

Q2 (a) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x^2}$ in the form of indeterminate.

Answer

Q.2.a. Soln:

$$\text{Let } y = \lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x^2}$$

$$\log y = \lim_{x \rightarrow 0} \log \left(\frac{\tan x}{x} \right)^{1/x^2}$$

$$= \lim_{x \rightarrow 0} \frac{1}{x^2} \log \frac{\tan x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\log \tan x - \log x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{2}{\sin 2x} - \frac{1}{x}}{2x}$$

$$= \lim_{x \rightarrow 0} \frac{2x - \sin 2x}{2x^2 \cdot \sin 2x}$$

$$= \lim_{x \rightarrow 0} \frac{2 - 2 \cos 2x}{4x^2 \cos 2x + 4x \sin 2x}$$

$$= \lim_{x \rightarrow 0} \frac{4 \sin 2x}{-8x^2 \sin 2x + 16x \cos 2x + 4 \sin 2x}$$

$$= \lim_{x \rightarrow 0} \frac{8 \cos 2x}{-16x^2 \cos 2x - 48x \sin 2x + 24 \cos 2x}$$

$$= \frac{1}{3}$$

\therefore The required L.T. is $e^{1/3}$ Ans.

Q2 (b) Use Taylor's theorem, expand $\sqrt{1 + \sin x}$ upto sixth power of x.

Answer

Q.2.b. Soln

$$f(x) = \sqrt{1 + \sin x}$$

$$= \sqrt{1 + \cos \left(\frac{\pi}{2} - x \right)}$$

$$= \sqrt{1 + 2 \cos^2 \left(\frac{\pi}{4} - \frac{x}{2} \right) - 1}$$

$$= \sqrt{2 \cos \left(\frac{\pi}{4} - \frac{x}{2} \right)}$$

by Taylor's Series,

$$f(a+h) = f(a) + h f'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + \dots$$

$$\sqrt{1 + \sin x} = \sqrt{2} \cos \left(\frac{\pi}{4} - \frac{x}{2} \right)$$

$$= \sqrt{2} \left[\cos \frac{\pi}{4} + \left(-\frac{x}{2} \right) \left(-\sin \frac{\pi}{4} \right) + \frac{1}{2!} \left(-\frac{x}{2} \right)^2 \times \left(-\cos \frac{\pi}{4} \right) \right. \\ \left. + \frac{1}{3!} \left(-\frac{x}{2} \right)^3 \cdot \left(\sin \frac{\pi}{4} \right) + \dots \right]$$

$$= \sqrt{2} \left[\frac{1}{\sqrt{2}} + \frac{x}{2} \left(\frac{1}{\sqrt{2}} \right) - \frac{1}{2} \cdot \frac{x^2}{4} \cdot \frac{1}{\sqrt{2}} - \frac{1}{3!} \frac{x^3}{8} \cdot \frac{1}{\sqrt{2}} + \dots \right]$$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{48} \dots \dots \dots \text{Ans.}$$

Q3 (a) Compute the are length of the curve $ay^2 = x^3$ from $x = 0$ to a point having $x = 0$ to a point having $x = 5a$.

Answer

Q.3.a. Soln. MODERATION-I

$$ay^2 = x^3$$

$$2ay \frac{dy}{dx} = 3x^2 \quad \text{or} \quad \frac{dy}{dx} = \frac{3x^2}{2ay} = \frac{3x^2}{2a \left(\frac{x^2}{a}\right)^{1/2}}$$

$$\frac{dy}{dx} = \frac{3x^2}{2a^{1/2} x^{1/2}} = \frac{3x^{3/2}}{2a^{1/2}}$$

Required length = $\int_0^{5a} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

$$= \int_0^{5a} \sqrt{1 + \frac{9}{4} \cdot \frac{x}{a}} dx$$

$$= \frac{1}{2\sqrt{a}} \int_0^{5a} \sqrt{4a + 9x} dx$$

$$= \frac{1}{2\sqrt{a}} \times \frac{2}{3 \times 9} \left[(4a + 9x)^{3/2} \right]_0^{5a}$$

$$= \frac{1}{27a^{1/2}} \left[(4a + 9 \times 5a)^{3/2} - (4a)^{3/2} \right]$$

$$= \frac{1}{27a^{1/2}} \left[343a^{3/2} - 8^{3/2} \right]$$

$$= \frac{335a}{27} \text{ Ans.}$$

Q3 (b) Find the length of an arch of the cycloid whose equations are $x = a(\theta + \sin \theta)$ and $y = a(1 + \cos \theta)$.

