Q2 (a) Define the following terms:

(i) Joint probability (ii) Conditional probability
(iii) Probability mass function (iv) Statistical independence

Answer

(i) Join Probability :- The probability of an event such as Ai Bj that is the intersection of events from sub experiments is called the joint probability of the events and is denoted by p(Ai,Bj)

(ii) Conditional Probability: - The probability of occurrence of even Bj may depend on the occurrence of a related event Ai. The occurrence of event Bj on the second sub experiments is conditional on the occurrence of event Ai on the first sub experiments we denote the probability of event Bj occurring given the event Ai is known to have occurrence by conditional probability p(Bj/Ai).

The expression of conditional probability in term of joint probability is

$$P(B/A) = \frac{P(AB)}{P(A)}$$

Where P (A) is marginal probability.

(iii) Probability mass function: - A discrete random variable X is characterized by set of allowed values of $(x_1, x_2 ....x_n)$ and probabilities of random variables taking of on one of these values based on out come of random experiment. The probability that x= $x_i = x(x_i)$ is denoted by $p(x = x_i)$ and is called probability mass function.

This probability mass function has the following properties.

(i) $\sum_{i=1}^{n} P(x = x_i) = 1$

(ii) $P(x \le x\ ) = \sum_{\substack{all \\ x_i \le x}} p(x = x_i)$

p(x=x) is called cumulative probability distribution function of X and is denoted by fx(x).

(iv) Statistical independence: - Suppose that Ai & Bj are events associated with outcome of two experiments. Suppose that event B is independent of Ai so that the occurrence of Ai does not influence the occurrence of Bj and vice versa. The events are statistically independent. Here precisely , we say that two events Ai and Bj are statistically independent if

$$P(Ai / Bj) = P(Ai)P(Bj)$$

or when

P(Ai/Bj) = P(Aj).

Q2 (b) The input of a binary communication systems denoted by random variable X, takes on one of the two values 0 or 1 with probabilities $3/4$ and $1/4$ respectively. Due to error caused by noise in the system, the output Y differs from the input X occasionally. The behaviour of the communication system is modelled by the conditional probabilities.

$$P(Y=1|X=1)=3/4 \text{ \& } P(Y=0|X=0)=\frac{7}{8}$$

Find

(i) P(Y = 1) and P(Y=0)

(ii) $P(X=1|Y=1)$

Answer

We Know that

$$P(x=x_i)=\sum_{j=1}^{m}P(x=x_i)(y=y_j)$$

$$=\sum_{i=1}^{m}\sum_{j=1}^{m}P(x=x_i/y=y_j)p(y=y_j)$$

(a)

$$P(y=1)=P(y=1|x=0)P(x=0)+p(y=1|x=1)P(x=1)$$

$$=\left(1-\frac{7}{8}\right)\frac{3}{4}+\frac{3}{4}.\frac{1}{4}=\frac{9}{32}$$

$$P(y=0)=1-P(y=1)=\frac{23}{32}$$

Using Bayes Rule

(b)

$$P(x=1|y=1)\frac{P(y=1/x=1)P(x=1)}{P(y=1)}$$

$$=\frac{(1/4)(3/4)}{9/32}=2/3$$

Q3 (a) Explain the three models for continuous random variables.

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Q3 (b) X and Y are two independent random variables, each having a Gaussian probability distribution function with a mean of zero and a variance of one.

(i) Find $P(|X|>3)$ **using Q(4) and also obtain an upper bound. Given that** $Q(0)=\frac{1}{2}, Q(3)=0.0013$.

(ii) Find the joint PDF of $Z=\sqrt{x^2+y^2}$ **&** $\omega=\tan^{-1}\left(\frac{y}{x}\right)$

(iii) Find P(z > 3)

Answer

$P(|X|>3)=2P(x>3)$

$=2\varphi(3)=2.\times(0.0013)$

$=0.0026$

Using chebyshev`s identity

$P(|X|>3)\le\left(\frac{1}{3}\right)^2=0.111$

(ii) Since x and y are independent, the joint probability distribution function of x and y is equal to the product of Pdf of x and y

$$f_{xy}(x,y)=\frac{1}{2\pi}e^{-(x^2+y^2)/2}$$

$-\infty<x,y<\infty.$

Solving for x & y in terms of 2 and w

$x_1=2\cos w$

$y_1=2\sin w$

$$J_1=\begin{vmatrix}\frac{\partial x_i}{\partial 2} & \frac{\partial x_i}{\partial w}\\ \frac{\partial y_i}{\partial 2} & \frac{\partial y_i}{\partial w}\end{vmatrix}$$

$$=\begin{vmatrix}\cos w - 2\sin w\\ \sin w.....2\cos w\end{vmatrix}=2$$

$(J_1)=2$

$\sin ce\quad z>0$

The joint pdf of 2 & w is $f2,w(2,w)=f_{x4}(x_i,4_j)(Ji)$

$$f2w(2\omega)=\frac{2}{2\pi}e^{-z^2/2}\quad 0<2<\infty,\text{-}\pi<\text{w}<\pi$$

$$\text{f}_2(2)=2e^{-2^2/2}$$

$$fw(w)=\frac{1}{2\pi}$$

$$\therefore\ f2,w(2,w)=\ f_2(2)f_w(w), Hence\ 2\ and\ \text{w are independent}$$

