Q2 (a) **If y =** $e^{ax}.\sin bx$, **then prove that** $y_2 - 2ay_1 + (a^2 + b^2)y = 0$ **.**

Answer

Solution.

Q.2.a. given eqn. is $y = e^{ax}$

Differentiating the above eqn. w.r.t. x, we get

$y_1 = e^{ax}.\cos bx + b \sin bx . a e^{ax}$ 2

$\therefore y_1 = (b \cos bx + a \sin bx) e^{ax}$ ——— (i)

Differentiating the above equation w.r.t. 'x' we get

$y_1 = (b \cos bx + a \sin bx) a e^{ax} + e^{ax}(b(-b \sin bx) + ab \cos bx)$

$y_2 = e^{ax}(ab \cos bx + a^2 \sin bx - b^2 \sin bx + ab \cos bx)$

$y_2 = [2ab \cos bx + (a^2 - b^2). \sin bx] e^{ax}$ ——— (ii) 2

Substituting the values of y_1, y_2 and y in the LHS we get

LHS $= [2ab \cos bx + (a^2 - b^2) \sin bx] e^{ax} - 2a(b \cos bx + a \sin bx) e^{ax}$

$+ (a^2 + b^2) e^{ax}. \sin bx = 0$

$\Rightarrow e^{ax}[2ab \cos bx + (a^2 - b^2) \sin bx - 2ab \cos bx - 2a^2 \sin bx$

$+ (a^2 + b^2) \sin bx] = 0$

$\Rightarrow e^{ax}\{a^2 - b^2 - 2a^2 + a^2 + b^2\} \sin bx = 0$

$\Rightarrow e^{ax}\{a^2 - b^2 - 2a^2 + a^2 + b^2\} \sin bx = 0$ this is the reqd. solution. 4

Q2 (b) **Find the equation of the tangent to the curve** $y^2 = 3 - 5x$ **parallel to the lines 5x – 4y + 13 = 0**

Answer

2. b. Given eqn. is $y^2 = 3-5x$

Differentiating the eqn. of the above curve, w.r.t. x,

we get, $2y\frac{dy}{dx} = 0-5$

$\therefore \frac{dy}{dx} = -\frac{5}{2y}$

Since the tangent is parallel to the line $5x-4y+13=0$

slope of the tangent and the line are equal.

Hence slope of the tangent $= \frac{5}{4}$

$\therefore -\frac{5}{2y} = \frac{5}{4}$ i.e $2y = -4$, i.e $y = -2$ 2

Substituting $y=-2$ in $y^2 = 3-5x$, we obtain $(-2)^2 = 3-5x$

$5x = -1$ or $x = -\frac{1}{5}$ 2

Hence $(-\frac{1}{5}, -2)$ is a point on the line whose the tangent is parallel to the given line.

Required eqn. of the tangent is

$y-(-2) = -\frac{5}{4}\left[x-\left(-\frac{1}{5}\right)\right]$ 4

that is, $5x-4y-7=0$ Ans

Q3 (a) **Evaluate** $\int e^{2x}.\sin 3x dx$

Answer

Q3.a. Soln.

Let $I = \int e^{2x} \cdot \sin 3x \, dx$, then

$$I = \int \underset{\mathrm{I}}{e^{2x}} \cdot \underset{\mathrm{II}}{\sin 3x} \, dx$$

