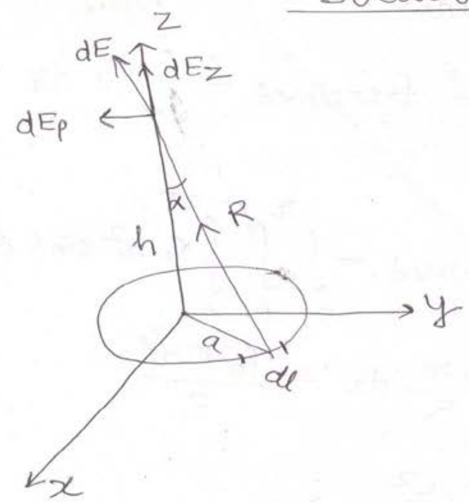
Q.2 a. A circular ring of radius a carries a uniform charge ρ_L C/m and is placed on the xy – plane with axis the same as the z – axis. Show that

$$\vec{E}(0,0,h) = \frac{\rho_L\, ah}{2\epsilon_0\left[h^2 + a^2\right]^{3/2}}\hat{a}_z$$

Answer:

Solution

2a



Consider the system shown in Fig:

We can calculate $\vec{E}$ by

$$\vec{E} = \int_L \frac{\rho_L\, dl}{4\pi\epsilon_0 R^2}\, a_R$$

In this case

$$dl = a\, d\phi$$

$$\vec{R} = a(-a_\rho) + h\, a_z$$

$$R = |\vec{R}| = (a^2+h^2)^{\frac{1}{2}}$$

and $a_R = \dfrac{\vec{R}}{R}$.

$$\frac{a_R}{R^2} = \frac{\vec{R}}{|\vec{R}|^3} = \frac{-a\, a_\rho + h\, a_z}{(a^2+h^2)^{3/2}}$$

so $$\vec{E} = \frac{\rho_L}{4\pi\epsilon_0}\int_{\phi=0}^{2\pi} \frac{(-a\, a_\rho + h\, a_z)}{(a^2+h^2)^{3/2}}\, a\, d\phi$$

by symmetry the contributions along a_ρ add up to zero. So

Thus we have only z component

$$\vec{E} = \frac{\rho_L\, a\, h\, a_z}{4\pi\epsilon_0 (a^2+h^2)^{\frac{3}{2}}}\int_0^{2\pi} d\phi$$

$$= \frac{\rho_L\, a\, h\, a_z}{2\epsilon_0 (a^2+h^2)^{3/2}}$$

b. **A charge distribution with spherical symmetry has density** $\rho_V = \begin{cases} \dfrac{\rho_0 r}{R} & 0 \le r \le R \\ 0 & r > R \end{cases}$. **Determine** $\vec{E}$ **everywhere**

Answer:

(2b) using Gauss's law

$$\varepsilon_0 \oint \vec{E} \cdot \vec{ds} = Q_{enclosed} = \int_V \rho_V \, dV$$

(a) for $r < R$

$$\varepsilon_0 E_r 4\pi r^2 = Q_{enclosed} = \int_0^r \int_0^\pi \int_0^{2\pi} \rho_V r^2 \sin\theta \, d\phi \, d\theta \, dr$$

$$= \int_0^r 4\pi r^2 \frac{\rho_0 r}{R} dr = \frac{\rho_0 \pi r^4}{R}$$

or $$\vec{E} = \frac{\rho_0 r^2}{4\varepsilon_0 R} a_r$$

(b) for $r > R$.

$$\varepsilon_0 E_r 4\pi r^2 = Q_{enclosed} = \int_0^r \int_0^\pi \int_0^{2\pi} \rho_V r^2 \sin\theta \, d\phi \, d\theta \, dr$$

$$= \int_0^R \frac{\rho_0 r}{R} 4\pi r^2 dr + \int_R^r 0 \cdot 4\pi r^2 dr$$

$$= \pi \rho_0 R^3.$$

$$\vec{E} = \frac{\rho_0 R^3}{4\varepsilon_0 r^2} a_r$$

Q.3 a. Discuss the boundary condition at a conductor and free space boundary in electrostatics.

