Q2 (a) Show that the real and imaginary parts of the function w = log z satisfy the Cauchy Riemann equations when z is not zero.

Answer

a). Here $\omega = \log z = \log(x+iy)$

Put $x = r\cos\theta$, $y = r\sin\theta$

and $\omega = u + iv$

$\therefore u + iv = \log(r\cos\theta + ir\sin\theta)$

$= \log r(\cos\theta + i\sin\theta)$

$= \log r e^{i\theta}$

$= \log r + \log e^{i\theta}$

$= \log r + i\theta$

$= \log\sqrt{x^2+y^2} + i\tan^{-1}\frac{y}{x}$

$\therefore u = \log\sqrt{x^2+y^2}$ & $v = \tan^{-1}\frac{y}{x}$

On differentiating, we get –

$$\frac{\partial u}{\partial x} = \frac{1}{2}\frac{1}{(x^2+y^2)}2x = \frac{x}{x^2+y^2} \longrightarrow (1)$$

$$\frac{\partial v}{\partial y} = \frac{1}{1+\frac{y^2}{x^2}}\left(\frac{1}{x}\right) = \frac{x}{x^2+y^2} \longrightarrow (2)$$

From (1) & (2), we have –

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \longrightarrow (3)$$

Again differentiating u & v, we get –

$$\frac{\partial u}{\partial y} = \frac{1}{2}\frac{1}{x^2+y^2}2y = \frac{y}{x^2+y^2} \longrightarrow (4)$$

& $$\frac{\partial v}{\partial x} = \frac{1}{1+\frac{y^2}{x^2}}\left(-\frac{y}{x^2}\right) = -\frac{y}{x^2+y^2} \longrightarrow (5)$$

From (4) & (5), we get –

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \longrightarrow (6)$$

From (3) & (6), $\omega = \log z$ satisfy Cauchy Riemann equations when z is not zero.

Now $x^2 + y^2 = 0 \Rightarrow x = 0 = y$

or $x + iy = 0 \Rightarrow z = 0$

$\therefore$ Cauchy-Riemann eqns are not satisfied when $x^2+y^2=0$ i.e. when $z = 0$

(Proved)

Q2 (b) Find the bilinear transformation which maps z = 1, i, -1 onto w = i, 0, -i respectively. **

Answer

(2) b). The bilinear transformation which maps $z = z_1, z_2, z_3$ into $w = w_1, w_2, w_3$ respectively is given by —

$$\frac{(w-w_1)(w_2-w_3)}{(w_1-w_2)(w_3-w)} = \frac{(z-z_1)(z_2-z_3)}{(z_1-z_2)(z_3-z)}$$

Substituting $z_1 = 1, z_2 = i, z_3 = -1$; $w_1 = i, w_2 = 0, w_3 = -i$, we get —

$$\frac{(w-i)(0+i)}{(i-0)(-i-w)} = \frac{(z-1)(i+1)}{(1-i)(-1-z)}$$

$$-\frac{w-i}{w+i} = -\frac{(1+i)(z-1)}{(1-i)(z+1)} = -\frac{(1+i)^2(z-1)}{(1-i)(1+i)(z+1)}$$

or $$\frac{w-i}{w+i} = \frac{2i}{2}\,\frac{z-1}{z+1} = i\,\frac{z-1}{z+1} = \frac{iz-i}{z+1}$$

$$\frac{(w-i)+(w+i)}{(w+i)-(w-i)} = \frac{(iz-i)+(z+1)}{(z+1)-(iz-i)}$$

$$\frac{w}{i} = \frac{z(1+i)+(1-i)}{z(1-i)+(1+i)}$$

or $$w = i\,\frac{1+i}{1-i}\;\frac{z+\frac{1-i}{1+i}}{z+\frac{1+i}{1-i}}$$

$$w = i\cdot i\,\frac{z-i}{z+i}$$

or $$\boxed{w = -\frac{z-i}{z+i}}$$

Ans

Q3 (a) Evaluate the following integral using Cauchy integral formula $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$ **where C is the circle** $|z| = \frac{3}{2}$**.**