Answer

Q.3.b. $x = a(\theta + \sin \theta), \frac{dx}{d\theta} = a(1 + \cos \theta)$
 $y = a(1 + \cos \theta), \frac{dy}{d\theta} = -a \sin \theta$

The limits for half of the curve $\theta = 0$ and $\theta = \pi$

\therefore The required length of the arch

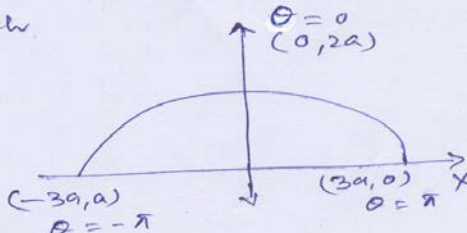
$$= 2 \int_0^{\pi} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta$$

$$= 2a \int_0^{\pi} \sqrt{(1 + \cos \theta)^2 + \sin^2 \theta} d\theta$$

$$= 2a \int_0^{\pi} \sqrt{1 + \cos^2 \theta + 2\cos \theta + \sin^2 \theta} d\theta$$

$$= 2\sqrt{2}a \int_0^{\pi} \sqrt{1 + \cos \theta} d\theta$$

$$= 2\sqrt{2} \sqrt{2} a \int_0^{\pi} \cos \frac{\theta}{2} d\theta$$

$$= 8a \left[\sin \frac{\theta}{2} \right]_0^{\pi} = 8a \text{ Ans.}$$


Q4 (a) If $|z_1 + z_2| = |z_1 - z_2|$, prove that the difference of amplitudes of z_1 and z_2 is $\frac{\pi}{2}$.

Answer

Q(4) (a). If $|z_1 + z_2| = |z_1 - z_2|$, prove that the difference of amplitudes of z_1 and z_2 is $\frac{\pi}{2}$ (8)

Solⁿ Ex \rightarrow (19.8) | P \rightarrow 707

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Marking \rightarrow

$$z_1 + z_2 = r(\cos\theta + j\sin\theta)$$

$$z_1 - z_2 = r(\cos\phi - j\sin\phi)$$

$$\text{amp}(z_1) = \frac{\theta + \phi}{2} \quad \underline{3}$$

$$\text{amp}(z_2) = \frac{\pi}{2} + \frac{\theta + \phi}{2} \quad \underline{3}$$

$$\text{amp}(z_2) - \text{amp}(z_1) = \frac{\pi}{2} \quad \underline{1}$$

Q4 (b) A resistance of 20 ohms an inductance of 0.2 Henry and a capacitance of 100 micro-farad are connected in series across 220 volts, 50 cycle/sec main. Calculate, (i) Impedance, (ii) Current, (iii) Voltage across, L, R and C.

Answer

4. b. soln here $R = 20$ ohms, $L = 0.2$ Henry
 $C = 100 \times 10^{-6}$ farad
 Voltage $V = 200$ volts, $\omega = 2\pi \times 50$ radian/sec

$\therefore X_L = L\omega = 0.2 \times 100\pi = 62.8319$ Ohms. \uparrow
 and $X_C = \frac{1}{C\omega} = \frac{1}{100 \times 10^{-6} \times 100\pi} = 31.8319$ Ohms. \uparrow

(i) let Z be the impedance of the circuit
 then, $Z = R + j(X_L - X_C) = 20 + j(62.8319 - 31.8319)$
 $Z = 20 + 31j$
 $\therefore |Z| = \sqrt{(20)^2 + (31)^2} = 36.89$ Ohms Ans 2

(ii) let I be the current in the circuit. then,
 $I = \frac{V}{|Z|} = \frac{200}{36.89} = 5.42$ Amp. 2

(iii) voltage across $L = V_L = I \times X_L = 5.42(62.8319)$
 $= 340.55$ volts Ans
 voltage across $R = V_R = I R = 5.42 \times 20 = 108.4$ volts Ans
 voltage across $C = V_C = I X_C = 5.42 \times 31.8319$
 $= 172.52$ volts Ans 2

Q5 (a) A rigid body is rotating with angular velocity 2 radian / sec about an axis OR, where R is $2i - j + k$ and O is the origin. Find the velocity of the point $3i + 2j - k$ on the body.

