Q2 (a) Use mathematical induction to prove that for all positive integers n, $n(n^2+5)$ is an integer multiple of 6.

Answer

Base n=1, 1(1+5) =6 is an integer multiple of 6.

Hypothesis: let us assume that it is true for n induction step: we need to prove that it is true for n+1.

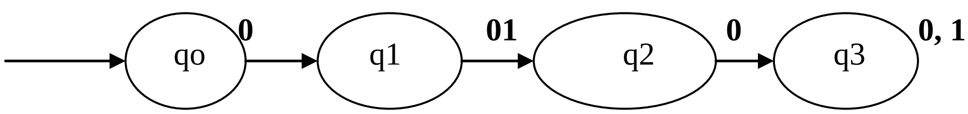
Consider (n+1) $(n+1)^2+5) = (n+1)(n^2+1+2n+5) = n(n^2+5)+(n^2+2n^2+3n+6)$

$$= n(n^2+5)+6\left(\frac{n^2}{2}\frac{n}{2}+1\right) = n(n^2+5)+6\left(\frac{n(n+1}{2}+1\right)$$

By hypothesis we know that $n(n^2+5)$ is divisible by 6. Clearly the last expression is divisible by 6. Therefore for all n, $n(n^2+5)$ is an integer multiple of 6.

Q2 (b) Define the terms - alphabet, power of alphabet, string and language. Provide one example for each.

Answer The state table can be represented by the state diagram as:



q0	[q0 ,q1]	[q0]
[q0,q1]	[q0,q1,q2]	[q0,q2]
[q0,q2]	[q0,q1,q3]	[q0]
[q0,q1,q2]	[q0,q1,q2,q3]	[q0,q2]
[q0,q1,q3]	[q0,q1,q2,q3]	[q0,q2,q3]
[q0,q1,q2,q3]	[q0,q1,q2,q3]	[q0,q2,q3]
[q0,q2,q3]	[q0,q1,q3]	[q0,q3]
[q0,q3]	[q0,q1,q3]	[q0,q3]

Q3 (a) For the following NFA, find the equivalent DF(A)

	0	**1**
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$
q_2	$\{q_3\}$	$\{q_3\}$
q_3	ϕ	ϕ

Answer

q0	[q0 ,q1]	[q0]
[q0,q1]	[q0,q1,q2]	[q0,q2]
[q0,q2]	[q0,q1,q3]	[q0,q3]
[q0,q1,q2]	[q0,q1,q2,q3]	[q0,q2,q3]
[q0,q1,q3]	[q0,q1,q2,q3]	[q0,q2]
[q0,q3]	[q0,q1]	[q0]
[q0,q1,q2,q3]	[q0,q1,q2,q3]	[q0,q2,q3]
[q0,q2,q3]	[q0,q1,q3]	[q0,q3]

Q3 (b) Write regular expression for the language defined over alphabet {a, b} as "The set of strings having at most one pair of consecutive a's and at most one pair of consecutive b's.

Answer

One pair of a's but no pair of b's $B_1 = (a+\in)(ab)^*aa(ba)^*(b+\in)$

One pair of b's but no pair of a's $B_2 = (a+\in)(ba)^*bb(ab)^*(a+\in)$

B= $B_1 + B_2$

Aa occurring before

$C = (b+\in)(ab)^*aa(ba)^*bb(ab)^*(a+\in)$

bb accruing before aa

$D = (a+\in)(ba)^*bb(ab)^*aa(ba)^*(a+\in)$

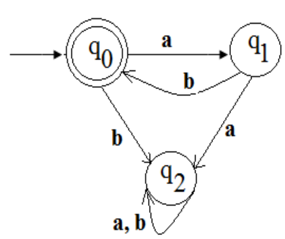
No pair of a's & b's

$E = (b+\in)(ba)^*(a+\in)$

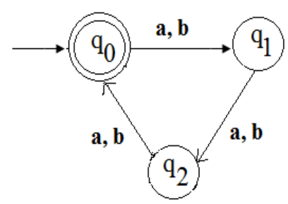
Required regular expression is B+C+D+E.