2

$$\Rightarrow I = e^{2x} \cdot \left(-\frac{\cos 3x}{3}\right) - \int 2e^{2x}\left(-\frac{\cos 3x}{3}\right) dx$$ 2

$$\Rightarrow I = -\frac{1}{3} e^{2x} \cos 3x + \frac{2}{3} \int e^{2x} \cdot \cos 3x \, dx$$

$$\Rightarrow I = -\frac{1}{3} e^{2x} + \frac{2}{3} \left[e^{2x} \cdot \frac{\sin 3x}{3} - \int 2 e^{2x} \frac{\sin 3x}{3} dx \right)$$

$$\Rightarrow I = -\frac{1}{3} e^{2x} \cdot \cos 3x + \frac{2}{9} e^{2x} \cdot \sin 3x - \frac{4}{9} \int e^{2x} \cdot \sin 3x \, dx$$ 2

$$\Rightarrow I = -\frac{1}{3} e^{3x} \cdot \cos 3x + \frac{2}{9} e^{2x} \cdot \sin 3x - \frac{4}{9} I$$

$$\Rightarrow \frac{13}{9} I = \frac{e^{2x}}{9} (2 \sin 3x - 3 \cos 3x)$$ 2

$$\Rightarrow I = \frac{e^{2x}}{13} (2 \sin 3x - 3 \cos 3x) + C \quad \text{Ans.}$$

Q3 (b) Evaluate $\displaystyle\int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx$

Answer

b. We have, $I = \int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$

$\Rightarrow I = 5\int_1^2 \frac{x^2}{x^2+4x+3}\,dx + 5\int_1^2 \left(1 - \frac{4x+3}{x^2+4x+3}\right)dx$ 2

$\Rightarrow I = 5\int_1^2 1\cdot dx - 5\int_1^2 \frac{4x+3}{x^2+4x+3}\,dx$

$= 5\int_1^2 1\cdot dx - 5\int_1^2 \frac{2(2x+4)-5}{x^2+4x+3}\,dx$ 2

$\Rightarrow I = 5\int_1^2 1\cdot dx - 5\left[\int_1^2 \left\{\frac{2(2x+4)}{x^2+4x+3} - \frac{5}{x^2+4x+3}\right\}dx\right]$

$\Rightarrow I = 5\int_1^2 1\cdot dx - 10\int_1^2 \frac{2x+4}{x^2+4x+3}\,dx + 25\int_1^2 \frac{1}{x^2+4x+3}\,dx$

$\Rightarrow I = 5\int_1^2 1\cdot dx - 10\int_1^2 \frac{2x+4}{x^2+4x+3}\,dx + 25\int_1^2 \frac{1}{(x+2)^2-(1)^2}\,dx$ 2

$\Rightarrow I = 5\left[x\right]_1^2 - 10\left[\log(x^2+4x+3)\right]_1^2 + 25\cdot\frac{1}{2(1)}\left[\log\left|\frac{x+2-1}{x+2+1}\right|\right]_1^2$

$\Rightarrow I = 5(2-1) - 10\left[\log 15 - \log 8\right] + \frac{25}{2}\left[\log\frac{3}{5} - \log\frac{2}{4}\right]$

$\Rightarrow I = 5 - 10\log\left(\frac{15}{8}\right) + \frac{25}{2}\log\left(\frac{3}{5}\times\frac{4}{2}\right)$

$\Rightarrow I = 5 - 10\log\frac{15}{8} + \frac{25}{2}\log\frac{6}{5}$ Ans 2

Q4 (a) **Let A =** $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}, C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ **find a matrix D such that CD – AB = 0**

Answer

Q.4. a. Soln:

Let $D = \begin{bmatrix} a & b \\ x & y \end{bmatrix}$, then $CD - AB = 0$

$\Rightarrow CD = AB$

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & b \\ x & y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ 2

$\Rightarrow \begin{bmatrix} 2a+5x & 2b+5y \\ 3a+8x & 3b+8y \end{bmatrix} = \begin{bmatrix} 10-7 & 4-4 \\ 15+28 & 6+16 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+5x & 2b+5y \\ 3a+8x & 3b+8y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ 2

$\Rightarrow 2a+5x = 3,\ 3a+8x = 43,\ 2b+5y = 0$

and $3b+8y = 22$

Solving $2a+5x=3$ and $3a+8x = 43$ 2

we get, $a = -191$ and $x = 77$

Solving $2b+5y=0$ and $3b+8y=22$, we get

$b = -110$ and $y = 44$

$\therefore\ D = \begin{bmatrix} a & b \\ x & y \end{bmatrix} = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$ 2 Ans: $\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$

