Q.2 a. Define the open loop control system and closed loop control system. Write Comparison between open loop control system and closed loop control system.(8)

Answer:

Ans: → Open loop control system is one in which the control action is independent of the output.

1 marks

→ Closed loop control system is one which the control action is some how dependent on the output

1 marks

Open loop control system	Closed loop control system
1. No Feedback. Hence feedback elements absent	1. Feedback exists. Hence feedback elements exists.
2. No error detector	2. Error detector is present
3. It is inaccurate	3. It is accurate.
4. Highly sensitive to parameter changes.	4. Less sensitive to parameter changes.
5. small bandwidth	5. Large bandwidth.
6. stable	6. may become unstable
7. Economical	7. costly.
8. Example: coffee maker, automatic ~~toast~~ toaster, Hand drier. etc. ...	8. Examples. Guided missile, Temperature control of oven, Respiration, servo voltage stabilizers etc. ...

4 marks

2 marks

b. Define Servomechanisms and regulators. Explain and Draw the Schematics and Block diagram for servomechanism of Water valve positioning with help of Potentiometer. (8)

Answer:

Answer:

1 marks

servomechanisms: A servo mechanism is power-amplifying feedback control system in which the controlled variable e is mechanical position, or a time derivative of position such as velocity or acceleration.

~~Regulated~~

2 marks working & construction

- At the input of the system there is rotating-type potentiometer connected across battery voltage source. Its movable terminal is calibrated in terms of angular position
- This output terminal is electrically connected to one terminal of a voltage amplifier called servo amplifier.
- The servoamplifier supplies enough power to operate an electric motor called a seromotor.
- The seromotor is linked with the water valve in manner which permits the valve to be opened or closed by the motor.
- Assume the loading effect of the valve on motor is negligible it does not 'resist' the motor.
- A 360' rotation of the motor shaft completely opens the valve.
- In addition, the movable terminal of second potentiometer connected in parallel at its fixed terminal with input potentiometer is mechanically connected to motershaft.
- It is electrically connected to the remaing input terminal of the servoamplifier. The potentiometer ratio are set so that they are equal. When the valve is closed.

Q.2 (B) Answer continue...

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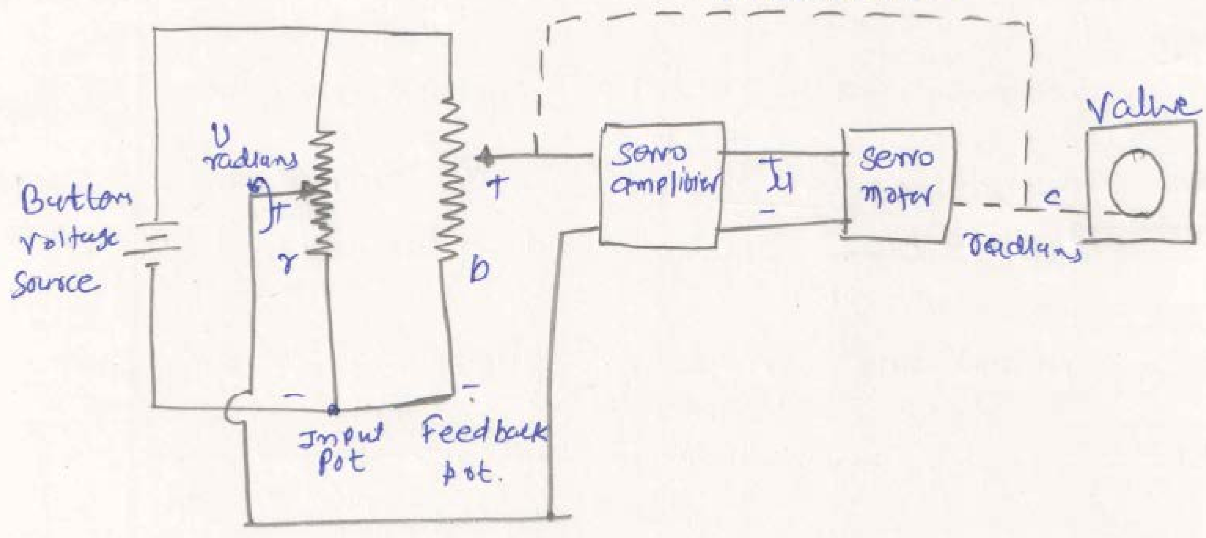
Page 3

- When a command is given to open the valve, servomotor rotates in the appropriate direction.
- As the valve opens, the second potentiometer, called the feedback potentiometer, rotates in the same direction as the input potentiometer. It stops when the potentiometer ratio are again equal.

1 marks

* Schematics of servomechanism of water level positioning

2 marks.



Pot. potentiometer

fig. 1 schematic diagram.

