Q2 (a) Prove that A − (B ∩ C) = (A − B) ∪ (A − C)

Answer

In order to prove this let x be any element of A-(B∩**C),** then

=x∈A-(B ∩ C)== x∈A and x∉(B∩**C)**

=x∈A and [**x**∉B or **x**∉ **C]**

=(x∈A and x∉B) or **(x**∈A and x∉c)

=(x∈A-B) or **(x**∈A-c)

This implies that

=A- (B∩**C)** ⊆(A-B)U(A-C) and

=(A-B)U(A-B)U(A-C) and

=(A-B)U(A-C) ⊆ A- (B∩**C)**

= thus A-(B∩**C)=(A-B)U(A-C)**

Q2 (b) A die is tossed thrice. Find the probability of getting an odd number at

Answer

Total no of outcomes = 6*6*6=216

Probability of not getting an odd number even once = probability of getting an even number in all the three tosses

No of ways in getting 3 evens =3*3*3=27

=probability of getting all even =27/216=1/8

➔Probability of getting at least one add =1-probability of getting all even =1-1/8=7/8

Q2 (c) A speaks truth in 80% of the cases and B speaks truth in 60% of the cases. Find the probability of the cases of which they are likely to contradict each other in stating the same fact.

Answer

Probability that A speaks Truth: P(A=T)=8

Probability that A speaks FALSE: P(A=F)=2

Probability that B speaks Truth: P(B=T)=6

Probability that A speaks False: P(B=F)=4

Now probability that A and B likely to contradict is

=P(A=T)*P(B=F)+P(B=T)*P(A=F)

=8*4+6*2=32+12=44

Q3 (b) What are tautologies and contradictions? Prove that, for any propositions P, Q, R the following compound propositions are tautologies:

(i) $[(P \to Q) \wedge (P \to R)] \to (P \to R)$

(ii) $[P \to (Q \to R)] \to [(P \to Q) \to (P \to R)]$

Answer

Let (L,<) be a poset . if every subset{x,y} containing any two elements, of L, has a glb(Infimum) and a lub (Supremum), then the poset (L,<)is called a lattice . the poset in (i) is a lattice because for every pair of elements in it, there a glb and lub in the poset . Similarly poset in (ii) is also a lattice . Poset of (iii) is also a lattice.

Q5 (b) Prove by induction
1.1! + 2.2! + ……. +n.n! = (n+1)! – 1

Answer
The result is obvious for n=1. When n=1, we have 1.1!=2!-1
Next suppose that the given result id true for n = k. i.e.
1.1!+2.2!+3.3!+………….+K.K!=(K+1)!-1
Now when n=k+1, we get
1.1!+2.2!+3.3!+………….kk!+(k+1).(k+1)!=(k+1)!-1+(k+1).(k+1)!
=(k+1)!{1+k+1}-1
=(k+2)!-1
this shown that whenever result is true for n =k , it is true for n=k+1 as well . Hence by induction it true for all n.

Q7 (a) Prove that the inverse of a linear function is also linear and the two slopes are reciprocals of each other.

Answer
Let f(x)=mx+b: if m=0; the function is very is badly not on-to-one and the inverse does exist.
If m !=0; then the inverse is y=mx+b solve for x
y-b=mx for m!=0
(y-b)/=mx
=x=(1/m)y-b/m
=f-1 (x)=(1/m)x-(b/m)
The inverse is a line with slope 1/m

Q7 (b) A toy manufacturer has a new product to sell. The number of units to be sold, x, is a function of the price p: n(p)=30-25p . The revenue earned is a function of the number of units sold: r(x) = 1000-(1/4)x^2. Find the function for revenue in terms of price, p.

Answer r (n (p)) =-156.25 p2+375 p+9775

Q8 (a) Let G be a cyclic group of order 6. How many of its elements generate G?

Answer G {1, g, g2, g3, g4, g5} and g5 and g are the only elements of order 6, thus the only possible generators for G

Q9 (a) Let R be a ring with a unity element 1. Show that the set R* of units in R is a group under Multiplication.

Answer

Let R be a ring with a unity element 1. Then by definition of Ring.

If a, $b \in R^*, then ab \in R^*$ (multiplication binary composition.)

$a.(b.c) = (a.b).c \forall R^*$

As R* is a ring with unity, $\exists$ an element

1 E R*, x.1=1=1.x

That is 1 is the multiplicative identity in R*

As R* is set of units in R.

If uER*, than there exist an element $u^{-1} \varepsilon R^* s.t. uu^{-1} = 1 = u^{-1}u$ (multiplicative identity)

Each element in R* possess multiplicative inverse. R* is a group under multiplication.

Q9 (b) Give steps and an example to generate a parity check matrix.

Answer

In coding theory, a parity-check matrix of a linear block code C is a generator matrix of the dual code. As such, a codeword c is in C if and only if the matrix-vector product Hc=0. The rows of a parity check matrix are parity checks on the codeword equal Zero.

In order to generate the parity check matrix you must first have the generator matrix and the codeword to check and see if it is correct.

1. Place your generator in row reduction form
2. Get the basis vectors
3. Put the vectors together to get the parity check matrix
4. Check it b multiplying the codewords by the parity = 0

For an example: 2*4 Generator Matrix

[1 0 1 1 0 1 1 0]

Rank = 2….therefore the number of columns is 2….Rank+X=# of columns of the Generator matrix

v1+v3+v4=0

v2+v3=0

v1=-r1-r2

v2=-r1

v3=r1

v4=r2

Parity = [-1,-1]

-1 0

1 0

0 1

<u>Text Book</u>

Discrete Mathematical Structures, D.S. Chandrasekharaiah, Prism Books Pvt. Ltd., 2005.