Q4 (b) Using Cramer's rule, solve the following system of liner equations,

(a + b) x – (a – b) y = 4ab

(a – b) x + (a + b) y = 2 $(a^2 - b^2)$

Answer

b. We have,

$$D = \begin{vmatrix} a+b & -(a-b) \\ a-b & a+b \end{vmatrix} = (a+b)^2 + (a-b)^2 = 2(a^2+b^2) \neq 0$$

So, the given system of eqn. has a unique solution 2

Now, $D_1 \begin{vmatrix} 4ab & -(a-b) \\ 2(a^2-b^2) & (a+b) \end{vmatrix}$

$$\Rightarrow D_1 = 2(a+b)\begin{vmatrix} 2ab & -(a-b) \\ a-b & 1 \end{vmatrix}$$ [Taking 2 common from C_1 and $(a+b)$ from R_2]

$$\Rightarrow D_1 = 2(a+b)\{2ab + (a-b)^2\}$$

$$\Rightarrow D_1 = 2(a+b)(a^2+b^2)$$ 2

and, $D_2 = \begin{vmatrix} a+b & 4ab \\ a-b & 2(a^2-b^2) \end{vmatrix}$

$$\Rightarrow D_2 = 2(a-b)\begin{vmatrix} a+b & 2ab \\ 1 & (a+b) \end{vmatrix}$$ [Taking $(a-b)$ common from R_2 and 2 from C_2]

$$\Rightarrow D_2 = 2(a-b)\{(a+b)^2 - 2ab\} = 2(a-b)(a^2+b^2)$$ 2

by Cramer's rule, we have,

$$x = \frac{D_1}{D} = \frac{2(a+b)(a^2+b^2)}{2(a^2+b^2)} = (a+b)$$

$$\text{and } y = \frac{D_2}{D} = \frac{2(a-b)(a^2+b^2)}{2(a^2+b^2)} = (a-b)$$

} Ans. 2

Q5 (a) Solve the differential equation (x + y) dy + (x – y) dx = 0 given that

y = 1 when x = 1

Answer

11)

Q.5.a. The given differential eqn. is

$$(x+y)dy + (x-y)dx = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x-y}{x+y}$$

2

$$\Rightarrow \frac{dy}{dx} = \frac{y-x}{x+y} \quad \text{—— (i)}$$

Since each of the functions $y-x$ and $x+y$ is a homogeneous function of degree 1. Therefore, equation (i) is a homogeneous diff. eqn., Putting $y = vx$ and $\frac{dy}{dx} = v + \frac{dv}{dx}$ in (i), we get

$$v + x\frac{dv}{dx} = \frac{vx - x}{x + vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v-1}{v+1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-1}{v+1} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\cancel{v}-1-v^2-\cancel{v}}{}$$

2

$$\Rightarrow x\frac{dv}{dx} = -\left(\frac{v^2+1}{v+1}\right)$$

$$\Rightarrow \frac{v+1}{v^2+1}dv = -\frac{dx}{x}, \quad x \neq 0 \text{ (by separating the variables)}$$

$$\Rightarrow \int\frac{v+1}{v^2+1}dv = -\int\frac{dx}{x} \quad [\text{Integrating both sides}]$$

$$\Rightarrow \int\frac{v}{v^2+1}dv + \int\frac{1}{v^2+1}dv = -\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\int\frac{2v}{v^2+1}dv + \int\frac{1}{v^2+1^2}dv = -\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\log(v^2+1) + \tan^{-1}v = -\log|x| + c$$

$$\Rightarrow \log(v^2+1) + 2\log|x| + 2\tan^{-1}v = 2c$$

$$\Rightarrow \log(v^2+1) + \log x^2 + 2\tan^{-1}v = k, \text{ where } k = 2c$$

$$\Rightarrow \log\{(v^2+1)x^2\} + 2\tan^{-1}v = k$$

$$\Rightarrow \log\{(y^2/x^2+1)x^2\} + \tan^{-1}\left(\frac{y}{x}\right) = k \quad \left[\because v = \frac{y}{x}\right]$$