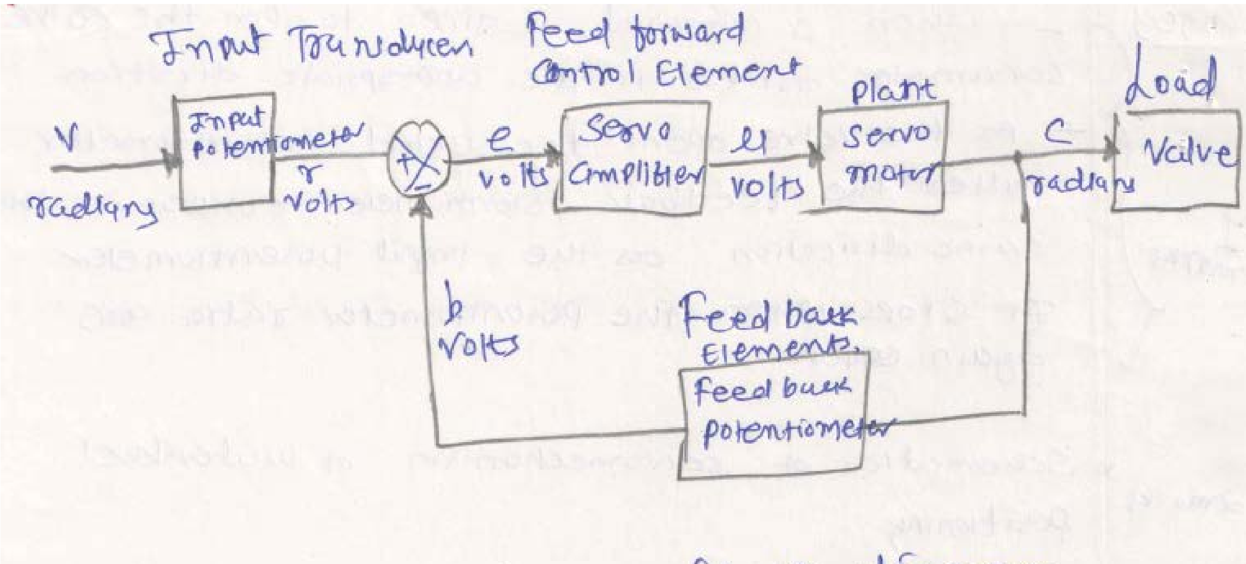


Fig. 2. Block diagram

Q.3 a. Derive the Laplace Transform of the following function.

(i) e^{at}

(ii) Sinωt **(8)**

Answer:

Answer: 1. $f(t) = E^{at}$ — 1 marks

$$L f(t) = \int_0^{\infty} e^{at} \cdot e^{-st} dt$$ — 2 marks

$$= \int_0^{\infty} e^{(a-s)t} dt$$ — 0.5 marks

$$\boxed{L e^{at} = \frac{1}{(s-a)}}$$ — 0.5 marks. — ①

2. $\sin\omega t$

$\Rightarrow$ - $f(t) = \sin\omega t$

- In function $f(t) = e^{at}$ put $a = j\omega$.

$e^{at} = e^{j\omega t}$ 1marks

Hence $f(t) = e^{j\omega t}$

therefor using Eq. ①

$$L e^{j\omega t} = \frac{1}{(s-j\omega)}$$ . . 1 marks

$\because$ $e^{j\omega t} = (\cos\omega t + j\sin\omega t)$.

$$\therefore L(\cos\omega t + j\sin\omega t) = \frac{1}{(s-j\omega)}$$

$$= \frac{1}{(s-j\omega)} \times \frac{(s+j\omega)}{(s+j\omega)}$$

$$= \frac{(s+j\omega)}{s^2+\omega^2}$$

1 marks.

separating into real & Imaginary parts.

$$\mathcal{L}\cos\omega t = \frac{s}{s^2+\omega^2}$$

$$\boxed{\mathcal{L}\sin\omega t = \frac{\omega}{s^2+\omega^2}}$$ 1 marks.

b. **Obtain the Solution of the differential equation given below**

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0$$

Given $x(0+) = 0$ **and** $x'(0+) = 1$ **(8)**

Answer:

Solution

~~Take~~ $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0$ ——— (1)

2 marks

Take Laplace transform. both side of above eq^n. (1)

$$\mathcal{L}\frac{d^2x}{dt^2} + \mathcal{L}\,2\left(\frac{dx}{dt}\right) + \mathcal{L}(2x) = 0$$

1 marks

$$s^2X(s) - sx(0+) - x'(0+) + [2sX(s) - x(0+)] + 2[X(s)] = 0$$

2 marks

– substituting $x(0+) = 0$ and $x'(0) = 1$

$$\Rightarrow [s^2X(s) - s\cdot 0 - 1] + 2[sX(s) - 0] + 2[X(s)] = 0$$

~~$X(s) = 1$~~

$$\Rightarrow s^2X(s) - 1 - 2sX(s) + 2X(s) = 0$$

$$X(s) = \frac{1}{(s^2+2s+2)}$$

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– on completing the square, the term (s^2+2s+2) can be expressed as $[(s+1)^2+(1)^2]$

1 marks

$$X(s) = \frac{1}{[(s+1)^2+(1)^2]}$$

Taking inverse Laplace transform on both sides

2 marks

$$\mathcal{L}^{-1}X(s) = \mathcal{L}^{-1}\left[\frac{1}{(s+1)^2+(1)^2}\right]$$

$$\boxed{x(t) = e^{-t}\sin t}$$

Ans.

Q.4 a. Define the Transfer function and Derive it for the given electrical network. (8)

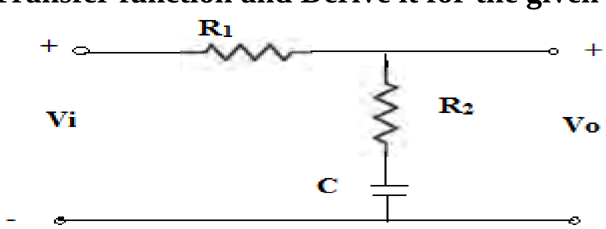
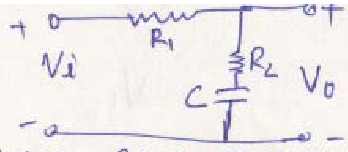


Figure 1

Answer:

Answer:



Transfer function:

The transfer function $P(s)$ of a continuous system is defined as that factor in the equation for $Y(s)$ multiplying the transform of the input $U(s)$. For Transfer function is $P(s) = \sum_{i=0}^{m} b_i s^i \Big/ \sum_{i=0}^{n} a_i s^i$

$$= \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0}$$

or

Transfer function: The transfer function of linear, time invariant differential equation system is ~~def~~ defined as the ratio of the Laplace transform of the output to Laplace transform of the input under assumption that all initial condition are zero.