Answer

a). Let $I = \int_C \frac{4-3z}{z(z-1)(z-2)}\,dz$

$$C : |z| = \frac{3}{2}$$

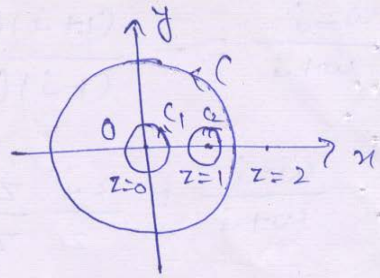
Poles of the integrand are given by —

$$z(z-1)(z-2) = 0 \Rightarrow z = 0, 1, 2$$

The integrand has 3 simple poles at $z = 0, 1, 2$

The given circle $|z| = \frac{3}{2}$ with centre at $z = 0$ and radius $\frac{3}{2}$ encloses two poles $z = 0$ and $z = 1$

$$\therefore I = \int_C \frac{4-3z}{z(z-1)(z-2)}\,dz$$



$$= \int_{C_1} \frac{\frac{4-3z}{(z-1)(z-2)}}{z}\,dz + \int_{C_2} \frac{\frac{4-3z}{z(z-2)}}{z-1}\,dz$$

$$= 2\pi i \left[\frac{4-3z}{(z-1)(z-2)}\right]_{z=0} + 2\pi i \left[\frac{4-3z}{z(z-2)}\right]_{z=1}$$, by Cauchy Integral formula

$$= 2\pi i \cdot \frac{4}{(-1)(-2)} + 2\pi i \,\frac{4-3}{1\,(1-2)}$$

$$= 2\pi i\,(2-1)$$

$$= 2\pi i$$

Ans

Q3 (b) Obtain the Taylor's or Laurent's series which represents the function

$$f(z) = \frac{1}{(1+z^2)(z+2)}$$

when-

(i) $1<|z|<2$ **(ii)** $|z|>2$

Answer

Q3 b). Here–

$$f(z) = \frac{1}{(1+z^2)(z+2)} = \frac{-\frac{z}{5}+\frac{2}{5}}{1+z^2} + \frac{\frac{1}{5}}{z+2}$$

$$= -\frac{1}{5}\frac{z-2}{1+z^2} + \frac{1}{5}\frac{1}{z+2}$$

(i) Here $1<|z|<2$

$$\therefore f(z) = -\frac{1}{5}\frac{1}{z^2}\frac{z-2}{1+\frac{1}{z^2}} + \frac{1}{5}\cdot\frac{1}{2}\frac{1}{1+\frac{z}{2}}$$

, since $\left|\frac{1}{z^2}\right|$ and $\left|\frac{z}{2}\right|$ are less than 1.

$$\therefore f(z) = -\frac{1}{5z^2}(z-2)\left(1+\frac{1}{z^2}\right)^{-1} + \frac{1}{10}\left(1+\frac{z}{2}\right)^{-1}$$

$$= -\frac{1}{5}\left(\frac{1}{z} - \frac{2}{z^2}\right)\left(1 - \frac{1}{z^2} + \frac{1}{z^4} - \frac{1}{z^6} + \cdots\right)$$

$$+ \frac{1}{10}\left(1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + \cdots\right)$$

$$= \frac{1}{5}\left[-\frac{1}{z} + \frac{1}{z^3} - \frac{1}{z^5} + \frac{1}{z^7} + \cdots + \frac{2}{z^2} - \frac{2}{z^4} + \frac{6}{z^6} - \frac{2}{z^8} + \cdots + \frac{1}{2} - \frac{z}{4} + \frac{z^2}{8} - \frac{z^3}{16} - \cdots\right]$$