2

$$\Rightarrow \log(x^2+y^2) + 2\tan^{-1}\left(\frac{y}{x}\right) = k \quad \text{—— (ii)}$$

It is given that $y = 1$, when $x = 1$, Putting $x=1, y=1$ in (ii)

we get, $\log 2 + 2\tan^{-1}(1) = k \Rightarrow k = \log 2 + 2(\pi/4)$

$$= (\pi/2) + \log 2$$

Substituting the value of k in (ii), we get

$\log(x^2+y^2) + 2\tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{2} + \log 2$ as reqd. soln. 2

Q5 (b) Solve the equation cos x(1 + cos y) dx – sin y (1 + sin x) dy = 0

Answer

Q.5.b. We have, $\cos x(1+\cos y)\,dx - \sin y(1+\sin x)\,dy = 0$ 11

$\Rightarrow \dfrac{\cos x}{1+\sin x}\,dx - \dfrac{\sin y}{1+\cos y}\,dy = 0$ 2

$\Rightarrow \displaystyle\int \frac{\cos x}{1+\sin x}\,dx - \int \frac{-\sin y}{1+\cos y}\,dy = 0$ [Integrating both Sides]

$\Rightarrow \log|1+\sin x| + \log|1+\cos y| = \log c$ 2

$\Rightarrow \log\{|1+\sin x|\,|1+\cos y|\} = \log c$

$\Rightarrow |1+\sin x|\,|1+\cos y| = c$ 2

$\Rightarrow (1+\sin x)(1+\cos y) = c$, which is the reqd. Soln. 2

Q6 (a) **Prove that the coefficient of x^n is expansion of $\left(\dfrac{1+x}{1-x}\right)^2$ is 4n**

Answer

Q.6.a. We have, $\left(\dfrac{1+x}{1-x}\right)^2 = (1+x)^2(1-x)^{-2}$ 2

$\Rightarrow \left(\dfrac{1+x}{1-x}\right)^2 = (1+2x+x^2)\{1+2x+3x^2+4x^3+\cdots+(n-1)x^{n-2}$
$+ nx^{n-1} + (n-1)x^n + \cdots\}$ 2

$\therefore$ Coefficient of x^n in $\left(\dfrac{1+x}{1-x}\right)^2 = 1\times(n+1)+2\times(n)+1\times(n-1)$ 2

$\Rightarrow$ Coefficient x^n in $\left(\dfrac{1+x}{1-x}\right)^n = n+1+2n+n-1 = 4n$ 2

Hence proved.

Q6 (b) **Let S_n denote the sum of the first n terms of an A.P. If $S_{2n} = 3S_n$, then prove that**

$$\frac{S_{3n}}{S_n} = 6$$

Answer

b. Let a be the first term and d the common difference of the given A.P. Then,

$$S_{2n} = 3S_n$$

$$\Rightarrow \frac{2n}{2}[a+(2n-1)d] = 3\times\frac{n}{2}[a+(n-1)d] \quad 2$$

$$\Rightarrow 2[a+(2n-1)d] = 3[a(n-1)d]$$

$$\Rightarrow 2a - (3n-3-4n+2)d = 0$$

$$\Rightarrow 2a - (n+1)d = 0 \quad 2$$

$$\Rightarrow 2a = (n+1)d \quad \text{(i)}$$

Now,

$$\frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{n}{2}[2a+(n-1)d]}$$

$$\Rightarrow \frac{S_{3n}}{S_n} = \frac{3[(n+1)d+(3n-1)d]}{[(n+1)d+(n-1)d]} \quad \text{[using (i)]} \quad 2$$

$$\Rightarrow \frac{S_{3n}}{S_n} = \frac{12nd}{2nd} = 6 \quad \text{Hence proved} \quad 2$$