2 marks

Transfer function $= P(s) = \left.\dfrac{\mathcal{L}(\text{output})}{\mathcal{L}(\text{input})}\right|_{\text{initial condition zero}}$

$$= \frac{Y(s)}{X(s) \text{ or } V(s)}.$$

~~P(s) =~~.

$$\boxed{P(s) = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_0}{a_0 s^n + b_1 s^{n-1} + \cdots + a_{n-1} s + a_0}}$$

Answer: Q.4 (A)

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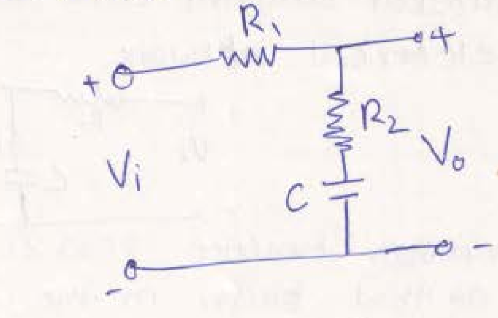
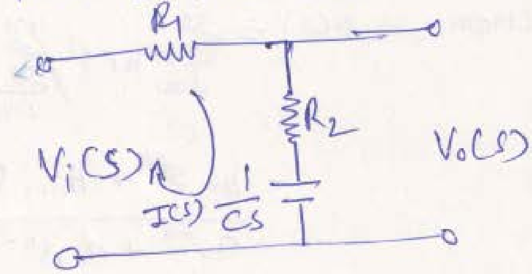


Applying KVL for the loop



$$\rightarrow R_1 I(S) + R_2 I(S) + \frac{1}{CS} I(S) = V_i(S) \quad \text{---} (1)$$

$$\left(R_1 + R_2 + \frac{1}{CS}\right) I(S) = V_i(S).$$

$$V_i(S) = \left[\frac{(R_1+R_2)CS + 1}{CS}\right] I(S) \quad \text{---} (2)$$

$$\rightarrow V_o(S) = \left(R_2 + \frac{1}{CS}\right) I(S)$$

$$= \left[\frac{R_2 CS + 1}{CS}\right] I(S) \quad \text{---} (3)$$

$\rightarrow$ Multiplying (1) & (3) the transfer function given by

$$T.F. = \frac{V_o(S)}{V_i(S)} = \frac{\frac{(R_2 CS + 1)}{CS} I(S)}{\left[\frac{(R_1+R_2)CS + 1}{CS}\right] I(S)}$$

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$$T.F = \frac{V_0(s)}{V_i(s)} = \frac{R_2 Cs + 1}{(R_1+R_2)Cs + 1}$$

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Answer

b. For the block diagram shown in fig. 2, determine the overall transfer function using block diagram reduction technique. (8)

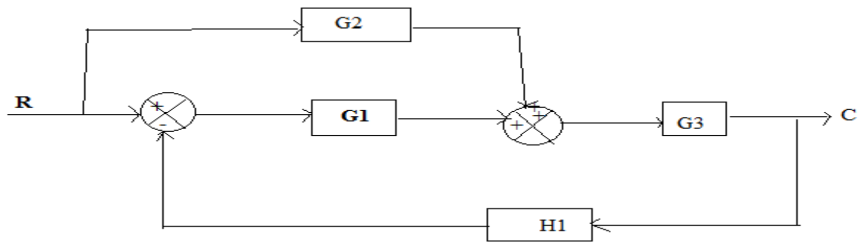

Figure 2

Answer:

Answer: (i) shifting a summing point after a block, we get the diagram as shown in fig. 1.

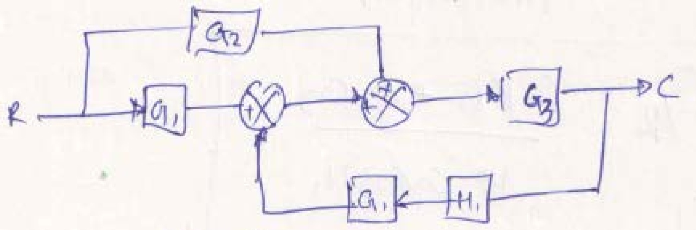

Fig. 1

(2) Interchanging consecutive summing points and combining the blocks connected in cascade.

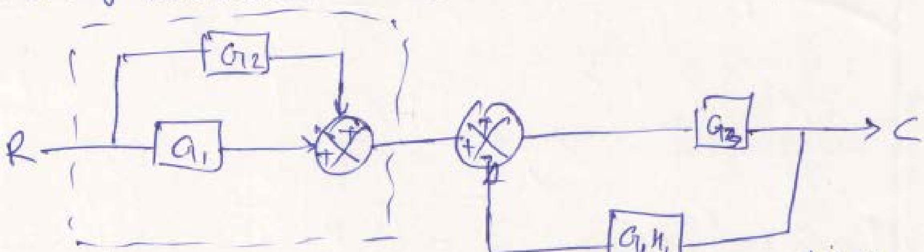

fig. 2

(iii) Combining block connected in parallel

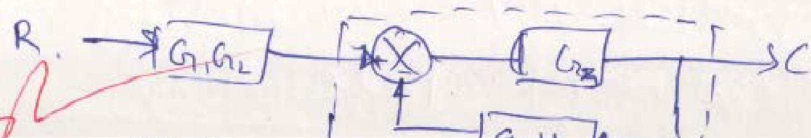

fig. 3

(iv) Eliminating feedback loop.