Q4 (a) Describe the languages accepted by the following DFAs

(i)



(ii)



Answer

(i)

$L_1 = \{(ab)^n \mid n \geq 0\}$

(ii)

$L_2 = setofstringin(a,b)^n\ whoselengthisdivisible$

$by 3.$

Q4 (b) Show that concatenation of two regular expression is a regular expression.

Answer

L_1 is not regular suppose L_1 is regular . Lets n be the constant of pumping Lemme, consider $O^n \| O^n$,The pump will occure in the first n o's . So we shall get $O^n \| O^n \in L, m \neq n$ Which is contradiction

(ii) L_2 is regular .

L_2 can be represented by regular expression .

0(0+1) +0+1(0+1) +1

Q5 (a) Prove following is not a regular language:

$$L = \{xx^R \big| x \in \{0,1\}^+\}$$

Answer

$L = \{O^i 1^j \mid ged(i,j)=1\}$ **is not regular .**

Suppose L is regular, consider the sets of primer {p1, p2,}.

This is an infinity set, consider the set of strings $O^{p1}, O^{p2}, O^{p3}, ...$ by my fill Nerode theorem, all of them can't be in different equivalence classes, for some Pi and pi, O^{pi} and O^{pj} must be in same equivalence class.

$O^{P_i} \approx, O^{P_i}$

$O^{p^i | P^j} O^{pj} |^{pi}$

$O^{pi} |^{pj} \in L$

Wherease

$O^{pi} |^{pj} \notin L.$

Hence we have a contradiction .L is not regular .

Q5 (b)If L is a Regular language then show that reverse of L i.e. L^R is also regular.

Answer

M=({q0,q1,q2,q3,q4}{a,b,c,d},{a,c,\$}, δ ,q0,\$,{q4})

Transitions are:

δ (q0,a, φ)contains (q0,,a\$)

δ (q0,a,a) contains (q0,aa)

δ (q0,b,a)contains (q1, $\in$)

δ (q1,b,a)contains (q2, $\in$)

δ (q1,c,\$)contains (q2,c\$)

δ (q2,c,c)contains (q2,cc)
δ (q2,d,c)contains (q3, $\in$)
δ (q3,d,c)contains (q3, $\in$)
δ (q3, $\in$,\$)contains (q4, $\in$)
This m/c accepts L by empty stack, taking 9/4 as the final state L is accepted by final also,

Q6 (a)Let L = $\{a^n b^n c^m d^m \mid n, m \geq 1\}$**. Draw a PDA that accepts L.**

Answer
M=({q0,q1,q2},{0,13,{x,\$3, δ,q0,\$, ϕ)
Where δ is given by
δ (q0,0,\$) contains (q0,x\$)
δ (q0,o,x) contains(q0,x,x)
δ (q0,1,\$) contains(q0,x,\$)
δ (q0,1,x) contains(q0,xx)
δ (q0,0,\$) contains(q1,\$)
δ (q0,o,x) contains(q1,x)
δ (q1,o,x) contains(q1, $\in$)
δ (q1,1,x) contains (q1, $\in$)
δ (q1,1,\$)contains (q2, $\in$)

Acceptance is by empty stack. We can also look at it as acceptance by final state by taking q2 as the final state,

Q6 (b) Define a Context Free Grammar that generates the language:
$L = \{a^i b^j c^k d^\ell \mid i, j, k, \ell \geq 1, i = \ell, j = k\}$ **Draw a PDA that accepts L.**

Answer
(i) L_2 is CFL generated by :
s→asd, S→a Ad, A→bAc, A→bc

(ii) L_3 is not context free. Then since the family of CFL is closed under intersection with regular sets
$L_3 \cap a^* b^* c^*$ is regular.
This is $\{a^n b^n c^n \mid n \geq 03\}$
We have shown that tis is not context free. So L_3 is not context free.