Q7 (a) If A, B, C are the angles of a triangle, then prove that,

sin2A + sin2B + sin2C = 4 sinA.sinB.sinC

Answer

Q.7-a. Soln:

We have, $LHS = \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A+2B}{2}\right) . \cos\left(\frac{2A-2B}{2}\right) + \sin 2C$ 2

$= 2\sin(A+B) . \cos(A-B) + \sin 2C$

$= 2\sin(\pi - C) . \cos(A-B) + 2\sin C . \cos C$ 2

$= 2\sin C . \cos(A-B) + 2\sin C . \cos C$ | $\because A+B+C=\pi$

$= 2\sin C\{\cos(A-B) + \cos C\}$ | $\therefore A+B=\pi-C$

$= 2\sin C\{\cos(A-B) - \cos(A+B)\}$ 2 | $\cos(A+B) = \cos(\pi-C) = -\cos C$

$= 2\sin C\{(\cos A . \cos B + \sin A . \sin B) - (\cos A . \cos B - \sin A . \sin B)\}$

$= 2\sin C(2\sin A . \sin B)$

$= 4\sin A . \sin B . \sin C$ 2

$= RHS$. proved

Q7 (b) **Prove that, cos20º.cos60º.cos40º.cos80º =** $\frac{1}{16}$

Answer

b. $LHS = \cos 20^\circ . \cos 40^\circ . \cos 60^\circ . \cos 80^\circ$

$= \frac{1}{2}(\cos 20^\circ . \cos 40^\circ . \cos 80^\circ)$ 2

$= \frac{1}{2}[\cos A . \cos 2A . \cos 2^2 A]$, where $A = 20$

$= \frac{1}{2}\left(\frac{\sin 2^3 A}{2^3 \sin A}\right) = \frac{1}{2^4} \frac{\sin 8A}{\sin A}$ 2

$= \frac{1}{2^4} . \frac{\sin 160^\circ}{\sin 20^\circ} = \frac{1}{2^4} . \frac{\sin(180^\circ - 20)}{\sin 20^\circ}$ 2

$= \frac{1}{2^4} . \frac{\sin 20^\circ}{\sin 20^\circ} = \frac{1}{2^4} = \frac{1}{16} = RHS$. 2

Q8 (a) **Find the equation of the two straight lines through (7, 9) and making an angle of 60º with the line** $x - \sqrt{3}y - 2\sqrt{3} = 0$

Answer

Q.8.a. Soln:

We know that the equations of two straight lines which pass through a point (x_1, y_1) and made a given angle α with the given straight line $y = mx + c$ are,

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m \tan\alpha}(x - x_1)$$

Here, $x_1 = 7, y = 9, \alpha = 60^\circ$ and m = (slope of the line $x - \sqrt{3}y - 2\sqrt{3} = 0$) $= \frac{1}{\sqrt{3}}$ 2

So, the eqn. of the required lines are

$$y - 9 = \frac{\frac{1}{\sqrt{3}} + \tan 60^\circ}{1 - \frac{1}{\sqrt{3}}\tan 60^\circ}(x-7) \quad \text{and} \quad y - 9 = \frac{\frac{1}{\sqrt{3}} - \tan 60^\circ}{1 + \frac{1}{\sqrt{3}}\tan 60^\circ}(x-7)$$ 2

or $(y-9)\left(1 - \frac{1}{\sqrt{3}}\tan 60^\circ\right) = \left(\frac{1}{\sqrt{3}} + \tan 60^\circ\right)(x-7)$ 2

and, $(y-9)\left(1 + \frac{1}{\sqrt{3}}\tan 60^\circ\right) = \left(\frac{1}{\sqrt{3}} - \tan 60^\circ\right)(x-7)$

or $0 = \left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)(x-7) \Rightarrow x + \sqrt{3}y = 7 + 9\sqrt{3}$