$$R \rightarrow \boxed{G_1 + G_2} \rightarrow \boxed{\frac{G_3}{1+G_1G_3H_1}} \rightarrow C$$

Fig. 4

(v) Combining the blocks connected in cascade

$$R \rightarrow \boxed{\frac{G_3(G_1+G_2)}{1+G_1G_3H_1}} \rightarrow C$$

Fig. 5

Hence overall Transfer function is given by

$$C/R = \frac{G_3(G_1+G_2)}{1+G_1G_3H_1}$$

$$\boxed{C/R = \frac{G_1G_3 + G_2G_3}{1+G_1G_3H_1}}$$

Q.5 **a. Find the Transfer function C/R for the system shown in fig.3 using Mason's gain formula.** **(8)**

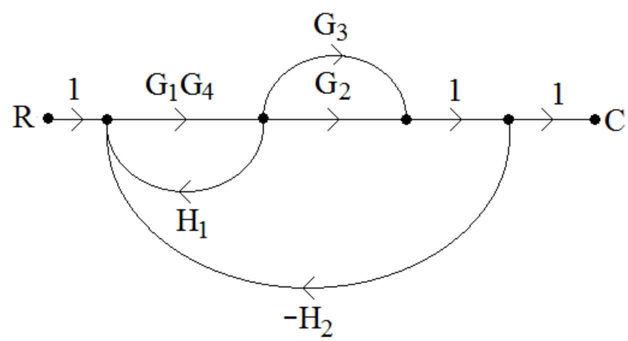

Figure 3

Answer:

Answer There are three feed back loops

3 marks $P_{11} = G_1 G_4 H_1$

$P_{21} = -G_1 G_2 G_4 H_2$

$P_{31} = -G_1 G_3 G_4 H_2$

There are no non touching loops and all loop touch both forward paths, then

1 marks $\Delta_1 = 1 \qquad \Delta_2 = 1$

Therefore the control ratio is

2 marks $$T = C/R = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

1 marks $$= \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

1 marks. $$\boxed{\frac{C}{R} = \frac{G_1G_4(G_2+G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}}$$

b. Explain transfer function computation of cascaded components using suitable example. (8)

Answer:

8.7 TRANSFER FUNCTION COMPUTATION OF CASCADED COMPONENTS

Loading effects of interacting components require little special attention using signal flow graphs. Simply combine the graphs of the components at their normal joining points (output node of one to the input node of another), account for loading by adding new loops at the joined nodes, and compute the overall gain using Equation (*8.2*). This procedure is best illustrated by example.

EXAMPLE 8.9. Assume that two identical resistance networks are to be cascaded and used as the control elements in the forward loop of a control system. The networks are simple voltage dividers of the form given in Fig. 8-20.

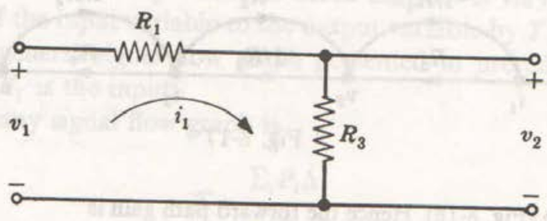

Fig. 8-20

Two independent equations for this network are

$$i_1 = \left(\frac{1}{R_1}\right)v_1 - \left(\frac{1}{R_1}\right)v_2 \qquad \text{and} \qquad v_2 = R_3 i_1$$

The signal flow graph is easily drawn (Fig. 8-21). The gain of this network is, by inspection, equal to

$$\frac{v_2}{v_1} = \frac{R_3}{R_1 + R_3}$$

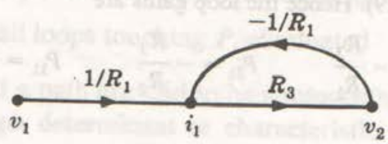

Fig. 8-21

If we were to ignore loading, the overall gain of two cascaded networks would simply be determined by multiplying the individual gains:

$$\left(\frac{v_2}{v_1}\right)^2 = \frac{R_3^2}{R_1^2 + R_3^2 + 2R_1R_3}$$

This answer is incorrect. We prove this in the following manner. When the two identical networks are cascaded, we note that the result is equivalent to the network of Example 8.6, with $R_2 = R_1$ and $R_4 = R_3$ (Fig. 8-22).

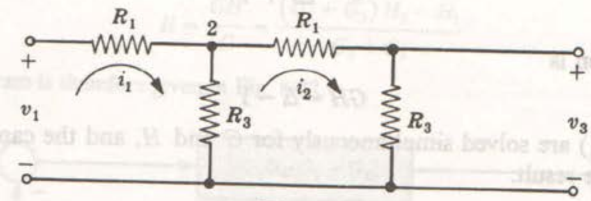

Fig. 8-22

The signal flow graph of this network was also determined in Example 8.6 (Fig. 8-23).