$$f(z) = \frac{1}{5}\left[\cdots - 2z^{-8} + z^{-7} + 2z^{-6} - z^{-5} - 2z^{-4} + z^{-3} + 2z^{-2} - z^{-1} + \frac{1}{2} - \frac{z}{4} + \frac{z^2}{8} - \frac{z^3}{16} \cdots\right]$$

ii) Here $|z| > 2$

$$f(z) = -\frac{1}{5}\frac{z-2}{1+z^2} + \frac{1}{5}\frac{1}{z+2}$$

$$= -\frac{1}{5}\frac{1}{z^2}(z-2)\left(1+\frac{1}{z^2}\right)^{-1} + \frac{1}{5z}\left(1+\frac{2}{z}\right)^{-1}$$

$$= \frac{1}{5}\left(-\frac{1}{z} + \frac{2}{z^2}\right)\left(1 - \frac{1}{z^2} + \frac{1}{z^4} - \frac{1}{z^6} + \cdots\right)$$

$$+ \frac{1}{5z}\left(1 - \frac{2}{z} + \frac{4}{z^2} - \frac{8}{z^3} + \frac{16}{z^4} + \cdots\right)$$

$$= \frac{1}{5}\left(-\frac{1}{z} + \frac{1}{z^3} - \frac{1}{z^5} + \frac{1}{z^7} + \cdots + \frac{2}{z^2} - \frac{2}{z^4} - \frac{2}{z^6} + \frac{2}{z^8} + \cdots + \frac{1}{z} - \frac{2}{z^2} + \frac{4}{z^3} - \frac{8}{z^4} + \cdots\right]$$

Ans

Q4 (a) Find the directional derivative of the divergence o $f(x,y,z) = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$ **at the point (2, 1, 2) in the direction of the outer normal to the sphere,** $x^2 + y^2 + z^2 = 9$

Answer

(4) a). Given– $f(x,y,z) = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$

Divergence of $f(x,y,z) = \nabla \cdot f$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot (xy\hat{i} + xy^2\hat{j} + z^2\hat{k})$$

$$= y + 2xy + 2z$$

Directional derivative of divergence of $(y + 2xy + 2z)$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(y + 2xy + 2z)$$

$$= 2y\hat{i} + (1+2x)\hat{j} + 2\hat{k}$$

Directional derivative at the point (2,1,1)

$$= 2\hat{i} + 5\hat{j} + 2\hat{k}$$

Normal to the sphere $x^2 + y^2 + z^2 = 9$

is $= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2 + y^2 + z^2 - 9)$

$$= 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

Normal at the point $(2,1,2) = 4\hat{i} + 2\hat{j} + 4\hat{k}$

Directional derivative along normal at (2,1,2)

$$= (2\hat{i} + 5\hat{j} + 2\hat{k}) \cdot \frac{4\hat{i} + 2\hat{j} + 4\hat{k}}{\sqrt{16+4+16}}$$

$$= \frac{1}{6}(8 + 10 + 8) = \frac{13}{3}$$

Ans

Q4 (b) A vector $\vec{r}$ is defined by $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. If $|\vec{r}|$ = r then show that the vector $r^n\vec{r}$ is irrotational.

Answer

(4) b). Curl $\vec{F} = \nabla \times \vec{F}$

$$= \nabla \times r^n \vec{r}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times (x^2+y^2+z^2)^{n/2}(x\hat{i}+y\hat{j}+z\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2+y^2+z^2)^{n/2}x & (x^2+y^2+z^2)^{n/2}y & (x^2+y^2+z^2)^{n/2}z \end{vmatrix}$$

$$= \left[\frac{n}{2}(x^2+y^2+z^2)^{n/2-1}(2yz) - \frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}(2yz)\right]\hat{i}$$

$$- \left[\frac{n}{2}(x^2+y^2+z^2)^{n/2-1}(2xz) - \frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}(2xz)\right]\hat{j}$$

$$+ \left[\frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}(2xy) - \frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}(2xy)\right]\hat{k}$$

$$= 0$$

Hence $r^n\vec{r}$ is irrotational.

