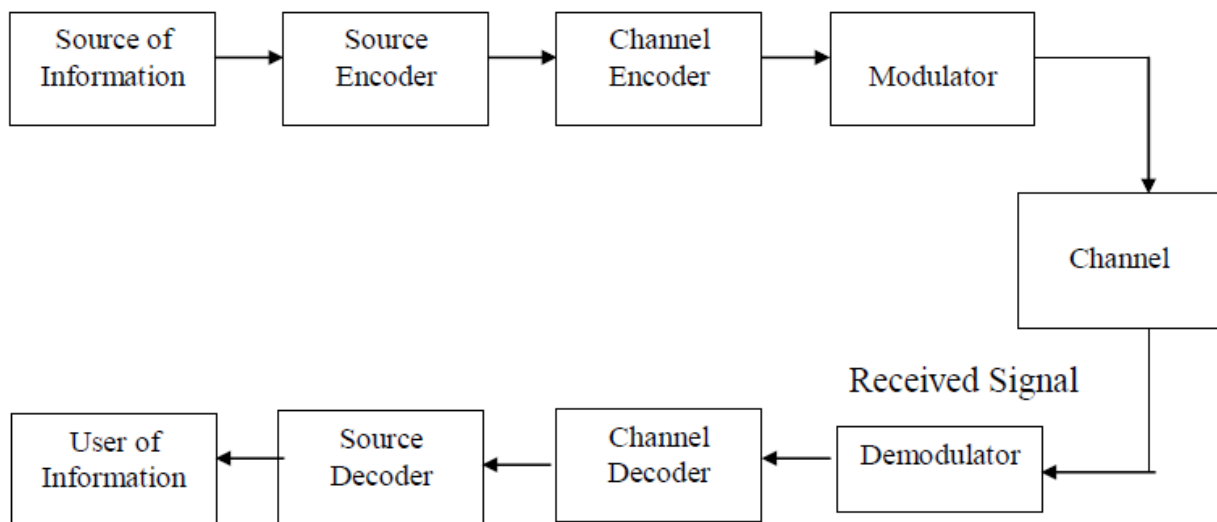


**Q.2 a. Draw the block diagram of digital communication system and explain the function of each block. (8)**

**Answer:**

The block diagram of the digital communication system is shown below

**Digital Communication System**



The figure shows the functional elements of a digital communication system.

Source of Information:

1. Analog Information Sources.
2. Digital Information Sources.

**Analog Information Sources** → Microphone actuated by a speech, TV Camera scanning a scene, continuous amplitude signals.

**Digital Information Sources** → These are teletype or the numerical output of computer which consists of a sequence of discrete symbols or letters. Analog information is transformed into discrete information through the process of sampling and quantizing.

**SOURCE ENCODER / DECODER:** The Source encoder ( or Source coder) converts the input i.e. symbol sequence into a binary sequence of 0's and 1's by assigning code words to the symbols in the input sequence. For eg. :-If a source set is having hundred symbols, then the number of bits used to represent each symbol will be 7 because  $2^7=128$  unique combinations are available. The important parameters of a source encoder are block size, code word lengths, average data rate and the efficiency of the coder (i.e. actual output data rate compared to the minimum achievable rate) at the receiver; the source decoder converts the binary output of the channel decoder into a symbol sequence. The decoder for a system using fixed – length code words are quite simple, but the decoder for a system using variable – length code words will be very complex. Aim of the source coding is to remove the redundancy in the transmitting information, so that bandwidth required for transmission is minimized. Based on the probability of the symbol code word is assigned. Higher the probability, shorter is the codeword. Ex: Huffman coding.

**CHANNEL ENCODER / DECODER:** Error control is accomplished by the channel coding operation that consists of systematically adding extra bits to the output of the source coder. These extra bits do not convey any information but helps the receiver to detect and / or correct some of the errors in the information bearing bits. There are two methods of channel coding:

1. Block Coding: The encoder takes a block of “k” information bits from the source encoder and adds “r” error control bits, where “r” is dependent on “k” and error control capabilities desired.
2. Convolution Coding: The information bearing message stream is encoded in a continuous fashion by continuously interleaving information bits and error control bits. The Channel decoder recovers the information bearing bits from the coded binary stream. Error detection and possible correction is also performed by the channel decoder. The important parameters of coder / decoder are: Method of coding, efficiency, error control capabilities and complexity of the circuit.