Answer: Article 5.4, Page Number 124-125 of Text Book I

b. Given the potential V =
(i) Find the electric flux density D at (2, $\pi/2$, 0)
(ii) Calculate the work done in moving a 10 μC charge from the point A (1, 30^0, 120^0) to B (4, 90^0, 60^0).

Answer:

3b (i) $D = \varepsilon_0 E$

$$E = -\nabla V = -\left[\frac{\partial V}{\partial r} a_r + \frac{1}{r}\frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} a_\phi\right]$$

$$= \frac{20}{r^3}\sin\theta\cos\phi\, a_r - \frac{10}{r^3}\cos\theta\cos\phi\, a_\theta + \frac{10}{r^3}\sin\phi\, a_\phi$$

at $(2, \pi/2, 0)$

$$D = \varepsilon_0 E = \varepsilon_0\left(\frac{20}{8} a_r - 0\, a_\theta - 0\, a_\phi\right)$$

$$= 2.5\,\varepsilon_0\, a_r \ \mathrm{C/m^2}$$

$$= 22.1\, a_r \ \mathrm{pC/m^2}$$

(ii) $$W = -Q\int_A^B E\cdot dl = Q V_{AB}$$

$$= Q(V_B - V_A)$$

$$= 10\left(\frac{10}{16}\sin 90\cos 60 - \frac{10}{1}\sin 30\cos 120\right)\cdot 10^{-6}$$

$$= 28.125\ \mu J$$

Q.4 a. State and derive the uniqueness theorem.

Answer: Article 7.2, Page Number 191-192 of Text Book I

b. In spherical coordinates V = 0 at r = 0.2m and V = 200 at r = 4m. Calculate the potential in various regions. Assume free space between these concentric spherical shells.

Answer:

4b By Laplace equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)=0$$

if θ & ϕ are constant

$$\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)=0$$

or $r^2\frac{dV}{dr}=A=$ constant

so $\frac{dV}{dr}=\frac{A}{r^2}$

$$\int dV=\int\frac{A}{r^2}\,dr$$

or $V=-\frac{A}{r}+B$

at $r=.2$ m $V=0$

$$0=\frac{-A}{.2}+B$$

or $B=\frac{A}{.2}=5A$

at $r=4$m $V=200$

$$200=\frac{-A}{4}+B$$

so $A=38$ & $B=190.5$

so $V=\left(-\frac{38}{r}+190.5\right)$ volts

Q.5 a. Explain the scalar and magnetic potentials with the help of one example of each.

Answer: Article 8.6 of Text Book I

b. Given the volume current density distribution in cylindrical coordinates as.

J(r, ϕ, z) = 0 $\quad$ 0 < r < a

$$= J_0\left(\frac{r}{a}\right)a_z \quad \mathbf{a \le r \le b}$$

= 0 $\quad$ b < r < ∞

Find the magnetic field intensity H in various regions.

Answer:

5b for $0 < r < a$ $\quad J(r, \phi, z) = 0$

using ampere's Law

$$\oint H \cdot dl = \int_S J \cdot dS$$

$$= \int_{r=0}^{a} \int_{\phi=0}^{2\pi} J a_z \cdot r\, dr\, d\phi\, a_z$$

$$H \cdot 2\pi r = 0$$

$$H = 0$$

for $a < r < b$

$$J = J_0\left(\frac{r}{a}\right) a_z$$

$$\oint H \cdot dl = \int_{\phi=0}^{2\pi} \int_{a}^{r} J_0\left(\frac{r}{a}\right) r\, dr\, d\phi$$

$$H \cdot 2\pi r = \frac{J_0}{a} \cdot \frac{r^3}{3}\Big|_a^r \; \phi\Big|_0^{2\pi}$$

$$H = \frac{2\pi J_0}{3a}\left(r^3 - a^3\right) a_\phi$$

at $r = b$

$$H = \frac{J_0}{3ab}\left(b^3 - a^3\right) a_\phi$$

for $b < r < \infty$ $\quad J = 0$

$$\oint H \cdot dl = \int_{\phi=0}^{2\pi} \int_{r=a}^{b} \frac{J_0 r}{a}\, r\, dr\, d$$

$$H = \frac{J_0}{3ar}\left(b^3 - a^3\right) a_\phi$$

Q.6 **b. Given that** H_1 **= -2a$_x$ + 6 a$_y$ + 4 a$_z$ A/m in region y – x – 2 $\leq$ 0 where** $\mu_1 = 5\mu_0$
calculate
(i) M_1 & B_1
(ii) H_2 & B_2 **in region y – x –2** ≥ 0 **where** $\mu_2 = 2\mu_0$