(Proved)

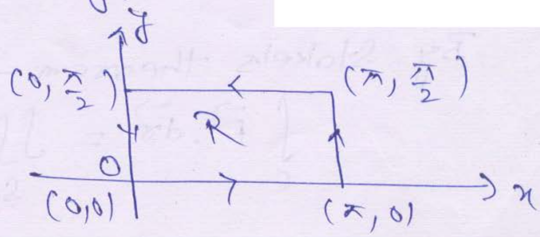
Q5 (a) Evaluate by Green's theorem in plane $\int_C (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy)$

Where C is the rectangle with vertices (0, 0), (π, 0), (π, $\frac{1}{2}\pi$), (0, $\frac{1}{2}\pi$)

Answer

5) a). By Green's theorem in plane, we have

$$\oint_C (M\,dx + N\,dy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$



Here $M = e^{-x}\sin y$, $N = e^{-x}\cos y$

$$\therefore \frac{\partial N}{\partial x} = -e^{-x}\cos y, \quad \frac{\partial M}{\partial y} = e^{-x}\cos y$$

Hence, the given line integral

$$= \iint_R (-e^{-x}\cos y - e^{-x}\cos y)\,dx\,dy,$$

where R is the region enclosed by the rectangle C

$$= \int_{x=0}^{\pi} \int_{y=0}^{\pi/2} (-2e^{-x}\cos y\,dx\,dy)$$

$$= \int_{x=0}^{\pi} -2e^{-x}(\sin y)_{y=0}^{\pi/2}\,dx$$

$$= \int_0^{\pi} -2e^{-x}\,dx = 2(e^{-x})_0^{\pi}$$

$$= 2(e^{-\pi} - 1)$$

Ans

Q5 (b) Using Stoke's theorem, evaluate $\int_c \left[(2x-y)dx - yz^2dy - y^2zdz\right]$ where C is the circle $x^2+y^2=1$, corresponding to the surface of sphere of unit radius.

Answer

5) b). The given integral $= \int_C [(2x-y)dx - yz^2dy - y^2zdz]$

$= \int_C [(2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}].(\hat{i}dx + \hat{j}dy + \hat{k}dz)$

By Stoke's theorem –

$\int_C \vec{F}.d\vec{r} = \iint_S \text{Curl}\vec{F}.\hat{n}\,dS \longrightarrow (1)$

Now Curl $\vec{F} = \nabla \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x-y & -yz^2 & -y^2z \end{vmatrix}$$

$= (-2yz+2yz)\hat{i} - (0-0)\hat{j} + (0+1)\hat{k}$

$= \hat{k}$

Putting this value in eqn (1), we get the required integral $= \iint \hat{k}.\hat{n}\,dS$

$= \iint \hat{k}.\hat{n}\frac{dx\,dy}{\hat{n}.\hat{k}}$

$\because dS = \frac{dx\,dy}{\hat{n}.\hat{k}}$

$= \iint dx\,dy$

$= \pi$

Ans

Q6 (a) Determine f(x) as a polynomial in for the following data:

x:	**-4**	**-1**	**0**	**2**	**5**
F(x):	**1245**	**33**	**5**	**9**	**1335**

by using Newton's divided difference formula

Answer

(6) a). We shall have to use Newton's divided difference formula. The divided difference table is—

x	$f(x)$	1st divided differences	2nd divided differences	3rd divided differences	4th divided differences
-4	1245				
		-404			
-1	33		94		
		-28		-14	
0	5		10		3
		2		13	
2	9		88		
		442			
5	1535				

Applying Newton's divided difference formula—

$$f(x) = f(x_0) + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] + \cdots$$

$$= 1245 + (x+4)(-404) + (x+4)(x+1)(94)$$

$$+ (x+4)(x+1)(x-0)(-14) + (x+4)(x+1)x(x-2)3$$

$$= 3x^4 - 5x^3 + 6x^2 - 14x + 5$$

Ans

Q6 (b) Find $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3^{rd}}$ and $\frac{3}{8^{th}}$ rule by dividing the range of integration into 6 equal parts. Hence obtain the approximate value of π in each case.