**MODULATOR:** The Modulator converts the input bit stream into an electrical waveform suitable for transmission over the communication channel. Modulator can be effectively used to minimize the effects of channel noise, to match the frequency spectrum of transmitted signal with channel characteristics, to provide the capability to multiplex many signals.

**DEMODULATOR:** The extraction of the message from the information bearing waveform produced by the modulation is accomplished by the demodulator. The output of the demodulator is bit stream. The important parameter is the method of demodulation.

**CHANNEL:** The Channel provides the electrical connection between the source and destination. The different channels are: Pair of wires, Coaxial cable, Optical fiber, Radio channel, Satellite channel or combination of any of these. The communication channels have only finite Bandwidth, non-ideal frequency response, the signal often suffers amplitude and phase distortion as it travels over the channel. Also, the signal power decreases due to the attenuation of the channel. The signal is corrupted by unwanted, unpredictable electrical signals referred to as noise. The important parameters of the channel are Signal to Noise power Ratio (SNR), usable bandwidth, amplitude and phase response and the statistical properties of noise.

- b. A message source generates one of four messages randomly every microsecond. The probabilities of these messages are 0.5, 0.4, 0.3 and 0.1. Each emitted message is independent of other messages in the sequence: (4)
- (i) What is the source entropy?
  - (ii) What is the rate of information generated by this source in bits per second?

**Answer:**

It is given that,

1. Number of messages,  $M = 4$ , let us denote them by  $m_1, m_2, m_3$  and  $m_4$ .
2. Their probabilities are  $p_1 = 0.5, p_2 = 0.4, p_3 = 0.3$  and  $p_4 = 0.1$ .
3. One message is transmitted per microsecond.

$$\therefore \text{Message transmission rate } r = \frac{1}{1 \times 10^{-6}} = 1 \times 10^6 \text{ messages/sec.}$$

- a. To obtain the source entropy (H) :

$$\begin{aligned}
 H &= \sum_{k=1}^4 p_k \log_2 (1/p_k) \\
 \therefore H &= p_1 \log_2 (1/p_1) + p_2 \log_2 (1/p_2) + p_3 \log_2 (1/p_3) + p_4 \log_2 (1/p_4) \\
 &= 0.5 \log_2 (1/0.5) + 0.4 \log_2 (1/0.4) + 0.3 \log_2 (1/0.3) + 0.1 \log_2 (1/0.1) \\
 \therefore H &= 1.871 \text{ bits/message} \quad \dots \text{Ans.}
 \end{aligned}$$

b. To obtain the information rate (R) :

$$R = H \times r = 1.871 \times 1 \times 10^6 = 1.871 \text{ M bits/sec} \quad \dots \text{Ans}$$

c. Derive an expression for channel capacity of a discrete memory less channel. (4)

Answer:

Consider a discrete memoryless channel with input alphabet  $\mathcal{X}$ , output alphabet  $\mathcal{Y}$ , and transition probabilities  $p(y_k|x_j)$ . The mutual information of the channel is defined by the first line of Eq. 2.46, which is reproduced here for convenience:

$$I(\mathcal{X};\mathcal{Y}) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j, y_k) \log_2 \left[ \frac{p(y_k|x_j)}{p(y_k)} \right] \quad (2.57)$$

Here we note that (see Eq. 2.36)

$$p(x_j, y_k) = p(y_k|x_j)p(x_j)$$

Also, from Eq. 2.37, we have

$$p(y_k) = \sum_{j=0}^{J-1} p(y_k|x_j)p(x_j)$$

From these three equations we see that it is necessary for us to know the input probability distribution  $\{p(x_j), j = 0, 1, \dots, J-1\}$  so that we may calculate the mutual information  $I(\mathcal{X};\mathcal{Y})$ . The mutual information of a channel, therefore, depends not only on the channel, but also on the way in which the channel is used.

The input probability distribution  $\{p(x_j)\}$  is obviously independent of the channel. We can then maximize the average mutual information  $I(\mathcal{X};\mathcal{Y})$  of the channel with respect to  $\{p(x_j)\}$ . Hence, we define the channel capacity of a discrete memoryless channel as the maximum average mutual information  $I(\mathcal{X};\mathcal{Y})$  in any single use of the channel (i.e., signaling interval), where the maximization is over all possible input probability distributions  $\{p(x_j)\}$  on  $\mathcal{X}$ . The channel capacity is commonly denoted by  $C$ . We thus write

