}$$

4

b. Find the Inverse Laplace transform of the following X(s)

$$X(s) = \frac{s+5}{(s^2+6s+10)}$$

Answer:

i) $$X(S) = \frac{s+5}{(S^2+6S+10)}$$

$$X(s) = \frac{(s+3)+2}{(s+3)^2+1^2} = \frac{(s+3)}{(s+3)^2+1^2} + \frac{2(1)}{(s+3)^2+1^2}$$

using relation

$$e^{-bt}\cos at\, u(t) \leftrightarrow \frac{s+b}{(s+b)^2+a^2} \quad and$$

$$e^{-bt}\sin at\, u(t) \leftrightarrow \frac{a}{(s+b)^2+a^2}$$

$$x(t) = e^{-3t}\cos tu(t) + 2e^{-3t}\sin t\, u(t)$$

4

c. State initial and final value theorem in Laplace transform.

Answer:

Initial Value theorem:

The initial value of x(t) i.e. x(0) directly from the transform X(S) without finding the inverse

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

Final value theorem:

The value of x(t) as $t \to \infty$ may be found directly from the LT X(s)

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

4

Q.8 a. Find the Z-transform of the following sequences and find their

ROC **(i)** $x[n]=\left[\frac{1}{2}\right]^{n-2}\left(\sin\Omega_0(n-2)\right)u[n-2]$ **(ii)** $x[n]=(5)^n u[-n-1]-(3\}^n u[n]$ **Answer:** $x[n]=\left[\frac{1}{2}\right]^{n}\sin\Omega_0 nu[n]$ $\sin\Omega_0 nu[n]\leftrightarrow\frac{z^{-1}\sin\Omega_0}{1-2z^{-1}\cos\Omega_0+z^{-2}}$ ROC $\|z\|>1$	2
Using scaling roperty $\left[\frac{1}{2}\right]^{n}\sin\Omega_0 nu[n]\leftrightarrow\frac{\{1/2)z^{-1}\sin\Omega_0}{1-z^{-1}\cos\Omega_0+\frac{1}{4}z^{-2}}$ ROC $\|z\|>\frac{1}{2}$ $x[n]=\left[\frac{1}{2}\right]^{n-2}\sin\Omega_0(n-2)u[n-2]\leftrightarrow\left[\frac{\{1/2)z^{-1}\sin\Omega_0}{1-z^{-1}\cos\Omega_0+\frac{1}{4}z^{-2}}\right]Z^{-2}$ ROC $\|z\|>\frac{1}{2}$	2
ii. $x[n]=(5)^n u[-n-1]-(3\}^n u[n]$ $X(z)=\sum_{n=-\infty}^{-1}(5)^n z^{-n}-\sum_{n=0}^{\infty}3^n z^{-n}$ $(5)^n u[-n-1]\leftrightarrow-\left[\frac{z}{z-5}\right]$ ROC: $\|Z\|<5$ $(3\}^n u[n]\leftrightarrow\left[\frac{z}{z-3}\right]$ $Roc:\|Z\|>3$ $X(z)=-\left[\frac{z}{z-5}\right]-\left[\frac{z}{z-3}\right]$ ROC: $\|z\|>3$ and $\|z\|<5$, ∴ Common ROC: $3<\|z\|<5$	3

b. State and prove the convolution property of Z- transform

Answer:

Convolution property:

Statement: convolving two signals in time domain is same as multiplying their Z-transforms in Z domain.