Answer

x	$y = \frac{1}{1+x^2}$
$x_0 = 0$	$1/1 = 1.0000000$
$x_0+h = \frac{1}{6}$	$36/37 = 0.9729729$
$x_0+2h = \frac{2}{6}$	$36/40 = 0.9000000$
$x_0+3h = \frac{3}{6}$	$36/45 = 0.8000000$
$x_0+4h = \frac{4}{6}$	$36/52 = 0.6923076$
$x_0+5h = \frac{5}{6}$	$36/61 = 0.5901639$
$x_0+6h = 1$	$1/2 = 0.5000000$

By Simpson's $\frac{1}{3}$rd rule, we get—

$$\int_0^1 \frac{dx}{1+x^2} = \int_{x_0}^{x_0+6h} \frac{dx}{1+x^2} = \frac{h}{3}\left[y_0 + y_6 + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{1}{18}\left[1.5000000 + 4(2.3631369) + 2(1.5923077)\right]$$

$$= \frac{1}{18}(14.137163)$$

$$= 0.7853979 \longrightarrow (1)$$

By Simpson's $\frac{3}{8}$th rule, we get—

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3}{8}h\left[y_0 + y_6 + 3(y_1+y_2+y_4+y_5) + 2y_3\right]$$

$$= \frac{1}{16}\left[1.5000000 + 3(3.1554446) + 2(0.8000000)\right]$$

$$= \frac{1}{16}(12.566334)$$

$$= 0.7853958 \quad \rightarrow (2)$$

But $\int_0^1 \frac{dx}{1+x^2} = \left(\tan^{-1}x\right)_0^1 = \tan^{-1}1 - \tan^{-1}0$

$$= \frac{\pi}{4} \quad \rightarrow (3)$$

From (1) & (3), we get –

$$\frac{\pi}{4} = 0.7853979$$

$$\Rightarrow \boxed{\pi = 3.1415916}$$

From (2) & (3), we get –

$$\frac{\pi}{4} = 0.7853958$$

$$\Rightarrow \boxed{\pi = 3.1415835}$$

Ans

Q7 (a) Use Lagrange's method to solve $x\left(y^2+z\right)p - y\left(x^2+z\right)q = z(x^2-y^2)$

Answer

(7) a). Here, Lagrange's subsidiary eqns for given eqns are—

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)} \longrightarrow (1)$$

Choosing $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as multipliers, each fraction of (1)

$$= \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{y^2+z-(x^2+z)+x^2-y^2}$$

$$= \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{0}$$

$$\Rightarrow \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$$

Integrating, we get—

$$\log x + \log y + \log z = \log c_1$$

$$\Rightarrow \log xyz = \log c_1$$

$$\Rightarrow xyz = c_1 \longrightarrow (2)$$

Choosing $x, y, -1$ as multipliers, each fraction of eqn (1)

$$= \frac{x\,dx + y\,dy - dz}{x^2(y^2+z) - y^2(x^2+z) - z(x^2-y^2)}$$

$$= \frac{x\,dx + y\,dy - dz}{0}$$

$$\Rightarrow x\,dx + y\,dy - dz = 0$$

Integrating, we get—

$$x^2 + y^2 - 2z = c_2 \longrightarrow (3)$$

2) & (3), the solution is—

$\phi(x^2+y^2-2z, xyz) = 0$, ϕ being an arbitrary function.

