Q.2 b. Write the dynamic equation in respect of the mechanical system given in Fig.1 below, also draw F-V analogous circuit.

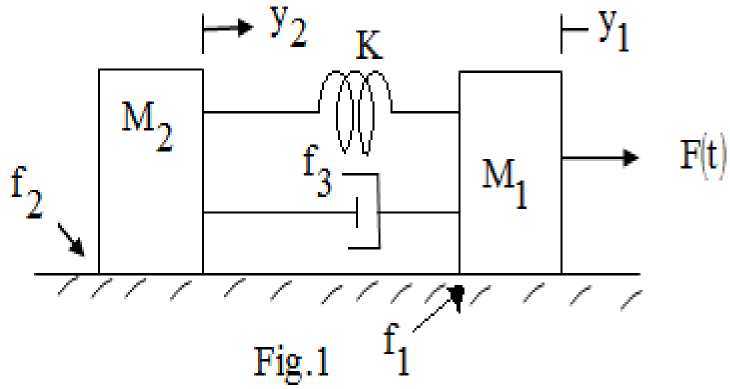
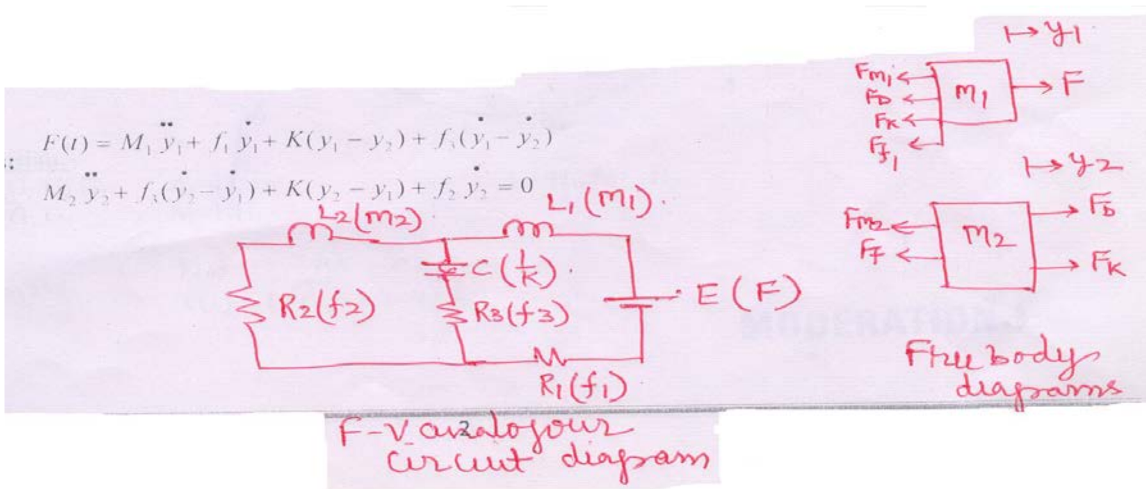


Fig.1

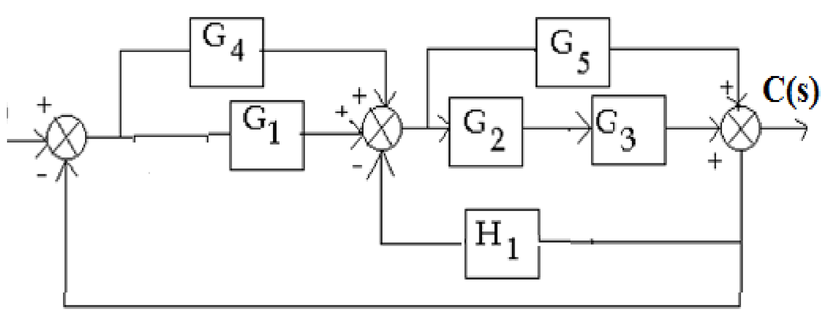
Answer:



$$F(t) = M_1 \ddot{y}_1 + f_1 \dot{y}_1 + K(y_1 - y_2) + f_3(\dot{y}_1 - \dot{y}_2)$$

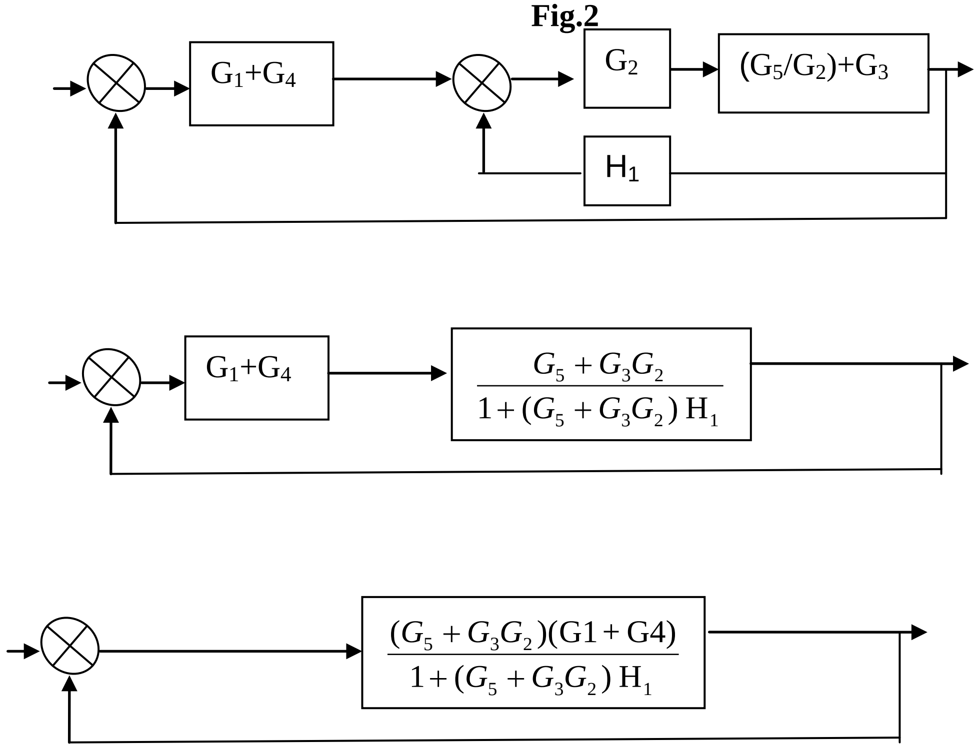
$$M_2 \ddot{y}_2 + f_3(\dot{y}_2 - \dot{y}_1) + K(y_2 - y_1) + f_2 \dot{y}_2 = 0$$