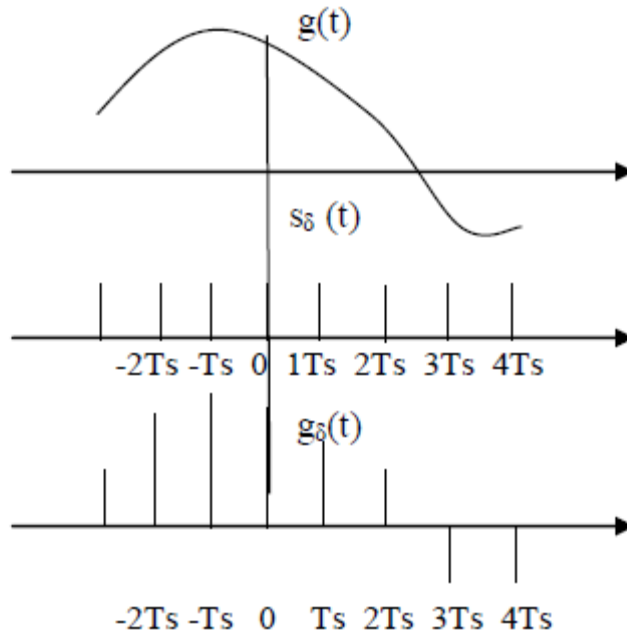
$$C = \max_{\{p(x_j)\}} I(\mathcal{X};\mathcal{Y}) \quad (2.58)$$

The channel capacity  $C$  is measured in bits per channel use.

**Q.3 a. State and prove sampling theorem for low pass signal and band pass signals. (8)**

Answer:

**Statement:** - "If a band-limited signal  $g(t)$  contains no frequency components for  $|f| > W$ , then it is completely described by instantaneous values  $g(kT_s)$  uniformly spaced in time with period  $T_s \leq 1/2W$ . If the sampling rate,  $f_s$  is equal to the Nyquist rate or greater ( $f_s \geq 2W$ ), the signal  $g(t)$  can be exactly reconstructed.



**Fig : Sampling process**

**Proof:**

Part - I : If a signal  $x(t)$  does not contain any frequency component beyond  $W$  Hz, then the signal is completely described by its instantaneous uniform samples with sampling interval (or period) of  $T_s < 1/(2W)$  sec.

Part – II The signal  $x(t)$  can be accurately reconstructed (recovered) from the set of uniform instantaneous samples by passing the samples sequentially through an ideal (brick-wall) lowpass filter with bandwidth  $B$ , where  $W \leq B < f_s - W$  and  $f_s = 1/(T_s)$ . If  $x(t)$  represents a continuous-time signal, the equivalent set of instantaneous uniform samples  $\{x(nT_s)\}$  may be represented as,

$$\{x(nT_s)\} \equiv x_s(t) = \sum x(t) \cdot \delta(t - nT_s) \text{ ----- 1.1}$$

where  $x(nT_s) = x(t)|_{t=nT_s}$ ,  $\delta(t)$  is a unit pulse singularity function and  $n$  is an integer. The continuous-time signal  $x(t)$  is multiplied by an (ideal) impulse train to obtain  $\{x(nT_s)\}$  and can be rewritten as,

$$x_s(t) = x(t) \cdot \sum \delta(t - nT_s) \text{ ----- 1.2}$$

Now, let  $X(f)$  denote the Fourier Transform  $F(T)$  of  $x(t)$ , i.e.

$$X(f) = \int_{-\infty}^{+\infty} x(t) \cdot \exp(-j2\pi ft) dt \text{ ----- 1.3}$$

Now, from the theory of Fourier Transform, we know that the F.T of  $\sum \delta(t- nTs)$ , the impulse train in time domain, is an impulse train in frequency domain:

$$F\{\sum \delta(t- nTs)\} = (1/Ts).\sum \delta(f- n/Ts) = fs.\sum \delta(f- nfs) \text{ ----- 1.4}$$

If  $X_s(f)$  denotes the Fourier transform of the energy signal  $x_s(t)$ , we can write using Eq. (1.2.4) and the convolution property:

$$\begin{aligned} X_s(f) &= X(f)* F\{\sum \delta(t- nTs)\} \\ &= X(f)*[fs.\sum \delta(f- nfs)] \\ &= fs.X(f)*\sum \delta(f- nfs)\text{-----1.5} \end{aligned}$$

