Q.2 a. Write down Euclid's algorithm for computing GCD of two non-negative integer numbers. What does this algorithm do for a pair of numbers in which the first number is smaller than the second one? Execute your algorithm on GCD(4121, 5369) (8)

Answer:

Euclid's algorithm in page 4 of the Text Book. (3)

GCD(m,n) = GCD(n, m mod n) if n is not zero

= m if n = 0 (2)

GCD(m,n) = GCD(n,m) if n > m

GCD(4121,5369)

=GCD(5369,4121)

= GCD(412,1248)

=GCD(1248,377)

=GCD(377,117) (3)

=GCD(117,26)

=GCD(26,13)

=GCD(13,0)

=13

b. There are n lockers in a hallway, numbered sequentially from 1 to n. Initially all the locker-doors are closed. You make n passes by the lockers, each time starting with locker no: 1. On the i^{th} pass (for i = 1 to n), you toggle the door of every i^{th} locker. That is, if the door is open you close it and if it is closed, you open it. Write a pseudo code to implement these operations. Execute your algorithm for n = 10 (8)

Answer:

Consider an array A of size n representing the door lockers whose initial values are L (locked). Now the operations can be implemented as follows:

```
for i = 1 to n
 {
  j = i
  while (j<=n)
   {
    toggle (A[j] )          (i.e. If A[j]= L, then A[j] =O else A[j]= L)
    j= j+ i
   }
}
```

4

(Initial Position)

L L L L L L L L L L

Pass 1

O O O O O O O O O O

Pass 2

O L O L O L O L O L

Pass 3

O L L L O O O L L L

Pass 4

O L L O O O O O L L

Pass 5

O L L O L O O O L O

Pass 6

O L L O L L O O L O

Pass 7

O L L O L L L O L O

Pass 8

O L L O L L L L L O

Pass 9

O L L O L L L L O O

(4)

Pass 10

O L L O L L L L O L

Q.3 a. Explain the different asymptotic notations used in expressing the complexity of algorithms? What is the complexity of an algorithm that has only sequential statements? Compare the two functions n^3 and 2^n for various values of n and determine when the second function will become larger than the first function. (8)

Answer:

Page 49 of the Text Book. (Answer available in any book on algorithms.) (4)

The complexity of an algorithm that has only sequential statements is always a constant. In other words O(1). (2)

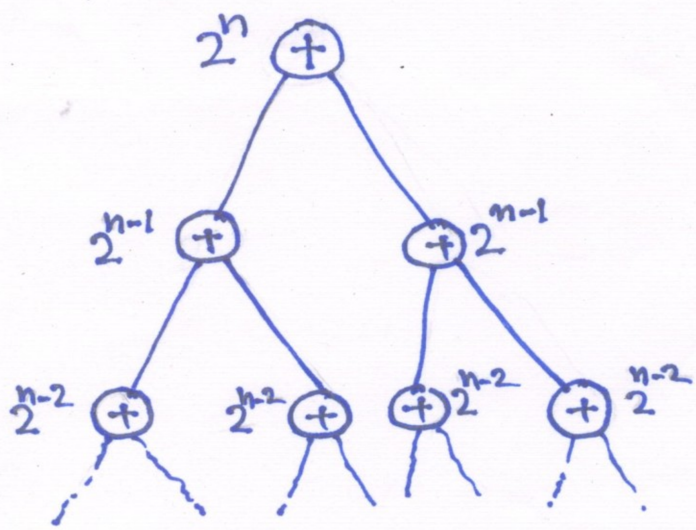
At n=10, second function will become greater than the first one (2)

b. Design a recursive algorithm for computing $2^n = 2^{n-1} + 2^{n-1}$. Draw a tree of recursive calls for this algorithm and compute the number of calls made for computing 2^n (8)

Answer:

```
exponentiation(n)
{
 if n = 0, return (1)
 else return (exponentiation(n-1) + exponentiation(n-1))
}
```

(4)



(4)

$(2^{n+1} - 1)$ calls

Q.4 a. You have to compute x mod m where x is a very large positive integer that your computer can not process. How will you overcome this issue? Write down a pseudo code for your algorithm. (6)

Answer:

Assume that x has n digits where n is large. Store the digits of x in an array A of size n. Now **x mod m** can be found as follows:

```
r = 0
for i = 1 to n
  {
   r = (r * 10 + A[i] ) mod m
  }
```