Ans

Q7 (b) Use Charpit's method to find complete integral of $q = (z+px)^2$

Answer

(7) b) Here given eqn is—

$$f(x,y,z,p,q) \equiv (z+px)^2 - q = 0 \quad \longrightarrow (1)$$

Charpit's auxiliary eqns are—

$$\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{-f_p} = \frac{dy}{-f_q}$$

or

$$\frac{dp}{2p(z+px)+2p(z+px)} = \frac{dq}{2q(z+px)} = \frac{dz}{-2px(z+px)+q}$$

$$= \frac{dx}{-2x(z+px)} = \frac{dy}{0}, \text{ by } (1)$$

Taking the 2nd and 4th fractions, ~~we~~ get—

$$\frac{1}{q}\,dq = -\frac{1}{x}\,dx$$

Integrating, we get—

$$\log q = \log a - \log x$$

$$\Rightarrow q = \frac{a}{x} \quad \longrightarrow (2)$$

Substituting this value of q in (1), we get

$$(z+px)^2 = \frac{a}{x} \text{ or } px = \frac{\sqrt{a}}{\sqrt{x}} - z$$

or $p = \frac{\sqrt{a}}{x\sqrt{x}} - \frac{z}{x}$

Now → $dz = p\,dx + q\,dy$

$\Rightarrow dz = \left(\frac{\sqrt{a}}{x\sqrt{x}} - \frac{z}{x}\right)dx + \frac{a}{x}dy$

, by (2) & (3)

$\therefore x\,dz = \sqrt{a}\,x^{-1/2}dx - z\,dx + a\,dy$

or $x\,dz + z\,dx = \sqrt{a}\,x^{-1/2}dx + a\,dy$

or $d(xz) = \sqrt{a}\,x^{-1/2}dx + a\,dy$

Integrating, we get —

$xz = 2\sqrt{a}\sqrt{x} + ay + b$

where a & b are arbitrary constants.

Ans

Q8 (a) A bag contains 6 white and 9 black balls. Four balls are drawn at a time. Find the probability for the first draw to give 4 white and the second draw to give 4 black balls in each of the following cases-

(i) The balls are replaced before the second daw

(ii) The balls are not replaced before the second draw

Answer

Q8) a) (i) In the first draw any 4 balls can be drawn out of $6+9=15$ balls in ${}^{15}C_4$ ways and 4 white balls can be drawn out of 6 white balls in ${}^{6}C_4$ ways.

$\therefore$ The probability that the 4 balls drawn in the first draw are white is

$$= \frac{{}^{6}C_4}{{}^{15}C_4} = \frac{15}{1356} = \frac{1}{91}$$

Before the second draw, the balls are replaced. Therefore we again have 6 white and 9 black balls. Any four balls can be drawn out of $6+9=15$ balls in ${}^{15}C_4$ ways and 4 black balls can be drawn out of 9 in ${}^{9}C_4$ ways.

$\therefore$ The probability that 4 black balls are drawn in the second draw $= \dfrac{{}^{9}C_4}{{}^{15}C_4}$

$$= \frac{126}{1365} = \frac{6}{65}$$

$\therefore$ Required probability $= \dfrac{1}{91} \times \dfrac{6}{65} = \dfrac{6}{5915}$

(ii) The probability of drawing 4 white balls in the first draw is $\dfrac{{}^{6}C_4}{{}^{15}C_4} = \dfrac{1}{91}$

Since the drawn balls are not replaced, we are left with 2 white balls and 9 black balls. The probability of drawing 4 black balls out of these 11 balls is

$$= \frac{{}^{9}C_4}{{}^{15}C_4} = \frac{21}{55}$$

$\therefore$ Required probability $= \dfrac{1}{91} \times \dfrac{21}{55} = \dfrac{3}{715}$

Ans

Q8 (b) Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. Find the chance of selecting 1 girl and 2 boys.

Answer

Q8) b) There are three ways of selecting (1)
1 girl and two boys –

I way: Girl is selected from first group, boy from second group and second boy from third group.