Answer

Soln → Angular vel. = 2 rad/sec

$\vec{OR} = 2i - j + k$

$\vec{\omega} = \frac{2(2i - j + k)}{\sqrt{4+4+1}}$

$= \frac{2}{3}(2i - j + k)$ $\frac{2}{3}$

$\vec{r} = \vec{OP} = 3i + 2j - k$ $\frac{1}{3}$

$\therefore \vec{v} = \vec{\omega} \times \vec{r}$ $\frac{1}{3}$

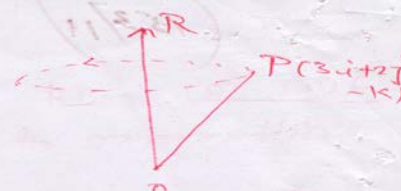
$= \frac{2}{3}(2i - j + k) \times (3i + 2j - k)$ $\frac{1}{3}$

$= \frac{2}{3} \begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix}$ $\frac{1}{3}$

$= \frac{2}{3} [(2-2)i - (-2-3)j + (4+6)k]$ $\frac{1}{3}$

$= \frac{2}{3} (5j + 10k)$ $\frac{1}{3}$

$= \frac{10}{3} (j + 2k)$ $\frac{1}{3}$



Q5 (b) A force of 15 units acts parallel to the line $i - 2j + 2k$ and passes through the points $2i - 2j - k$. Using vector method, find the magnitude of the moment of the force about the point $i + j - k$.

Answer

Q.5-b. Soln: The unit vector in the direction of $i - 2j + 2k$ is $\frac{i - 2j + 2k}{\sqrt{1+4+4}} = \frac{i - 2j + 2k}{3}$ 2

\therefore The force $\vec{F} = 15 \left(\frac{i - 2j + 2k}{3} \right) = 5(i - 2j + 2k)$ 1

Let $A = i + j - k$ and $P = 2i - 2j - k$

$\therefore \vec{r} = \vec{PA} = \text{P.v. of } A - \text{P.v. of } P$
 $= (2i - 2j - k) - (i + j - k)$
 $= i - 3j$ 2

Moment of \vec{F} about $P = \vec{m}$
 $= \vec{r} \times \vec{F}$

$= 5 \begin{vmatrix} i & j & k \\ 1 & -3 & 0 \\ 1 & -2 & 2 \end{vmatrix} = 5(-6i - 2j + k)$ 2

Magnitude of the moment $= |\vec{m}| = 5\sqrt{36+4+1} = 5\sqrt{41}$ Ans. 1

Q6 (a) Solve the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x + \sin x$

Answer

Q.6.a. Soln:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x + \sin x \quad D \equiv \frac{d}{dx}$$

The above eqn. can be written as

$$(D^2 + D - 2)y = x + \sin x$$

The auxiliary equation of the above differential eqn is

$$m^2 + m - 2 = 0$$

or $(m+2)(m-1) = 0$, which gives, $m = -2, 1$

C.F. = $C_1 e^{-2x} + C_2 e^x$

Now, P.I. = $\frac{1}{D^2 + D - 2} (x + \sin x)$

$$= \frac{1}{D^2 + D - 2} x + \frac{1}{D^2 + D - 2} \sin x$$

$$= \frac{1}{-2 \left[1 + \frac{D+D^2}{2} \right]} x + \frac{1}{-1 + D - 2} \sin x$$

$$= -\frac{1}{2} \left[1 - \frac{D+D^2}{2} \right]^{-1} x + \frac{1}{D-3} \sin x$$

$$= -\frac{1}{2} \left[1 + \frac{D+D^2}{2} + \dots \right] x + \frac{D+3}{D^2-9} \sin x$$

$$= -\frac{1}{2} \left[1 + \frac{D}{2} + \frac{D^2}{2} + \dots \right] x + \frac{D+3}{-1-9} \sin x$$

$$= -\frac{1}{2} \left[x + \frac{1}{2} \right] - \frac{1}{10} (D+3) \sin x$$

$$= -\frac{1}{4} (2x+1) - \frac{1}{10} (\cos x + 3 \sin x)$$

Hence the general solution is,

$$y = C_1 e^{-2x} + C_2 e^x - \frac{1}{4} (2x+1) - \frac{1}{10} (\cos x + 3 \sin x)$$

Ans.