(6)

b. Write down Strassen's algorithm for multiplying two matrices. Use the algorithm to compute the product of the following two matrices. How would you modify Strassen's algorithm to multiply n x n matrices in which n is not an exact power of 2? (10)

$$A = \begin{bmatrix} 2 & 8 \\ 13 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & -9 \\ -3 & 4 \end{bmatrix}$$

Answer:

Strassen's formulae in Page 139 of the Text Book (4)

multiplication steps (4)

If n is not a power of two, the matrices should be padded with rows and columns of zeros and make the size to the next power of two. (2)

Q.5 a. The first half of an array contains 1 in each cell and the second half contains 2 in each cell. Write an algorithm that shuffles the contents of the array properly with minimum number of exchanges. That is, after shuffling the contents of the array should be 1,2,1,2,1,2… and so on. (6)

Answer:

Let the size of the array A be 2n in which there are n number of 1s in the first half and n number of 2s in the second half. They can be properly shuffled using the following pseudo code:

```
i = n, j = n+1
while ( i > 1 and j < 2n)
{
  interchange (A[i] and A[j])
        i = i-2,  j = j+2
}
```

(6)

b. Write an algorithm for finding the k^{th} smallest element (k^{th} order statistic) in a list of n numbers. (10)

Answer:

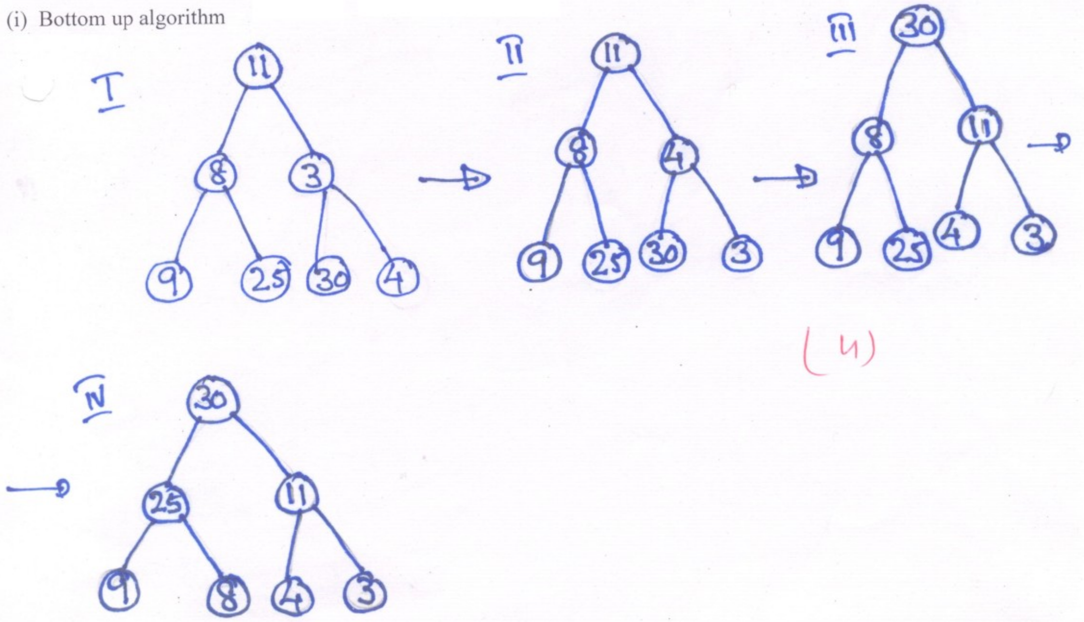
The technique used in Quick sort for placing the pivotal element in its proper position is to be used here. Thus if the first element of the list (which is used as the pivot) is placed at the i^{th} position in the list through this procedure, that element is the i^{th} smallest element. If $k = i$, the first element is the k^{th} smallest element. If $k < i$, repeat the procedure on the left sub-list of the list, otherwise on the right sub-list

Algo-(5)
Explanation-(5)

Q.6 a. Construct a heap for the list 11,8,3,9,25,30,4 by (i) Bottom-up algorithm (ii) Successive key insertion(Top down algorithm). Is it always true that the bottom up and successive key insertion algorithms yield the same heap for the same input? (10)