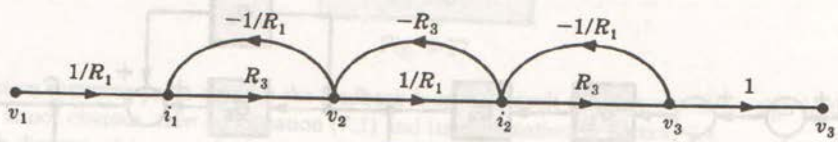

Fig. 8-23

We observe that the feedback branch $-R_3$ in Fig. 8-23 does not appear in the signal flow graph of the cascaded signal flow graphs of the individual networks connected from node v_2 to v_1' (Fig. 8-24). This means that, as a result of connecting the two networks, the second one loads the first, changing the equation for v_2 from

$$v_2 = R_3 i_1 \qquad \text{to} \qquad v_2 = R_3 i_1 - R_3 i_2$$

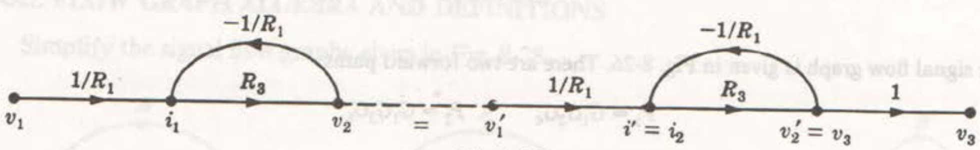

Fig. 8-24

This result could also have been obtained by directly writing the equations for the combined networks. In this case, only the equation for v_2 would have changed form.

The gain of the combined networks was determined in Example 8.8 as

$$\frac{v_3}{v_1} = \frac{R_3^2}{R_1^2 + R_3^2 + 3R_1R_3}$$

when R_2 is set equal to R_1 and R_4 is set equal to R_3. We observe that

$$\left(\frac{v_2}{v_1}\right)^2 = \frac{R_3^2}{R_1^2 + R_3^2 + 2R_1R_3} \neq \frac{v_3}{v_1}$$

It is good general practice to calculate the gain of cascaded networks directly from the *combined* signal flow graph. Most practical control system components load each other when connected in series.

Q.6 a. Derive Static error constant K_p, K_v, and K_a for unit step, unit ramp and unit parabolic input. (8)

Answer:

Answer (1) unit - step input (position step).

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For unit - step input signal

$r(t) = u(t)$

and $R(s) = 1/s$

The steady state error of the system for unit-step input is.

$$e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1+G(s)}$$

$$= \lim_{s \to 0} \frac{s \cdot 1/s}{1+G(s)} \quad \left[\because R(s) = 1/s \right]$$

$$= \lim_{s \to 0} \frac{1}{1+G(s)}$$

$$= \frac{1}{1+\lim_{s \to 0} G(s)}$$

$$= \frac{1}{1+G(0)}$$

$$\boxed{e_{ss} = \frac{1}{1+K_p}} \quad \text{——— (1)}$$

where $K_p = \lim_{s \to 0} G(s) = G(0)$

is defined as static position constant

(2) unit ramp input

$r(t) = t$, since $\frac{dr(t)}{dt} = 1$,

this ramp input is also termed the velocity input, which is just like the unit-step input. The Laplace transform of unit-ramp input is given by

$R(s) = 1/s^2$ ——— (2)

In this case steady state error is given by

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$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} \quad \text{———— (3)}$$

Put the value of R(s) in eqn. (3)
we get

$$e_{ss} = \lim_{s\to 0} \frac{s \times 1/s^2}{1+G(s)}$$

$$= \lim_{s\to 0} \frac{1}{s+sG(s)}$$

$$= \frac{1}{\lim_{s\to 0} sG(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_v}} \quad \text{———— (4)}$$

~~where~~ where $K_v = \lim_{s\to 0} sG(s)$ is defined as the static velocity error constant.

(3) Unit Parabolic Input:

$r(t) = t^2/2$

$$\frac{dr(t)}{dt} = \frac{2t}{2} = t \quad \text{———— (5)}$$

and $\frac{d^2r(t)}{dt^2} = 1$ since $\frac{d^2r(t)}{dt^2} = 1$, the unit-parabolic input $t^2/2$ is defined the unit acceleration input.

The Laplace transform of unit parabolic input

$R(s) = 1/s^3$

– The steady state error is given by

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} \quad \text{———— (6)}$$

Put the valle of R(s) in above eqn. (6)

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$$e_{ss} = \lim_{s \to 0} \frac{s \cdot 1/s^3}{1+G(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)}$$

$$= \frac{1}{\lim_{s \to 0} s^2 G(s)}$$

$$= \frac{1}{K_a}$$

$$\boxed{e_{ss} = \frac{1}{K_a}}$$

wher

$K_a = \lim_{s \to 0} s^2 G(s)$ is defined as the static acceleration error Constant

b. **For a Unity feedback control system the forward path transfer function is given by**

$$G(s) = \frac{20}{s(s+2)(s^2+2s+20)}$$

Determine the Steady state error of the system, when the inputs are: **(8)**

(i) 5 **(ii) 5t** **(iii)** $\frac{3t^2}{2}$.

Answer:

(i) $r(t) = 5 \quad \therefore R(t) = 5/s$

$$e_{ss} = \lim_{s \to 0} s \cdot E(s) = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{5}{s} \frac{1}{1+\left(\frac{20}{s(s+2)(s^2+2s+20)}\right)}$$

$$= \lim_{s \to 0} \frac{5s(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20)+20}$$

$$\boxed{\therefore e_{ss} = 0.}$$

(ii) $R(s) = 5/s^2$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{5}{s^2} \cdot \frac{1}{1+\frac{20}{s(s+2)(s^2+2s+20)}}$$

$$= \lim_{s \to 0} s \cdot \frac{5}{s^2} \cdot \frac{s(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20)}$$

$$\boxed{\therefore e_{ss} = 10}$$

(3) $R(s) = 3/s^3$

$$e_{ss} = \lim_{s \to 0} \frac{3}{s^3} \cdot \frac{s(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20)+20}$$

$$\boxed{e_{ss} = \infty}$$

Q.7 a. (i) Explain the Nyquist Stability criterion.
(ii) Write the Properties of POLAR plots. **(2x4)**

Answer:

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- Nyquist stability criterion.