Q6 (b) A body weighing 10 kg is hung from a spring. A pull of 20 kg. Wt. will stretch the spring to 10 cm. The body is pulled down to 20 cm below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position at time t sec., the maximum velocity and the period of oscillation.

Answer

Qs(6)(b) Ex (14.7) | P → 555/556 MODERATION-I

Highest Engg. mathematics by B.S. Grewal (8)

Marking → $k \rightarrow$ restoring force

$k = 200 \text{ kg/m}$ 1

O → fixed end of spring

A → lower end of spring

B → equilibrium position of spring when a body weighing w is hung from A.

AB = 0.05 m 1

C → when the weight is pulled down to C

After any time (t), the weight is at P.

$T_p = 10 + 200x$ 1

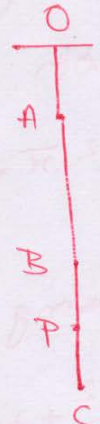
Equation of motion of the body →

$$\frac{d^2x}{dt^2} = -\mu^2 x, \quad \mu = 14 \quad \underline{2}$$

Period of oscillation = $\frac{2\pi}{\mu} = \underline{0.45 \text{ sec}}$ 1

Displacement of the body from B at time t is $x = 0.2 \cos(14t) \text{ m}$ 1

Maximum velocity = 2.8 m/sec.



Q7 (a) Find a Fourier series for the function $f(x) = x$ in the interval $[-\pi, \pi]$

Answer

Q.7.a. Clearly, $f(x) = x$ is an odd function.
 Therefore, Fourier series for $f(x)$ is purely a sine series
 given by $\sum_{n=1}^{\infty} b_n \sin nx$ (i)

where, $b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$

Since $f(x) = x$ is continuous for all $x \in (-\pi, \pi)$
 therefore, Fourier series of $f(x)$ converges to $f(x)$ for all
 $x \in (-\pi, \pi)$

Hence, $x = \sum_{n=1}^{\infty} b_n \sin nx$

Computation of b_n : we have

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

$$\Rightarrow b_n = \frac{2}{\pi} \int_0^{\pi} x \sin nx \, dx$$

$$\Rightarrow b_n = \frac{2}{\pi} \left[-x \cdot \frac{\cos nx}{n} + \frac{\sin nx}{n^2} \right]_0^{\pi}$$

$$\Rightarrow b_n = \frac{2}{\pi} \left[\left(-\pi \frac{\cos n\pi}{n} + \frac{\sin n\pi}{n^2} \right) - 0 \right]$$

$$\Rightarrow b_n = -\frac{2}{\pi} \cos n\pi = -\frac{2}{\pi} (-1)^n = \frac{2}{\pi} (-1)^{n+1}$$

Substituting the value of b_n in (i), we obtain

$$\sum_{n=1}^{\infty} \frac{2}{\pi} (-1)^{n+1} \sin nx$$

or $2 \left\{ \sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \frac{1}{4} \sin 4x + \dots \right\}$
 as the Fourier series of $f(x)$ Ans

Q7 (b) Develop $f(x)$ in Fourier series in the interval $(0, 2)$, if

$$f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$$

Answer

Q.7.b.

Soln:

We know that the Fourier series of a function $f(x)$ defined on $[a, b]$ is given by

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos \left(\frac{2n\pi x}{b-a} \right) + b_n \sin \left(\frac{2n\pi x}{b-a} \right) \right\}, \text{ where.}$$

$$a_0 = \frac{2}{b-a} \int_a^b f(x) dx, \quad a_n = \frac{2}{b-a} \int_a^b f(x) \cos \left(\frac{2n\pi x}{b-a} \right) dx$$

$$\text{and } b_n = \frac{2}{b-a} \int_a^b f(x) \sin \left(\frac{2n\pi x}{b-a} \right) dx. \quad 2$$

Here, $a=0$, $b=2$.