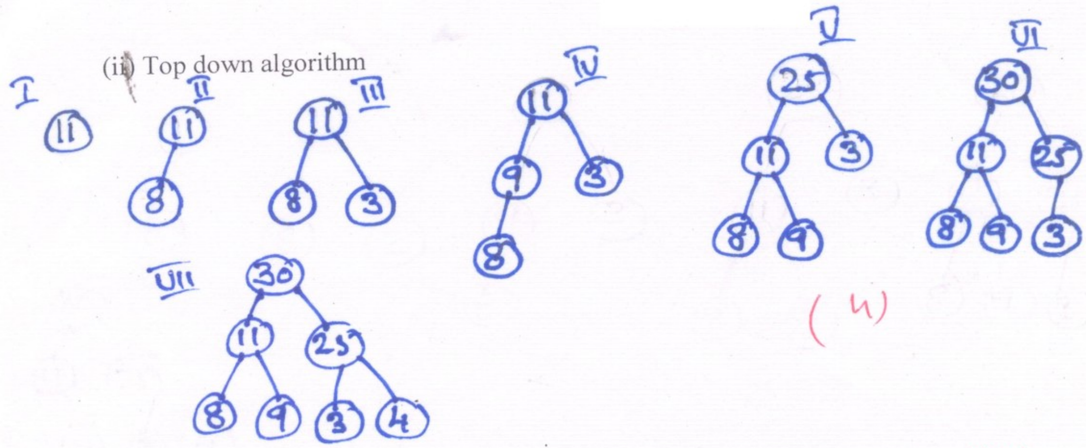
Answer:

(i) Bottom up algorithm



(4)

(ii) Top down algorithm



(4)

From the output itself it is very clear that both the algorithms may not yield the same heap. (2)

b. **Apply Horner's rule to evaluate the polynomial $P(x) = 3x^5 + 2x^4 - 5x^3 + x^2 + 7x+12$ at $x = 2$. How many multiplications and additions are required to evaluate a polynomial of degree n? (6)**

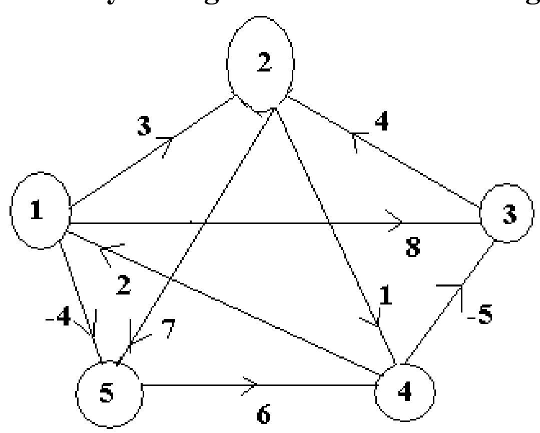
Answer:

$$\left.\begin{aligned}
&3x^5 + 2x^4 - 5x^3 + x^2 + 7x+12\\
&= (x(\ 3x^4 + 2x^3 - 5x^2 + x + 7\ )+12)\\
&= (x(\ x\ (3x^3 + 2x^2 - 5x + 1) + 7\ )+12)\\
&= (x(\ x\ (x\ (3x^2 + 2x - 5) + 1) + 7\ )+12)\\
&= (x(\ x\ (x\ (x(3x + 2) - 5) + 1) + 7\ )+12)
\end{aligned}\right\}\ 4$$

The evaluation of this polynomial can be done using 5 multiplications and 5 additions. For this purpose evaluation is to be done starting from the innermost parenthesis outwards. The evaluation the expression inside each set of parenthesis can be done using one multiplication and one addition. 2

The value of the expression at x=2 is 118

Q.7 a. Write down Floyd's algorithm to find the all pairs shortest path of a digraph. Execute your algorithm on the following graph. (8)



Answer:

Floyd's algorithm in Page 276 of the text book. → (5)

(5)

$$D^{(0)} = \begin{bmatrix} 0 & 3 & 8 & \alpha & -4 \\ \alpha & 0 & \alpha & 1 & 7 \\ \alpha & 4 & 0 & \alpha & \alpha \\ 2 & \alpha & -5 & 0 & \alpha \\ \alpha & \alpha & \alpha & 6 & 0 \end{bmatrix} \quad D^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \alpha & -4 \\ \alpha & 0 & \alpha & 1 & 7 \\ \alpha & 4 & 0 & \alpha & \alpha \\ 2 & 5 & -5 & 0 & -2 \\ \alpha & \alpha & \alpha & 6 & 0 \end{bmatrix} \quad D^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \alpha & 0 & \alpha & 1 & 7 \\ \alpha & 4 & 0 & 5 & 11 \\ 2 & 5 & -5 & 0 & -2 \\ \alpha & \alpha & \alpha & 6 & 0 \end{bmatrix}$$

$$D^{(3)} = \begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \alpha & 0 & \alpha & 1 & 7 \\ \alpha & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ \alpha & \alpha & \alpha & 6 & 0 \end{bmatrix} \quad D^{(4)} = \begin{bmatrix} 0 & 3 & -1 & 4 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ \alpha & \alpha & \alpha & 6 & 0 \end{bmatrix} \quad D^{5} = \begin{bmatrix} 0 & 1 & -3 & 2 & 4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$$

b. Write down the recursive algorithm for solving Knap Sack problem. Apply your algorithm to solve the following instance of the Knap Sack problem. Capacity of the Knap Sack is 5. (8)