Hence, the required line are $x = 7$ and $x + \sqrt{3}y = 7 + 9$

2 An

Q8 (b) Find the area of the triangle formed by the lines y =x, y = 2x and y = 3x +4

Answer

Q.8.b. Soln: The given equations are,

$y = x$ —— (i), $y = 2x$ —— (ii) and $y = 3x + 4$ —— (iii)

Suppose the eqns (i), (ii) and (iii) represent the sides AB, BC, and CA respectively of a triangle ABC,

Solving (i) and (ii), we get: $x = 0$ and $y = 0$ 2

Thus, AB and BC intersect at B (0,0)

Solving (ii) and (iii), we obtain: $x = -4, y = -8$ 2

Thus, BC and CA intersect at $C = (-4, -8)$

Solving (iii) and (i), we get, we get: $x = -2$, and $y = -2$

So, CA and AB intersect at $A(-2, -2)$ 2

Thus, the vertices of the triangle ABC are:

A (-2,2), B (0,0) and C (-4,-8)

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} -2 & -2 & 1 \\ 0 & 0 & 1 \\ -4 & 8 & 1 \end{vmatrix} = 4 \text{ Sq. units.}$$ 2

Q9 (a) Find the equation of the circle passing through the point (1, -2) & (4, -3) and which has its centre on the strength line $3x + 4y = 7$

Answer

Q.9.a. Let the eqn of the circle be

$x^2+y^2+2gx+2fy+c=0$ —— (i)

Since it passes through $(1,-2)$ and $(+4,-3)$

$\therefore\ 5+2g-4f+c=0$ —— (ii) 2

$25+8g-6f+c=0$ —— (iii)

Centre of the circle is $(-g,-f)$ and it lies on the

$3x+4y=7$ —— (iv) 2

$\therefore\ -3g-4f=7$

Solving (ii), (iii) and (iv)

we get $g=-\frac{47}{15}$, $f=\frac{3}{5}$ and $c=\frac{11}{3}$ 2

$\therefore$ The required equation of the circle is

$x^2+y^2-\frac{94}{15}x+\frac{6}{5}y+\frac{11}{3}=0$ Ans. 2

or $15(x^2+y^2)-94x+18y+55=0$

Q9 (b) Find the focus, vertex, axis, latus-rectum and directrix of the parabola $x^2+4x+2y=0$

Answer

Q.9.b. The equation of the parabola is
$x^2+4x+2y=0$ or $x^2+4x=-2y$
Adding 4 on both sides,
$x^2+4x+4=-2y+4$ or $(x+2)^2=-2(y-2)$ — (i) 2
Shifting the origin to the point $(-2, 2)$ i.e.

Let $X = x+2$ and $Y = y-2$ — (ii)
Then (i) becomes,
$X^2=-2Y$ or $X^2=-4\cdot\frac{1}{2}Y$ — (iii) $(X^2=-4aY)$
comparing it with $X^2=-4aY$
we have $a=\frac{1}{2}$
$\therefore$ Focus is $X=0$, $Y=-a=-\frac{1}{2}$
or $x+2=0$, $y-2=-\frac{1}{2}$ from (ii)
$\therefore$ $x=-2$, $y=2-\frac{1}{2}=\frac{3}{2}$
$\therefore$ Focus is $(-2,\frac{3}{2})$ — Ans. 2
Vertex is $(-2,2)$ or $x+2=0$, $y-2=0$ or $x=-2, y=2$
$\therefore$ Vertex is $(-2,2)$ —— Ans.
Latus-rectum $=4a=4\cdot\frac{1}{2}=2$ — Ans. 2
Equation of Axis is $X=0$ or $x+2=0$ Ans
Equation of directrix is $Y-a=0$ or $y-2-\frac{1}{2}=0$
or $2y-5=0$ Ans 2

<u>**Text Book**</u>

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