Therefore, the Fourier series of function $f(x)$ given by

$$f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$$

$$\text{is given by } \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos n\pi x + b_n \sin n\pi x \right\}$$

$$\text{where } a_0 = \int_0^2 f(x) dx, \quad a_n = \int_0^2 f(x) \cos n\pi x dx \quad \text{and}$$

$$b_n = \int_0^2 f(x) \sin n\pi x dx$$

Computation of a_0 , we have, $a_0 = \int_0^2 f(x) dx$

$$\Rightarrow a_0 = \int_0^1 \pi x dx + \int_1^2 \pi(2-x) dx = \int_0^1 f(x) dx + \int_1^2 f(x) dx$$

$$\Rightarrow a_0 = \frac{\pi}{2} + \frac{\pi}{2} = \pi \quad 2$$

Computation of a_n : we have,

$$a_n = \int_0^2 f(x) \cos n\pi x dx$$

$$\Rightarrow a_n = \int_0^1 f(x) \cos n\pi x dx + \int_1^2 f(x) \cos n\pi x dx$$

$$\Rightarrow a_n = \int_0^1 \pi x \cos n\pi x dx + \int_1^2 \pi(2-x) \cos n\pi x dx$$

$$\Rightarrow a_n = \pi \left[x \frac{\sin n\pi x}{n\pi} + \frac{\cos n\pi x}{n^2 \pi^2} \right]_0^1 + \pi \left[(2-x) \frac{\sin n\pi x}{n\pi} - \frac{\cos n\pi x}{n^2 \pi^2} \right]_1^2$$

$$\Rightarrow a_n = \pi \left[\frac{\cos n\pi}{n^2 \pi^2} - \frac{1}{n^2 \pi^2} \right] + \pi \left[-\frac{\cos 2n\pi}{n^2 \pi^2} + \frac{\cos n\pi}{n^2 \pi^2} \right]$$

$$\Rightarrow a_n = 2\pi \left\{ \frac{\cos n\pi - 1}{n^2 \pi^2} \right\}$$

$$\Rightarrow a_n = \frac{2}{n^2 \pi} \left\{ (-1)^n - 1 \right\}$$

$$\Rightarrow a_n = \begin{cases} -\frac{4}{n^2 \pi}, & \text{if } n \text{ is odd} \\ 0, & \text{if } n \text{ is even} \end{cases} \quad 2$$

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$$b_n = \int_0^2 f(x) \sin n\pi x \, dx$$

$$\Rightarrow b_n = \int_0^1 f(x) \sin n\pi x \, dx + \int_1^2 f(x) \sin n\pi x \, dx$$

$$\Rightarrow b_n = \int_0^1 \pi x \sin n\pi x \, dx + \int_1^2 \pi (2-x) \sin n\pi x \, dx$$

$$\Rightarrow b_n = \pi \left[-\frac{x}{n\pi} \cos n\pi x + \frac{\sin n\pi x}{n^2\pi^2} \right]_0^1$$

$$+ \pi \left[-(2-x) \frac{\cos n\pi x}{n\pi} - \frac{\sin n\pi x}{n^2\pi^2} \right]_1^2$$

$$\Rightarrow b_n = \pi \left\{ -\frac{\cos n\pi}{n\pi} \right\} + \pi \left\{ \frac{\cos n\pi}{n\pi} \right\} = 0$$

substituting the values of a_0 , a_n and b_n in (1) we obtain the series,

$$\frac{\pi}{2} - \frac{4}{\pi} \left\{ \frac{\cos n\pi}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \dots \right\}$$

As Fourier series of $f(x)$ 2

Q8 (a) Find the Laplace transform of $\cos t \cdot \cos 2t \cdot \cos 3t$.

Answer

Q8.a. Soln:

$$\cos t \cdot \cos 2t \cdot \cos 3t = \frac{1}{2} [(2 \cos t \cdot \cos 2t) \cos 3t] \quad 1$$

$$= \frac{1}{2} [(\cos 3t + \cos t) \cos 3t] \quad 1$$

$$= \frac{1}{4} [2 \cos^2 3t + 2 \cos 3t \cdot \cos t] \quad 1$$

$$= \frac{1}{4} [1 + \cos 6t + \cos 4t + \cos 2t] \quad 1$$

$$\therefore \mathcal{L} \{ \cos t \cdot \cos 2t \cdot \cos 3t \} = \frac{1}{4} [\mathcal{L} \{ 1 \} + \mathcal{L} \{ \cos 6t \}$$

$$+ \mathcal{L} \{ \cos 4t \} + \mathcal{L} \{ \cos 2t \}] \quad 2$$

$$= \frac{1}{4} \left\{ \frac{1}{s} + \frac{s}{s^2+36} + \frac{s}{s^2+16} + \frac{s}{s^2+4} \right\} \quad \text{Ans } 2$$