- The closed loop control system whose open loop T.F. is GH is stable if and only if

$$N = -P_0 \leq 0$$

where

$P_0 =$ { no of poles (≥ 0) of GH in RHP for continous systems. / no. of poles (≥ 0) GH outside the unit circle (of z-plane) for discrete-time system }

N = total no. of CW encirclements of the (-1,0) point (i.e $GH = -1$) in the GH-plane (continuous or discrete).

If $N > 0$, the number of CW encirclements of 1+GH in RHP for continuous systems, or outside the unit circle for discrete system is determined by

$$Z_0 = N + P_0$$

If $N \leq 0$, the (-1, 0) point is not enclosed by the Nyquist stability plot. Therefore $N \leq 0$ if the region to the right of the contour in the prescribed direction does not include the (-1, 0) point. Shading this region helps significally in determining whether $N \leq 0$

If $N \leq 0$ and $P_0 = 0$, then the system is absolutely stable if and only if $N = 0$: that if and only if the (-1, 0) point does not lie in the shaded region.

→ Properties of Polar plot $P(\omega)$ [$P(\omega j)$ or $P(e^{-j\omega T})$]

1. The Polar plot for

$$P(\omega)+a$$

where a is any complex constant, is identical to the plot of $P(\omega)$ with the origin of coordinates shifted to the point $-a$ = (Re a + j Im a)

2. The Polar plot of the transfer function of a time-invariant linear system exhibits conjugate symmetry. That is the graph for $-\infty < \omega < 0$ is the mirror Image about the horizontal axis of the graph for $0 \le \omega < \infty$

3. The Polar plot may be constructed directly from a Bode plot. If one is available. value of magnitude and phase angle at various frequency ω on the Bode plot represent pointings along the locus of the polar plot.

4. Constant increments of frequency are not generally separated by equal intervals along the polar plot.

b. Sketch the Nyquist Stability plot for the Open loop transfer function given by

$GH(s) = \frac{1}{(s+p_1)(s+p_2)}$ **$p_1, p_2 > 0$** **(8)**

Answer:

Answer: The Nyquist path for this type 0 system

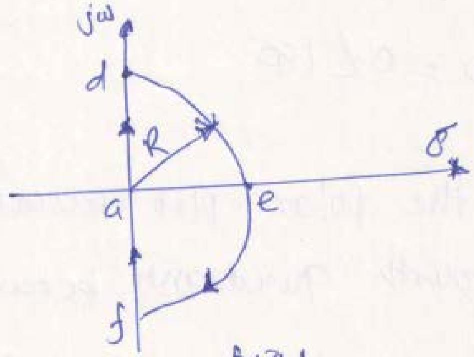

fig 1

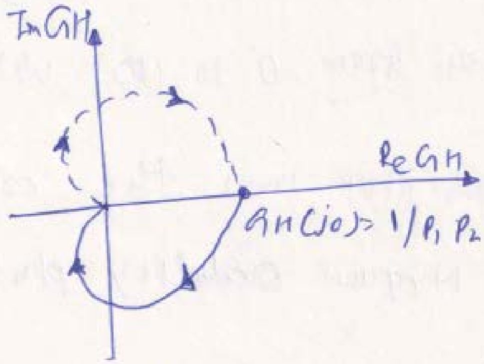

fig 2

Since there are no poles on the jω-axis the polar plot of GH(jω) yields the image of path $\overline{ad}$ (and hence $\overline{fad}$) in the GH(s) plane. Letting $s = j\omega$ for $0 < \omega < \infty$ we get

$$GH(j\omega) = \frac{1}{(j\omega + P_1)(j\omega + P_2)}.$$

$$= \frac{1}{\sqrt{(\omega^2+P_1^2)(\omega^2+P_2^2)}} \angle -\tan^{-1}\left(\frac{\omega}{P_1}\right) - \tan^{-1}\left(\frac{\omega}{P_2}\right)$$

$$GH(j0) = \frac{1}{P_1 P_2} \angle 0^\circ$$

$$\lim_{\omega \to \infty} GH(j\omega) = 0 \angle 180^\circ$$

For $0 < \omega < \infty$ the polar plot passes through the third and fourth quadrants because

$$\phi = -\tan^{-1}(\omega/P_1) + \tan^{-1}(\omega/P_2)\,]$$

varies from 0° to 180° when ω increase

— Path det plot into the origin $P(s) = 0$

Therefore the Nyquist stability plot is a replica of polar plot.

Q.8. a. A unity feedback control system has an open loop transfer function $G(S) = \frac{K}{s(s+4)}$

Draw the root locus and determine the value of K if the damping ratio ξ is to be 0.707 **(8)**

Answer:

Solution:-

1). The root loci start ($K=0$) From $s=0$ and $s=-4$

2). As there is no open loop zero root loci terminate ($K=\infty$) at infinity.

3). As the number of poles is 2 the number of root locus branches $N=2$.

4). The root locus on the real axis exists between $s=0$ and $s=-4$.

5). Break away points.