This equation, when interpreted appropriately, gives an intuitive proof to Nyquist theorems as stated above and also helps to appreciate their practical implications. Let us note that while writing Eq.(1.5), we assumed that  $x(t)$  is an energy signal so that its Fourier transform exists. With this setting, if we assume that  $x(t)$  has no appreciable frequency component greater than  $W$  Hz and if  $fs > 2W$ , then Eq.(1.5) implies that  $X_s(f)$ , the Fourier Transform of the sampled signal  $x_s(t)$  consists of infinite number of replicas of  $X(f)$ , centered at discrete frequencies  $n.fs$ ,  $-\infty < n < \infty$  and scaled by a constant  $fs = 1/Ts$

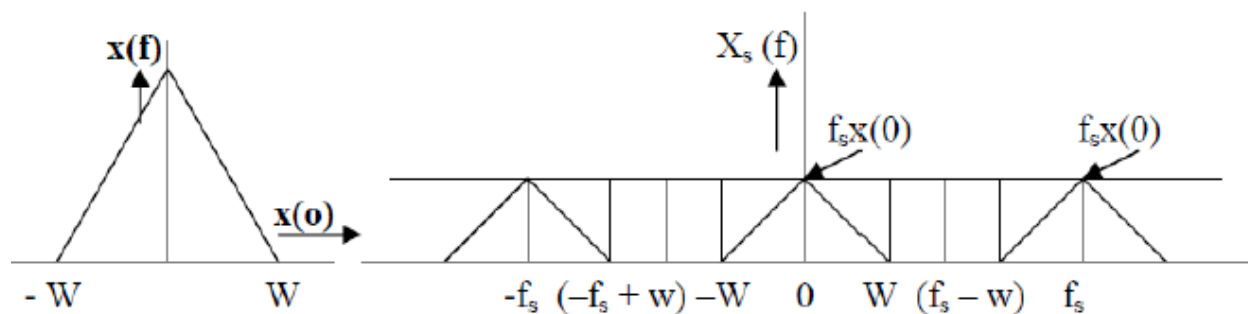


Fig.1

Fig. 1 indicates that the bandwidth of this instantaneously sampled wave  $x_s(t)$  is infinite while the spectrum of  $x(t)$  appears in a periodic manner, centered at discrete frequency values  $n.fs$ .

Part – I of the sampling theorem is about the condition  $fs > 2.W$  i.e.  $(fs - W) > W$  and  $(- fs + W) < -W$ . As seen from Fig. 1, when this condition is satisfied, the spectra of  $x_s(t)$ , centered at  $f = 0$  and  $f = \pm fs$  do not overlap and hence, the spectrum of  $x(t)$  is present in  $x_s(t)$  without any distortion.

This implies that  $x_s(t)$ , the appropriately sampled version of  $x(t)$ , contains all information about  $x(t)$  and thus represents  $x(t)$ .

The second part suggests a method of recovering  $x(t)$  from its sampled version  $x_s(t)$  by using an ideal lowpass filter. As indicated by dotted lines in Fig. 1, an ideal low pass filter (with brick-wall type response) with a bandwidth  $W \leq B < (fs - W)$ , when fed with  $x_s(t)$ , will allow the portion of  $X_s(f)$ , centered at  $f = 0$  and will reject all its replicas at  $f = n fs$ , for  $n \neq 0$ . This implies that the shape of the continuous time signal  $x_s(t)$ , will be retained at the output of the ideal filter.

- b. Draw Block diagram of PAM-TDM (pulse amplitude modulation-time division multiplexing) and explain the process in detail. (8)

Answer:

#### 4.7 TIME-DIVISION MULTIPLEXING

An important feature of pulse-amplitude modulation is a *conservation of time*. That is, for a given message signal, transmission of the associated PAM wave engages the communication channel for only a fraction of the sampling interval on a periodic basis. Hence, some of the time interval between adjacent pulses of the PAM wave is cleared for use by other independent message signals on a *time-shared basis*. By so doing, we obtain a *time-division multiplex system* (TDM), which enables the joint utilization of a common channel by a plurality of independent message signals without mutual interference.