Q8 (b) Find Laplace transform of $\cos^4 t$

Answer

Q.8.b. Soln: $\cos^4 t = \left(\frac{1 + \cos 2t}{2} \right)^2$

$$= \frac{1}{4} \{ 1 + 2 \cos 2t + \cos^2 2t \}$$

$$\Rightarrow \cos^4 t = \frac{1}{4} \left\{ 1 + 2 \cos 2t + \frac{1 + \cos 4t}{2} \right\}$$

$$\Rightarrow \cos^4 t = \frac{1}{8} \{ 3 + 4 \cos 2t + \cos 4t \}$$

$$\therefore \mathcal{L} \{ \cos^4 t \} = \frac{1}{8} [\mathcal{L} \{ 3 \} + 4 \mathcal{L} \{ \cos 2t \} + \mathcal{L} \{ \cos 4t \}]$$

$$\Rightarrow \mathcal{L} \{ \cos^4 t \} = \frac{1}{8} \left\{ \frac{3}{s} + \frac{4s}{s^2 + 4} + \frac{s}{s^2 + 16} \right\} \text{ Ans}$$

Q9 (a) Find $\mathcal{L}^{-1} \left\{ \frac{3s-2}{s^2-4s+20} \right\}$

Answer

Q.9.a. Soln:

We have, $\mathcal{L}^{-1} \left\{ \frac{2s-2}{s^2-4s+20} \right\} = \mathcal{L}^{-1} \left\{ \frac{3(s-2)+4}{(s-2)^2+4^2} \right\}$

$$= \mathcal{L}^{-1} \left\{ \frac{3(s-2)}{(s-2)^2+4^2} + \frac{4}{(s-2)^2+4^2} \right\}$$

$$= 3 \mathcal{L}^{-1} \left\{ \frac{(s-2)}{(s-2)^2+4^2} \right\} + 4 \mathcal{L}^{-1} \left\{ \frac{1}{(s-2)^2+4^2} \right\}$$

$$= 3 e^{2t} \cdot \mathcal{L}^{-1} \left\{ \frac{s}{s^2+4^2} \right\} + 4 e^{2t} \cdot \mathcal{L}^{-1} \left\{ \frac{1}{s^2+4^2} \right\}$$

$$= 3 e^{2t} \cdot \cos 4t + 4 e^{2t} \cdot \left(\frac{\sin 4t}{4} \right)$$

$$= 3 e^{2t} \cdot (3 \cos 4t + \sin 2t) \text{ Ans}$$

Q9 (b) Solve the equation

$$\frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 5x = e^{-t} \sin t, x(0) = 0, x'(0) = 1$$

Using Laplace transform.

Answer

Q. (a) Solve the equation—

$$\frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 5x = e^{-t} \sin t,$$

$x(0) = 0, x'(0) = 1$

Soln → Using Laplace transform

Taking the Laplace transform of both sides, we get—

$$[s^2 \bar{x} - sx(0) - x'(0)] + 2[s\bar{x} - x(0)] + 5\bar{x} = \frac{1}{(s+1)^2 + 1}$$

Using the given conditions, it reduces to—

$$(s^2 + 2s + 5)\bar{x} - 1 = \frac{1}{(s+1)^2 + 1}$$

$$\bar{x} = \frac{1}{(s^2 + 2s + 2)(s^2 + 2s + 5)} + \frac{1}{(s+1)^2 + 1}$$

$$= \frac{1}{3} \left[\frac{1}{s^2 + 2s + 2} - \frac{1}{s^2 + 2s + 5} \right] + \frac{1}{(s+1)^2 + 1}$$

$$= \frac{1}{3} \left[\frac{1}{s^2 + 2s + 2} + \frac{2}{s^2 + 2s + 5} \right]$$

$$= \frac{1}{3} \left[\frac{1}{(s+1)^2 + 1} + \frac{2}{(s+1)^2 + 2^2} \right]$$

Taking inverse Laplace transform of both sides, we get—

$$x = \frac{1}{3} L^{-1} \left\{ \frac{1}{(s+1)^2 + 1} + 2 \cdot \frac{1}{(s+1)^2 + 2^2} \right\}$$

$$x = \frac{1}{3} \left[e^{-t} \sin t + 2 \cdot \frac{1}{2} e^{-t} \sin 2t \right]$$

$$x = \frac{1}{3} e^{-t} (\sin t + \sin 2t)$$

Text Books

1. Engineering Mathematics- Dr. B S Srewal, 12th Edition 2007, Khanna Publishes Delhi.
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