The characteristic equation is ;

$$s(s+4)+K=0$$

$$K=-(s^2+4s)$$

$$\therefore \frac{dK}{ds}=-(2s+4)$$

Put $\frac{dK}{ds}=0$

$$\therefore (2s+4)=0$$

$s=-2$ is a break away point

6). The angle of asymptotes is given by

$$\frac{(2K+1)180^\circ}{P-Z} \quad \text{where } K=0 \text{ and } 1$$

(i) $\frac{(2\times0\times1)180^\circ}{2-0}=90^\circ$

(ii) $\frac{(2 \times 1 \times 1) 180^\circ}{2 - 0} = 270^\circ$

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7). The symptotes intersect on reed axis at a point given by

$$x = \frac{\sum \text{Poles} - \sum \text{Zeros}}{P - Z}$$

$$= \frac{(0-4)-0}{2}$$

$$= 2$$

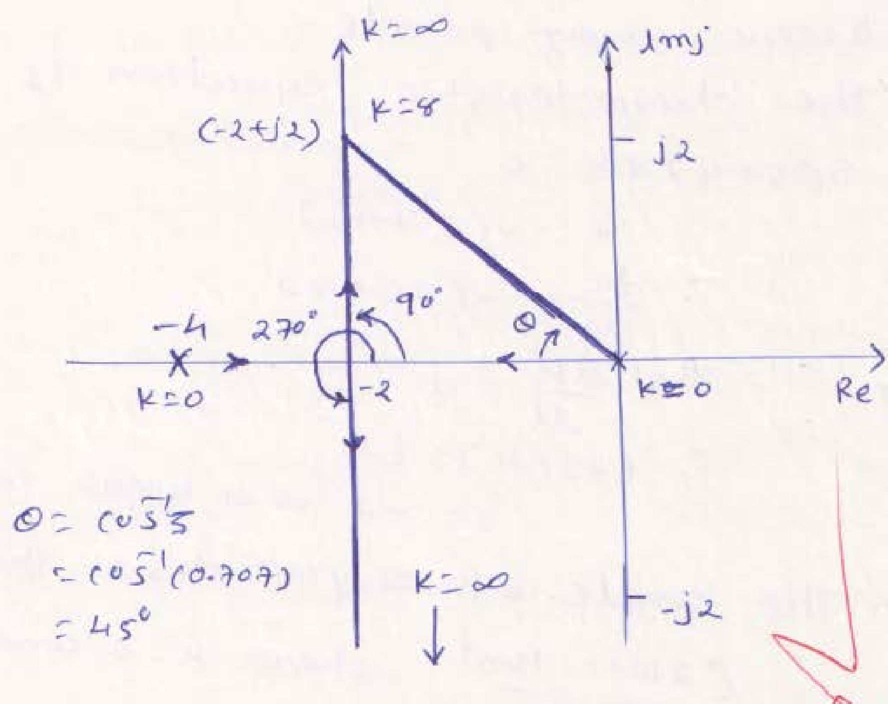
Using above data the root locus is plotted as under

Since $\zeta = 0.707$

and $\cos\theta = \zeta$

$\cos\theta = 0.707$

$\theta = 45^\circ$



Root locus for $G(s)H(s) = \frac{K}{s(s+4)}$

From the origin a line at angle $\theta = 45°$ is drawn as shown in fig. which intersects the root locus plot at $s = (-2+j2)$. As the point $s = -2+j2$ lies on the root locus the following equation should be satisfied at $s = (-2+j2)$

$$|G(s)H(s)| = 1$$

$$\therefore \left|\frac{K}{s(s+4)}\right| = 1$$

$$\left|\frac{K}{(-2+j2)[(-2+j2)+4]}\right| = 1$$

$$\left|\frac{K}{(-2+j2)(2+j2)}\right| = 1$$

$$\left|\frac{K}{\sqrt{(2^2+2^2)}\sqrt{(2^2+2^2)}}\right| = 1$$

$$\frac{K}{\sqrt{8}\sqrt{8}} = 1 \quad \text{or} \quad K = 8$$

b. Find break away point between 0 and 1 for $GH = \dfrac{K}{s(s+1)(s+3)(s+4)}$. **(4)**

Answer:

13.20. Find the breakaway point between 0 and -1 for

$$GH = \frac{K}{s(s+1)(s+3)(s+4)}$$

The breakaway point must satisfy

$$\frac{1}{\sigma_b} + \frac{1}{(\sigma_b+1)} + \frac{1}{(\sigma_b+3)} + \frac{1}{(\sigma_b+4)} = 0$$

If this equation were simplified, a third-order polynomial would be obtained. To avoid solving a third-order polynomial, the following procedure may be used. As a first guess, assume $\sigma_b = -0.5$ and use this value in the two terms for the poles furthest from the breakaway point. Then

$$\frac{1}{\sigma_b} + \frac{1}{\sigma_b+1} + \frac{1}{2.5} + \frac{1}{3.5} = 0$$

which simplifies to $\sigma_b^2 + 3.92\sigma_b + 1.46 = 0$ and has the root $\sigma_b = -0.43$ between 0 and -1. This value is used to obtain a better approximation as follows:

$$\frac{1}{\sigma_b} + \frac{1}{\sigma_b+1} + \frac{1}{2.57} + \frac{1}{3.57} = 0 \qquad \sigma_b^2 + 3.99\sigma_b + 1.496 = 0 \qquad \sigma_b = -0.424$$

The second computation did not result in a value much different from the first. A reasonable first guess can often result in a fairly accurate approximation with only one computation.

c. **Determine angle of departure for complex poles and angle of arrival for complex zeroes for** $GH = \dfrac{K(s+1+j)(s+1-j)}{s(s+2j)(s-2j)}$ **here K>0.** **(4)**

Answer:

13.25. Determine the departure angles from the complex poles and the arrival angles at the complex zeros for the open-loop transfer function

$$GH = \frac{K(s+1+j)(s+1-j)}{s(s+2j)(s-2j)} \qquad K > 0$$

For the complex pole at $s = 2j$,

$$\arg GH' = 45° + 71.6° - 90° - 90° = -63.4° \quad \text{and} \quad \theta_D = 180° - 63.4° = 116.6°$$

Since the root-locus is symmetric about the real axis, the departure angle from the pole at $s = -2j$ is $-116.6°$. For the complex zero $s = -1 + j$,

$$\arg GH'' = 90° - 108.4° - 135° - 225° = -18.4° \quad \text{and} \quad \theta_A = 180° - (-18.4°) = 198.4°$$

Thus the arrival angle at the complex zero $s = -1 - j$ is $\theta_A = -198.4°$.