Item	Weight	Value
1	2	Rs.12
2	1	Rs.10
3	3	Rs.20
4	2	Rs.15

Answer:

Recursive solution in Page 284 → (4)

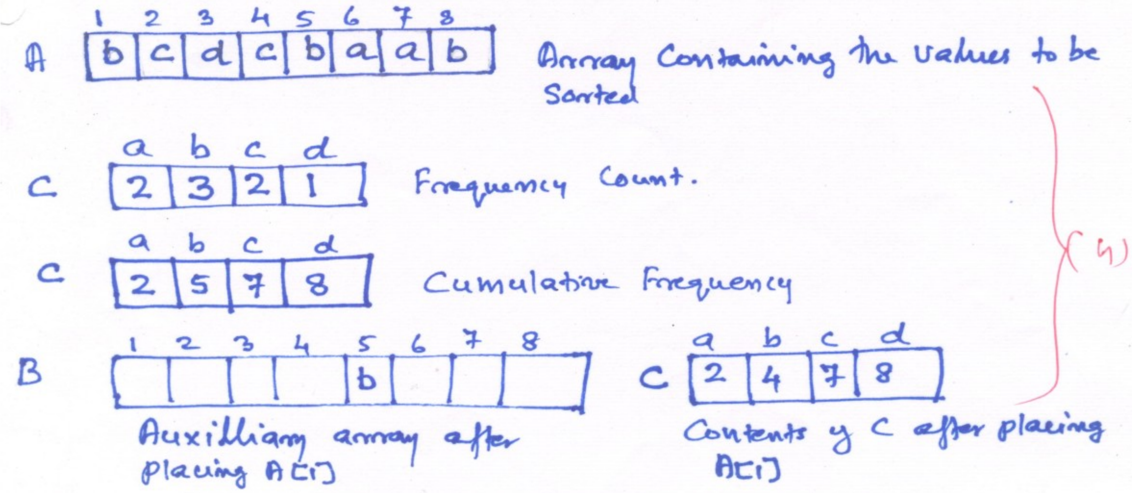
		0	1	2	3	4	5
	0	0	0	0	0	0	0
$W_1=2$, $V_1=12$	1	0	0	12	12	12	12
$W_2=1$ $V_2=10$	2	0	10	12	22	22	22
$W_3=3$ $V_3=20$	3	0	10	12	22	30	32
$W_4=2$ $V_4=15$	4	0	10	15	25	30	37

(4)

Q.8 a. Write down Distribution Counting Sort algorithm. Assuming that the set of possible list-values is [a,b,c,d], sort the following list in alphabetical order by the Distribution Counting Sort algorithm: b,c,d,c,b,a,a,b (8)

Answer:

Distribution Counting Sort algorithm in Page 240 of the text book. — (4)