(iii)

$$\text{P}(2>3)=\int_3^\infty ze^{-z^2/2}d2=/-e^{-2^2/2}/_3^\infty$$

$$=(-4.5)=0.011$$

Q4 (a) Explain Markoff statistical model for information sources.

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Q4 (b) A discrete source emits one of five symbols once every millisecond. The symbol probabilities are $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}$ and $\frac{1}{16}$ respectively. Find the source entropy and information rate.

Answer

$$H=\sum_{i=1} Pi\log_2\left(\frac{1}{P_2}\right)bits/symbol.$$

$$=\frac{1}{2}\log_2(2)+\frac{1}{4}\log_2(4)+\frac{1}{8}\log_2(8)+\frac{1}{16}\log_2(16)+\frac{1}{16}\log_2(16)$$

$$=0.5+0.5+0.375+0.25+0.25$$

$$=1.875 bits/symbol.$$

$$InformationRate$$

$$R=r_s Hbits/\sec$$

$$=1000\times1.875=1875 bits/\sec$$

Q6 (a) Explain Discrete Memory less channel.

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Q6 (b) A discrete memoryless source X has four symbols x_1, x_2, x_3, x_4 with probabilities $P(x_1)=0.4$, $P(x_2)=0.3$, $P(x_3)=0.2$, $P(x_4)=0.1$

(i) Calculate H(X)

(ii) Find the amount of information contained in the message $x_1x_2x_1x_3$ and $x_4x_3x_3x_2$.

Answer

The entropy is given by H(x) = $\sum_{i=1}^{4} P(x_i)\log_2[P(x_i)$

Substituting value and simplifying

$$H(x) = -0.4\log_2 0.4 - 0.3\log_2 0.3 - 0.2\log_2 0.2 - 0.1\log_2 0.1$$
$$= 1.85 b/symbol$$

$$(ii)$$
$$P(x_1, x_2, x_1, x_3) = (0.4)(0.3)(0.4)(0.2) = 0.0096$$
$$therefore, I(x_1, x_2, x_1, x_3) = -\log_2(0.0096)$$
$$= 6.7$$
$$b/symbols$$
$$I(x_1, x_2, x_1, x_3) < 7.4$$
$$P(x_4, x_3, x_3, x_2) = -\log_2(0.0012) = 9.7$$
$$b/symbol.$$

Q7 (a) Explain the following terms:
(i) Mutual information
(ii) Channel capacity

Answer

Mutual Information :- If is denoted by I(x:4) defined by I(x:4)=H(x)-H(x/4) b/ symbol Since H(x) represents the uncertainty about the channel input before the channel output is observed and H(x/y) represents the uncertainty about the channel input after the channel output is observed , the mutual information I(x:y) represents the un certainty about channel I/P that is resolved by observing the channel about.

(i) I(x;y)=I(y;x)
(ii) I(x;y) ≥ 0
(iii) I(x;y)=H(y)-H(y/x)
(iv) I(x;y)=H(x)+H(y)-H(x,y)

(ii) Channel Capacity: - The bandwidth and the noise power place a restriction upon the rate of information that can be transmitted by a channel. The channel capacity is passed as

$c = B\log_2(1 + S/N)$ for white Gaussian Noise

B= Channel bandwidth
S=signal power
N=Noise power

$$C = \frac{k}{2}\log_2(1 + S/N)$$

K-maximum pulse per second.

Q7 (b) Explain differential entropy and mutual information for continuous ensembles.

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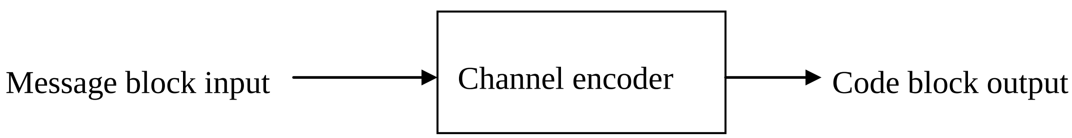
Q8 (a) What is linear block code? Explain the steps for determination of all code words for a linear block code.