Q.3 a. Determine the transfer function C(s) /R(s) for the block diagram shown in Fig.2 below.



Answer:

Fig.2



b. Reduce the signal flow graph shown in Fig.3 below using Mason's gain formula:

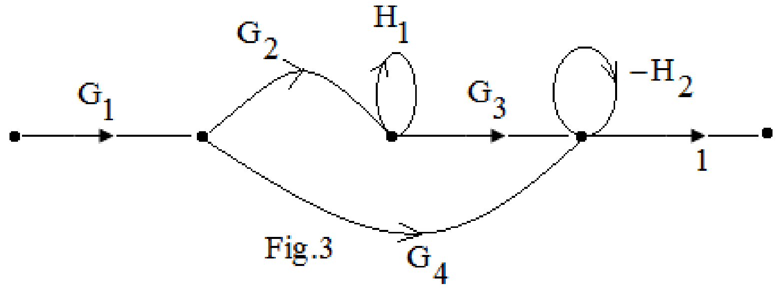


Fig.3

Answer:

$P_1 = G_1\, G_2\, G_3$, $\Delta_1 = 1$ $L_1 = H_1$, $L_2 = -H_2$

$P_1 = G_1\, G_4$, $\Delta_2 = 1 - H_1$

$$\frac{Y(s)}{X(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{1-(L_1+L_2)+(L_1L_2)}$$

Q.5 a. A unity feedback system is characterized by the open loop transfer function

$$G(s) = \frac{1}{s\,(0.5s+1)(0.2s+1)}$$

Determine the steady state error for unit step, unit ramp and unit acceleration input.

Answer:

(i) $K_p = \lim_{s\to 0} G(s)H(s) = \infty$, therefore e_{ss}(unit step)= $\frac{1}{1+K_p} = \frac{1}{\infty} = 0$

(ii) $K_v = \lim_{s\to 0} sG(s)H(s) = \frac{1}{1} = 1$, therefore e_{ss}(unit ramp)= $\frac{1}{K_v} = \frac{1}{1} = 1$

(ii) $K_a = \lim_{s\to 0} s^2G(s)H(s) = 0$, therefore e_{ss}(unit acceleration)=

$\frac{1}{K_a} = \frac{1}{0} = \infty$

b. Using Routh–Hurwitz stability criterion, determine the stability of the following characteristic polynomial:

$$F(s) = s^6 + 4s^5 + 12s^4 + 16s^3 + 41s^2 + 36s + 72$$

Answer:

s^6	1	12	41	72
s^5	4	16	36	
s^4	8	32	72	
s^3	32	64		
s^2	16	72		
s^1	-80			
s^0	72			

As there are two changes of sign in the first column of the Routh array, two roots lie in the RSH of s- plane

Q.6 Sketch the root loci for the system whose open loop transfer function is given by $G(s)H(s) = \frac{K}{s(s+2)(s^2+6s+25)}$

Answer:

1. Root axis exists only between s=0 and -2.
2. Four asymptotes with angles 45^o, 135^o, 225^o and 315^o
3. Centroid is σ_A= -2
4. Breakaway point at s=-0.8981

5. Intersection with imaginary line at s=±j2.5 with K=192.1875
6. Angle of departure from upper complex pole is given by -140.91^{o}.

Q.7 b. Given the open loop transfer function.

$$G(s)H(s) = \frac{20}{s(s+2)(s+10)}$$

Answer:

1. Corner frequencies: 2 and 10
2. Gain cross over frequency = 4.0
3. Phase cross over frequency=4.47
4. GM=21.6dB
5. PM=78^{o}

Sketch the Bode plot of the system and determine the following:
(i) gain margin (ii) phase margin

Q.9 b. The vector matrix differential equation describing the dynamics of the system is given by $\dot{X} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} X$.

Obtain the solution of the above equation if $X(0) = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$

Answer:

STM= $\Phi(t) = \begin{bmatrix} 3e^{-2t} - 2e^{-3t} & e^{-2t} - e^{-3t} \\ -6(e^{-2t} - 2e^{-3t}) & -2e^{-2t} + 3e^{-3t} \end{bmatrix}$

Therefore,

$$X(t) = \begin{bmatrix} 3e^{-2t} - 2e^{-3t} & e^{-2t} - e^{-3t} \\ -6(e^{-2t} - 2e^{-3t}) & -2e^{-2t} + 3e^{-3t} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X(t) = \begin{bmatrix} e^{-2t} - e^{-3t} \\ -2e^{-2t} + 3e^{-3t} \end{bmatrix}$$

Text Book

Control Systems Engineering, I.J. Nagrath and M. Gopal, 5th Edition (2007 New Age International Pvt. Ltd.)