∴ Probability of the selection of (Girl + Boy + Boy)

$$= \frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{18}{64}$$

II way: Boy is selected from first group, girl from second group and second boy from third group.

∴ Probability of the selection of (Boy + Girl + Boy)

$$= \frac{1}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{6}{64}$$

III way: Boy is selected from first group, second boy from second group and the girl from the third group.

∴ Probability of selection of (Boy + Boy + Girl)

$$= \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{2}{64}$$

∴ Total probability $= \frac{18}{64} + \frac{6}{64} + \frac{2}{64}$

$$= \frac{26}{64} = \frac{13}{32}$$

Ans

Q9 (a) A manufacturer knows that the condensers he makes contain on an average 1% of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensers?

Answer

(9) a) Here $P = 1\% = 0.01$, $n = 100$, $m = np$

$= 100 \times 0.01$

$= 1$

$$P(r) = \frac{e^{-m}(m)^r}{\lfloor r}$$

$$= \frac{e^{-1}(1)^r}{\lfloor r} = \frac{e^{-1}}{\lfloor r}$$

P (4 or more faulty condensers)

$$= P(4) + P(5) + \cdots + P(100)$$

$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 - \left[\frac{e^{-1}}{\lfloor 0} + \frac{e^{-1}}{\lfloor 1} + \frac{e^{-1}}{\lfloor 2} + \frac{e^{-1}}{\lfloor 3}\right]$$

$$= 1 - e^{-1}\left(1 + 1 + \frac{1}{2} + \frac{1}{6}\right)$$

$$= 1 - \frac{8}{3e} = 1 - 0.981 = 0.019$$

Q9 (b) A sample of 100 dry battery cells tested to find the length of life produced the following results-

$\bar{x} = 12$ hours, $\sigma = 3$ hours

Assuming the data to be normally distributed, what percentage of battery cells are expected to have life-

(i) More than 15 hours
(ii) Less than 6 hours
(iii) Between 10 and 14 hours

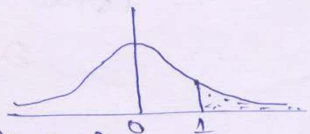
Answer

(9) b) Here x denotes the length of life of dry battery cells.

Also $z = \frac{x - \bar{x}}{\sigma} = \frac{x-12}{3}$

(i) when $x = 15$, $z = 1$

$\therefore P(x > 15) = P(z > 1)$



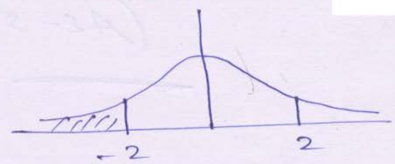
$= P(0 < z < \infty) - P(0 < z < 1)$

$= 0.5 - 0.3413 = 0.1587 = 15.87\%$

(ii) when $x = 6$, $z = -2$

$\therefore P(x < 6) = P(z < -2)$

$= P(z > 2)$



$= P(0 < z < \infty) - P(0 < z < 2)$

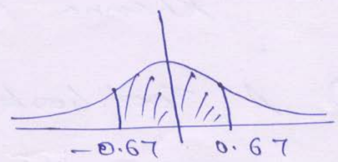
$= 0.5 - 0.4772$

$= 0.0228 = 2.28\%$

(iii) when $x = 10$, $z = -\frac{2}{3} = -0.67$

when $x = 14$, $z = \frac{2}{3} = 0.67$

$\therefore P(10 < x < 14)$



$= P(-0.67 < z < 0.67)$

$= 2P(0 < z < 0.67)$

$= 2 \times 0.2487$

$= 0.4974 = 49.74\%$

Ans

Text Books

1. Higher Engineering Mathematics-Dr. B.S. Grewal, 40th edition 2007, Khanna Publishers, Delhi.

2. A Text book of Engineering Mathematics-N.P. Bali & Manish Goyal, 7th edition 2007, Laxmi Publication (P) Ltd.