Answer
linear block code:
For the block of k message bits, (n-k) parity bits or check bits are added. This means that the total bits at the output of channel encoder are n. such type of codes are know as (x, k) block codes



Message

1 ← k bits →

Message	**Check bits**
← K →	**←(n-k)→**
←n--------------------bits--------→	

(Functional block diagram of a block coder)
A code is said to be linear if the sum of any two code vectors produces another code vector. Any code vector in expressed as a linear combination of other code vectors.

Q8 (b) The generator matrix for a (6, 3) block code is given as

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Find all code vectors of this code.

Answer
The message block size K for this code is 3 and length of the code vector n is 6. these are eight possible message blocks:(000),(001),(010),(011),(100),(101),(110),(111). The code vector for message block D= (111) is

$$C = DG = (111)\begin{bmatrix} 1 & 0 & 00 & 1 & 1 \\ 0 & 1 & 01 & 0 & 1 \\ 0 & 0 & 11 & 1 & 0 \end{bmatrix} = (111000)$$

Similary

Message	Code	Vectors
000	000	000
001	001	110
010	010	101
011	011	011
100	100	011
101	101	101
110	110	110

Q9 (a) Explain cyclic codes. Give their advantages and disadvantages.

Answer

Cycle Code: - may be described as the subclass of linear block codes. They have the property that a cycle code shifts of one code word produces another code word .e.g. consider an n bit code vector as shown

$X = (x_{n-1,} x_{n-2}, ...x_1, x_0)$

Where $x_{n-1,}\ x_{n-2}, ...x_1, x_0$ etc represent individual bits of the code x. The shift of code cyclically, generates another code vector x i.e.

$X^1 = (x_{n-2,} x_{n-3}, ...x_1, x_0, x_{n-1})$

One more cycle shift produces another code vector "x"

$X^{11} = (x_{n-3,} x_{n-4}, ...x_1, x_0, x_{n-1}, x_{n-2})$

Advantage:

1. The error correcting and decoding methods of cyclic codes are simpler and easy to implement.
2. The encoder's & decoders for these codes are simpler compared to non-cyclic codes.
3. These codes detect error burst that span many successive bits.
4. These codes eliminates storage needed for lookup table decoding

Disadvantage:

1. The detection is simpler but error correction is complicated since combinational logic circuits in error detector are complex.

Q9 (b) Obtain the convolutional coded output for the message '1100101'. The convolutional encoder is shown in Fig.1

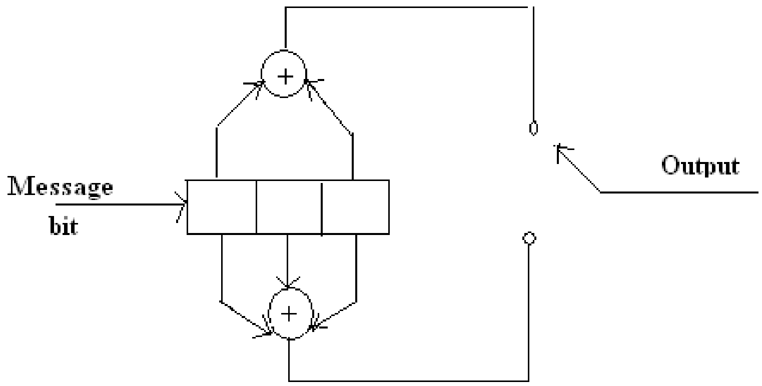


Fig.1

Answer

Message	**Probability**	**Code**	**No. of bits**
X1	**0.05**	**1110**	**4**
X2	**0.15**	**010**	**3**
X3	**0.2**	**10**	**2**
X4	**0.05**	**1111**	**4**
X5	**0.15**	**011**	**3**
X6	**0.03**	**00**	**2**
X7	**0.1**	**110**	**3**

Average length L= $\sum_{i=1}^{7} x_i \, p(x_i)$

L=4(0.05+0.05) +3(0.15+0.15+0.1) +2(0.2+0.3)

=2.6bits

Entropy,

$$f(x) = \sum_{i=1}^{7} p(x_i) \log_2 \frac{1}{p}(x_i)$$

$$= 0.3\log_2(1/0.3) + 0.2\log_2(1/0.2) + 0.3\log\ (1/0.15) + 0.1\log_2(1/0.3) + 0.1\log(1/0.05)$$

$$= 2.57 bits$$

$$\eta = \frac{H(x)}{L\log_2 M} = \frac{2.57}{2.6\log_2 2} = \frac{2.57}{2.6} = 98.85\%$$

<u>Text Books</u>

1. Digital and Analog Communication Systems by K. Sam Shanmugam, John Wiley India Edition, 2007 reprint.

2. Digital Communications by Simon Haykin, John Wiley & Sons, Student Edition.