Q7 (a) Prove that the following language is not context free,
$L_1 = \{a^p \mid p$ **is a prime}**

Answer

Suppose L_1 is context free, Then by pumping Lexma there exits K such that for all $Z \in L$, **and |Z|** $\geq$ k. Z can be written in the form uvxyz such that $uv^2xy^2z \in L$ for all $i \geq 0$ consider $p > k, a^p \in L_1$ a^p =uvxyz

Now u,v,x,y,z $\in a*$, therefore by pumping lemma;

Uxyz $(vy)^2 \in L_1$ *for* all $\mathrm{i} \geq 0$
let
$|vy| = r$
$uxz(a^r)^i \in L_1$, *for* all $\mathrm{i} \geq 0$
or
$\mathrm{z}(\mathrm{a}^{\mathrm{r}})^{z-1} \in L_1$, *for* all $\mathrm{i} \geq 0$
$\mathrm{a}^{\mathrm{p+r(i-1)}} \in L_1$, *for* all $\mathrm{i} \geq 0$

Choose I such that p+r(I-1) is not a prime. Select I-1=p, Therefore, I=p+1.
$a^{p+rp} \in L_1$
but
$a^{p+rp} = a^{p(r+1)}; p(r+1)$
is not a prime . so we come to conclusion that a^s where s is not a prime belong to L_1, This is contradiction , therefore L_1 is not CF.

Q7 (b) What is Chomsky Normal form? Explain how a grammar can be put in CNF. Use an example to illustrate.

Answer
$q_0$111100011
0$q_1$11100011
01$q_1$1100011
0111$q_1$00011
01111$q_2$0011
0111$q_3$10011
$q_3$01110011
0001111$q_2$11
No more for $(q_2,1)$, m/o halls with o/ps….0001111111…………..
The first block of 1's is shiffed feb by step to the right till it become adjacent to the second block of 1's.

Q8 (a) Consider the following TM M' with transitions as follows:

$\delta(q_0,1) = (q_1,0,R)$
$\delta(q_1,1) = (q_1,1,R)$
$\delta(q_1,0) = (q_2,1,R)$
$\delta(q_2,0) = (q_3,0,L)$
$\delta(q_3,0) = (q_0,0,R)$
$\delta(q_3,1) = (q_3,1,L)$

q_0 is the initial state and 0 is taken as blank symbol. Trace the sequence of moves when the machine scan starts on ...00 1111 000 11 00...

Answer

	0	**L**	**#**
q_0	(q_0,o,R)	(q_0,1,R)	(q_1,#,R)
q_1	(q_1,o,R)	(q_2,1,R)	-
q_2	(q_3,0,L)	-	-
q_3	-	-	-

Q8 (b) Construct a TM with three character 0, 1, and # which locates a '1'under the following conditions. There is only one # on the tape and somewhere to the right of it is a 'l'. The rest of the tape is blank. The head starts at or to the left of the #. When the TM halts, the tape is unchanged and head stops at the 'l'. Zero is taken as the blank symbol.

Answer

This is not RE.

Let S= { $L_1, L_2, ...\}$ *each* L in S is infinite.

$L_s = \{< M >| T(M) = L$ and $L \in S\}$

if L_s is RE, it should satisfy the there conditions of Rice .theorem for recursively enumerable index sets. But condition 2 is violated, L_s is not RE.

Q9 (a) Define a Recursively Enumerable language. Give an example of it. Give an example of a language that is not recursively enumerable.

Answer

f is a non total function and π^2 is a total function . Any algorithm to solve the problem with x_1, x_2 *and* x_3 should have two subroutines one computing f (x_1) and another computing $\pi^2(x_2, x_3)$. since f is non total for some arguments, the algorithm will not come out of the subroutine for computing f (x_1) and will not be able to half .Hence the problem is un decidable.

Q9 (b) Show that the following problem is undecidable.
"Given x_1, x_2 and x_3 determine whether $f(x_1) = \pi^2(x_2, x_3)$, where f is a fixed non total recursive function and π^2 is cantor numbering function".

Answer
The grammar G= (N, T, P, S), N={S, A}, T= {a, b, c}. P is given as:

S→aSc, S→aAc, A→bA, A→b

Rule 1 & 2 general equal no, of a's and c's ; note 2 makes sure at least one a and one c are generated , Rule 3 and 4 generate b's in the middle , Rule 4 makes sure at least . One b is generated, It is to be noted that a's and c's are generated first & b's after wards.

Text Books

Introduction to Automata Theory, Languages and Computation, Jhon E Hopcroft, Rajeev Motwani, Jeffery D. Ullman, Pearson Education, Third Edition, 2006