Q.9 a. Define Gain margin, Phase margin, Gain Cross Over frequency and Phase cross over frequency. **(2x4)**

Answer:

- Gain Margin : It is the amount of gain in decibels (dB) that can be increased to drive the system to the verge of instability. It is denoted by GM. Gain margin is measured at the phase cross over. — 2

- Phase Margin : It is defined as the amount of additional phase lag at the gain cross-over frequency required to bring the system to verge of instability. It is denoted by PM or ϕ_M. — 2

- Gain cross over frequency:

The gain cross-over frequency is the frequency of G(jω) at the gain cross over or where,

$$|G(j\omega)| = 1$$

or

$$20 \log_{10} |G(j\omega)| = 0$$

It is denoted by ω_{gc}. — 2

- Phase cross-over frequency : The phase cross-over frequency is the frequency at the phase cross over. It is denoted by ω_{pc}. — 2

b. Sketch the Bode Plot for the Open –loop transfer function for the unity feedback system given below and assess stability.

$$G(S) = \frac{50}{(s+1)(s+2)}$$ **(8)**

Answer:

Solution:- Rewrite the transfer function is Bode form.

$$\therefore \quad G(s) = \frac{\frac{50}{2}}{(s+1)(\frac{s}{2}+1)} = \frac{25}{(s+1)(0.5s+1)}$$

Sinusoidal form of the transfer function is

$$G(j\omega) = \frac{25}{(j\omega+1)(j0.5\omega+1)}$$

As the system is type 0, the initial slope of the Bode plot is 0. db/decade and the initial ordinate is given by

$$20 \log_{10} 25 = 27.9 \text{ db}$$

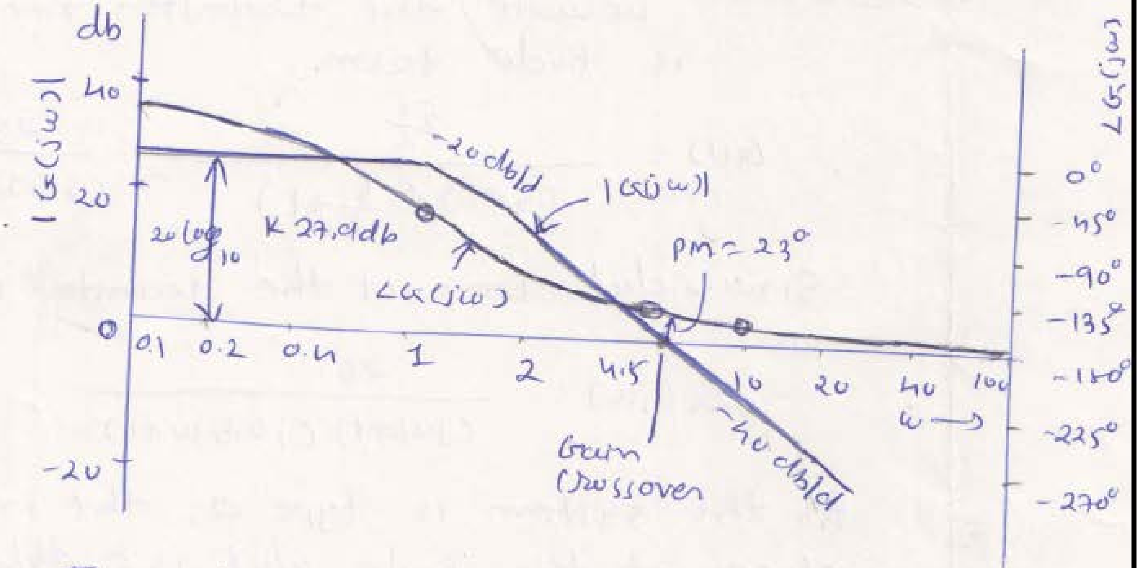
The two corner frequencies due to denominator terms are

$\omega = 1$ rad/sec and

$\omega = \frac{1}{0.5} = 2$ rad/sec

The slope of the magnitude plot after $\omega = 1$ rad/sec is $(0-20) = -20$ db/decade and after $\omega = 2$ rad/sec is $(-20-20) = -40$ db/decade.

Page-30 The frequency range of the plot is considered between $\omega = 0.1$ rad/sec and $\omega = 10$ rad/sec. The Bode plot is drawn as under.



The phase angle is given by

$$\angle G(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}(0.5\omega)$$

The phase angle for frequency range between $\omega = 0.1$ rad/sec and 10 rad/sec is calculated and given below.

ω rad/sec	0.1	1	5	10
$\angle G(j\omega)^\circ$	-8.5	-71.5	-146.8	-163.

The phase curve is asymptotic to $-180°$ line at high frequencies and the gain at high frequencies as such the gain margine is + infinite. At the gain cross-over frequency the phase angle is $-157°$

Hence phase margin is

$$P.M = 180° + (-157°) = 23°$$

→ The gain margin & phase margin both are positive, therefore the system is stable.