If $x[n] \leftrightarrow X(z)$

$h[n] \leftrightarrow H(z)$

Then $x[n] * h[n] \leftrightarrow X[z]H[z]$

Proof: $z\{x[n]*h[n]\} = \sum_{\eta=-\infty}^{\infty}[x[n]*h[n]]z^{-n}$

$$= \sum_{\eta=-\infty}^{\infty}\left[\sum_{k=-\infty}^{\infty}x[k]h[n-k]\right]z^{-n}$$

Let n-k= l, n=k+l

$$= \sum_{l=-\infty}^{\infty}\left[\sum_{k=-\infty}^{\infty}x[k]h[l]\right]z^{-(k+l)}$$

$$= \sum_{l=-\infty}^{\infty}h[l]z^{-l}\left[\sum_{k=-\infty}^{\infty}x[k]\right]z^{-k}$$

$$= X[Z]H[Z]$$

3

1

c. Find the Inverse Z-transform of

$$X(z) = \frac{z}{z^2 - 5z + 6}$$

ROC **(i)** $|z| > 3$

(ii) $|z| < 2$

(iii) $2 < |z| < 3$

Answer:

i) $X(z) = \dfrac{z}{z^2 - 5z + 6}$ ROC (i) $|z| > 3$ (ii) $|z| < 2$ $(iii) 2 < |z| < 3$

2

Solution:

$$X(z)=\frac{z}{z^2-5z+6}=\frac{z}{(z-3)(z-2)}$$ 1

$$\frac{X(z)}{z}=\frac{1}{(z-3)(z-2)}=\frac{A}{(z-3)}+\frac{B}{(z-2)}$$

Solving for A & B. A = 1 B = -1

$$\frac{X(z)}{z}=\frac{1}{z-3}-\frac{1}{z-2}$$

$$X(Z)=\frac{z}{z-(3)}-\frac{z}{z-(2)}$$

Taking IzT

(i) ROC $|z|>3$

$$x[n]=(3)^n u[n]-(2)^n u[n]$$

(ii) ROC $|z|<2$

$$x[n]=-(3)^n u[-n-1]+(2)^n u[-n-1]$$

ROC $2<|z|<3$ $\quad x[n]=-(3)^n u[-n-1]-(2)^n u[n]$

Q.9 a. The random variable x is expresses as its density function

$$f_x(x)=\begin{cases}\alpha e^{-\alpha x} & \text{x} > 0; \alpha = \text{cons tan t}\\ 0 & \text{otherwise}\end{cases}$$

Then, find expected values E[x] and E[x^2]. 2

Answer:

$$\mathrm{E}[\mathrm{x}]=\mu_{\mathrm{x}}=\int_{-\infty}^{\infty}\mathrm{x}f_{\mathrm{x}}(\mathrm{x})d\mathrm{x}$$

$$=\int_{-\infty}^{\infty}\alpha \mathrm{x} e^{-\alpha \mathrm{x}}d\mathrm{x}$$

$$=\alpha\left[\mathrm{x}\left(\frac{e^{-\alpha \mathrm{x}}}{-\alpha}\right)\Bigg|_0^{\infty}+\int_0^{\infty}\frac{e^{-\alpha \mathrm{x}}}{\alpha}d\mathrm{x}\right]$$ 2

$$= -\frac{1}{\alpha} e^{-\alpha x}\Big|_0^\infty = \frac{1}{\alpha}$$ **2**

And $$E[X^2] = \mu_x = \int_{-\infty}^{\infty} x f_x(x) dx$$

$$= \alpha \int_0^\infty x^2 \alpha e^{-\alpha x} dx$$

$$= \alpha \left[x \left(\frac{x^2 e^{-\alpha x}}{-\alpha} \right)\Big|_0^\infty - \int_0^\infty 2x e^{-\alpha x} dx \right]$$

$$E[X^2] = \frac{2}{\alpha}$$

b. What are the properties of wide sense stationary process?

4

Answer:

i) A random process X(t) is called wide sense stationary if it satisfies

a. Mean of the process is constant

b. autocorrelation function is independent of time

c. variance of the process is constant

c. Write properties of power spectral density.

Answer:

Article 1.7 Text book – II, Page 46 – 47. **4**

TEXT BOOKS

1. Signals and Systems, A.V. Oppenheim and A.S. Willsky with S. H. Nawab, Second Edition, PHI Private limited, 2006

2. Communication Systems, Simon Haykin, 4th Edition, Wiley Student Edition, 7th Reprint 2007