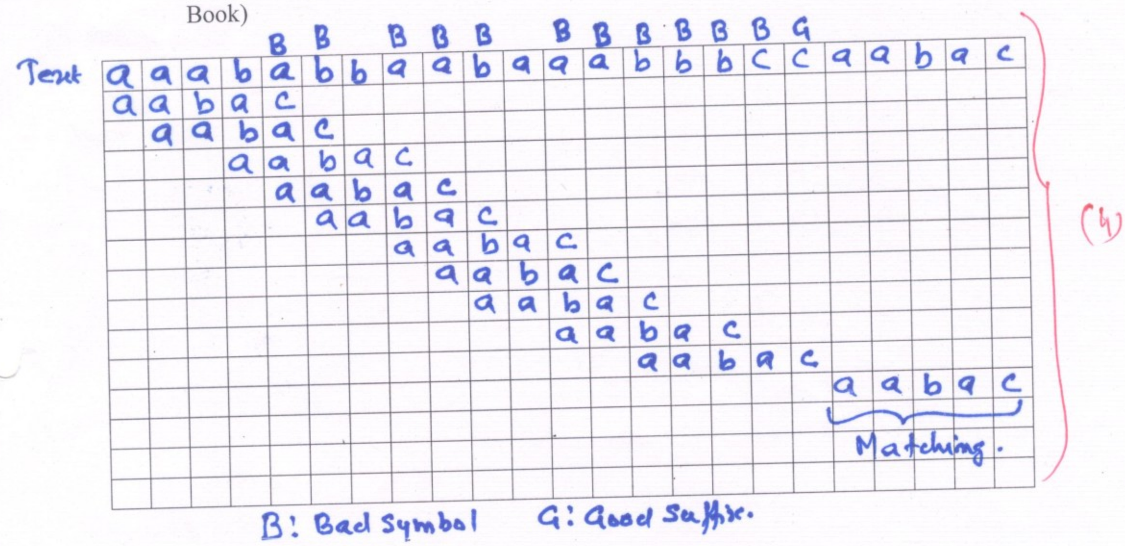


b. **Explain the two parameters that determine the shift size in Boyer Moore string matching algorithm. Apply Boyer Moore algorithm to search for the pattern aabac in the text aaababbaabaaabbbccaabac. (8)**

Answer:

Algo — (4)

(i) Bad Symbol shift and (ii) Good Suffix Shift (Details in Page 246 of Text Book)

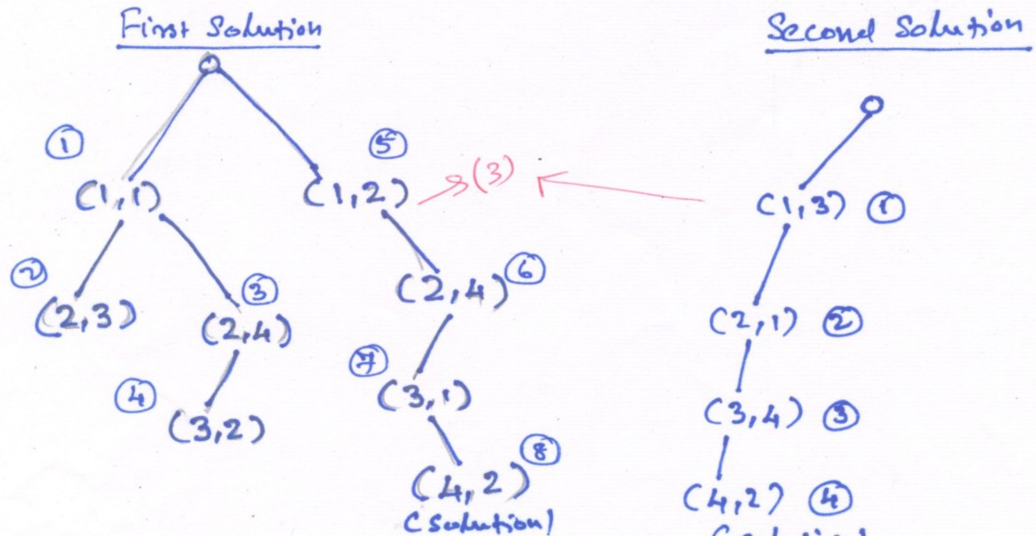


Q.9 a. Apply back tracking search for finding solution to the Four Queen's problem. Explain how this algorithm can be used to find the second solution to the problem. (8)

Answer:

Solution tree in page 396 of the text book. (3)

A second solution to the problem can be found by keeping the first Queen in the third column of the first row. Repeat the same procedure as that used for finding the first solution (2)



Note:-

$(x,4) = x^{th}$ row 4^{th} column

b. Write a pseudo code for the Bisection method for solving non-linear equations. Apply this method to find the root of the equation x^3-x-1 = 0. (8)

Answer:

Pseudo code in page 431 of the text book. (4)

n	a_n	b_n	x_n	$f(x_n)$
1	0.0	2.0	1.0	-1.0
2	1.0	2.0	1.5	0.875
3	1.0	1.5	1.25	-0.296875
4	1.25	1.5	1.375	0.224609
5	1.25	1.375	1.3125	-0.051514
6	1.3125	1.375	1.34375	0.082611
7	1.3125	1.34375	1.328125	0.014576
8	1.3125	~~1.34375~~ 1.328125	1.3203125	-0.018711

(4)

TEXT BOOK

I. Introduction to The Design & Analysis of Algorithms, Anany Levitin, Second Edition, Pearson Education, 2007 (TB-I)