Answer:

(6b) Plane is $f = y - x - 2$; Unit vector normal to the plane is given by.

$$a_n = \frac{\nabla f}{|\nabla f|} = \frac{a_y - a_x}{\sqrt{2}}$$

(a) $M_1 = \chi_{m1} H_1 = (\mu_{r1} - 1) H_1 = (5-1)(-2, 6, 4)$

$= -8a_x + 24a_y + 16a_z$ A/m

$B_1 = \mu_1 H_1 = \mu_0 \mu_{r1} H_1 = -12.57a_x + 37.7a_y + 25.13a_z$ (μWb/m^2)

(b) $H_{1n} = (H_1 \cdot a_n)\, a_n = -4a_x + 4a_y$

but $H_1 = H_{1n} + H_{1t}$

$H_{1t} = H_1 - H_{1n} = (-2, 6, 4) - (-4, 4, 0)$

$= 2a_x + 2a_y + 4a_z$

by boundary conditions.

$H_{2t} = H_{1t} = 2a_x + 2a_y + 4a_z$

$B_{2n} = B_{1n}$ or $\mu_2 H_{2n} = \mu_1 H_{1n}$

$H_{2n} = \frac{\mu_1}{\mu_2} H_{1n} = \frac{5}{2}(-4a_x + 4a_y)$

$= -10a_x + 10a_y$

so $H_2 = H_{2n} + H_{2t} = -8a_x + 12a_y + 4a_z$ A/m

and $B_2 = \mu_2 H_2 = \mu_0 \mu_{r2} H_2$

$= -20.11a_x + 30.16a_y + 10.05a_z$ μWb/m^2

Q.7 **b. A parallel – plate capacitor with plate area of** $5\,\text{cm}^2$ **and plate separation of 3 mm has a voltage 50 sin 1000t volt applied to its plates. Calculate the displacement current. Assume** $\epsilon = 2\epsilon_0$.

Answer:

$$D = \in E = \in \frac{V}{d}$$

$$Jd = \frac{\partial D}{\partial t} = \frac{\in}{d}\frac{dV}{dt}$$

So, $I_d = J_d . S = \frac{\in S}{d} . \frac{dV}{dt} = C\frac{dV}{dt}$

and $I_c = \frac{d\theta}{dt} = S\frac{dP_s}{dt} = S\frac{dD}{dt} = \in S\frac{dE}{dt}$

$$= \frac{\in S}{d} . \frac{dV}{dt} = C\frac{dV}{dt}$$

So, $I_d = 2 . \frac{10^{-9}}{36\pi} . \frac{5\times10^{-4}}{3\times10^{-3}} \times 10^3 \times 50 \cos 1000t$

$= 147.4 \cos 1000t$ nA

c. State and explain Faraday's law and find expression for the emf.

Answer: Article 10.5 of Text Book I

Q.8 Explain the following:

(i) Polarization of waves

Answer: Article 8.1.1, Page Number 208 of Text Book II

(ii) Troposphere scatter propagation

Answer: Article 8.2.4, Page Number 230 of Text Book II

(iii) Skip distance

Answer: Article 9.1.2, Page Number 238 of Text Book II

(iv) Radiation Resistance

Answer: Article 9.2.2, Page Number 241 of Text Book II

Q.9 a. Define the following:

(i) Directive gain **(ii) Resonant Antenna**
(iii) End-fire array **(iv) Horn Antenna**

Answer: Article 9.3.1 to 9.3.3 of Text Book II

b. Explain working principle and constructional features of Helical Antenna, also write its applications.

Answer: Article 9.8 of Text Book II

Text books

1) **Engineering Electromagnetics, W.H. Hayt & J.A. Buck, 7th Edition, Tata Mc Graw Hill, Special Indian Edition 2006**

2) **Electronic Communication Systems, George Kennedy & Bernard Davis, 4th Edition (1999), Tata McGraw Hill Publishing Company Ltd.**