The concept of TDM is illustrated by the block diagram shown in Fig. 4.19. Each input message signal is first restricted in bandwidth by a low-pass pre-alias filter to remove the frequencies that are nonessential to an adequate signal representation. The pre-alias filter outputs are then applied to a *commutator*, which is usually implemented using electronic switching circuitry. The function of the commutator is two-fold: (1) to take a narrow sample of each of the  $N$  input messages at a rate  $f_s$  that is slightly higher than  $2W$ , where  $W$  is the cutoff frequency of the pre-alias filter, and (2) to sequentially interleave these  $N$  samples inside a sampling interval  $T_s = 1/f_s$ . Indeed, this latter function is the

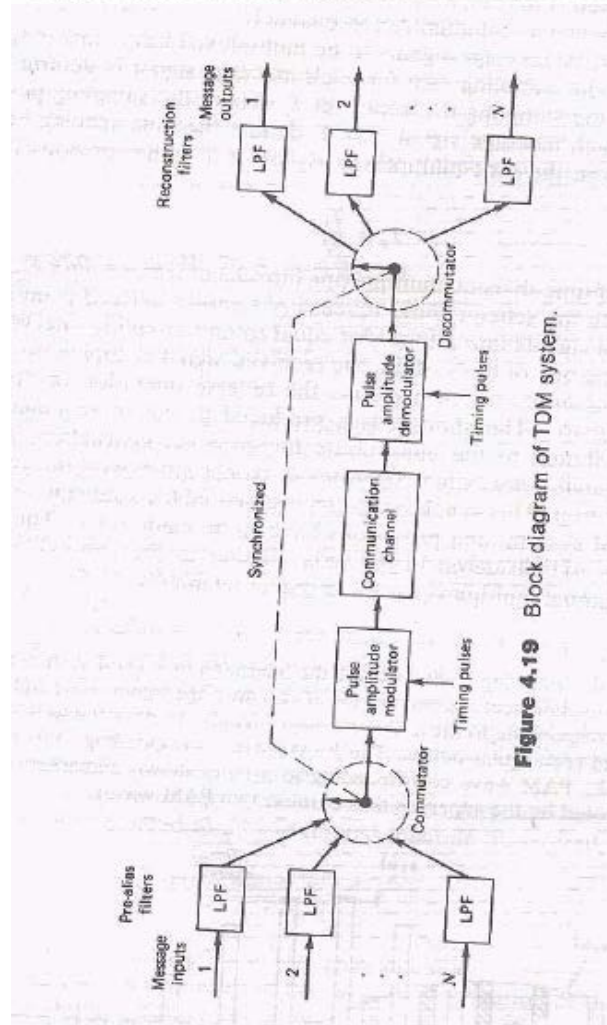


Figure 4.19 Block diagram of TDM system.



essence of the time-division multiplexing operation. Following the commutation process, the multiplexed signal is applied to a *pulse-amplitude modulator*, the purpose of which is to transform the multiplexed signal into a form suitable for transmission over the communication channel.

Suppose that the  $N$  message signals to be multiplexed have similar spectral properties. Then the sampling rate for each message signal is determined in accordance with the sampling theorem. Let  $T_s$  denote the sampling period so determined for each message signal. Let  $T_x$  denote the time spacing between adjacent samples in the time-multiplexed signal. It is rather obvious that we may set

$$T_x = \frac{T_s}{N} \quad (4.70)$$

Hence, the use of time-division multiplexing introduces a *bandwidth expansion factor*  $N$ , because the scheme must squeeze  $N$  samples derived from  $N$  independent message signals into a time slot equal to one sampling interval.

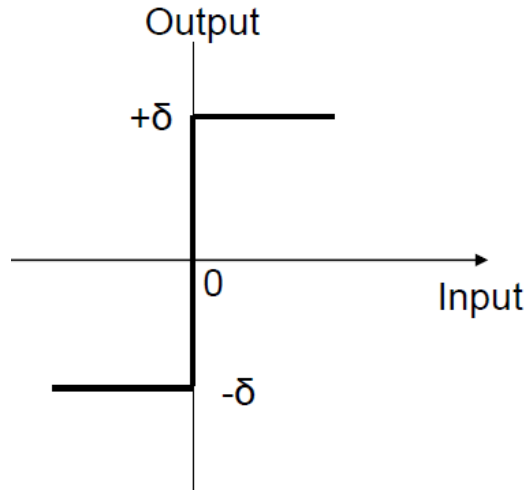
At the receiving end of the system, the received signal is applied to a *pulse-amplitude demodulator*, which performs the reverse operation of the pulse-amplitude modulator. The short pulses produced at the pulse demodulator output are distributed to the appropriate low-pass reconstruction filters by means of a *decommutator*, which operates in *synchronism* with the commutator in the transmitter. This synchronization is essential for a satisfactory operation of the TDM system, and provisions have to be made for it. The issue of synchronization is considered in the next chapter in the context of the T1 carrier, a 24-channel multiplexer used in digital telephony. //

**Q.4 a. Explain Delta Modulation (DM) in detail with the help of neat block diagram.**

**Also discuss its advantages and disadvantages. (8)**

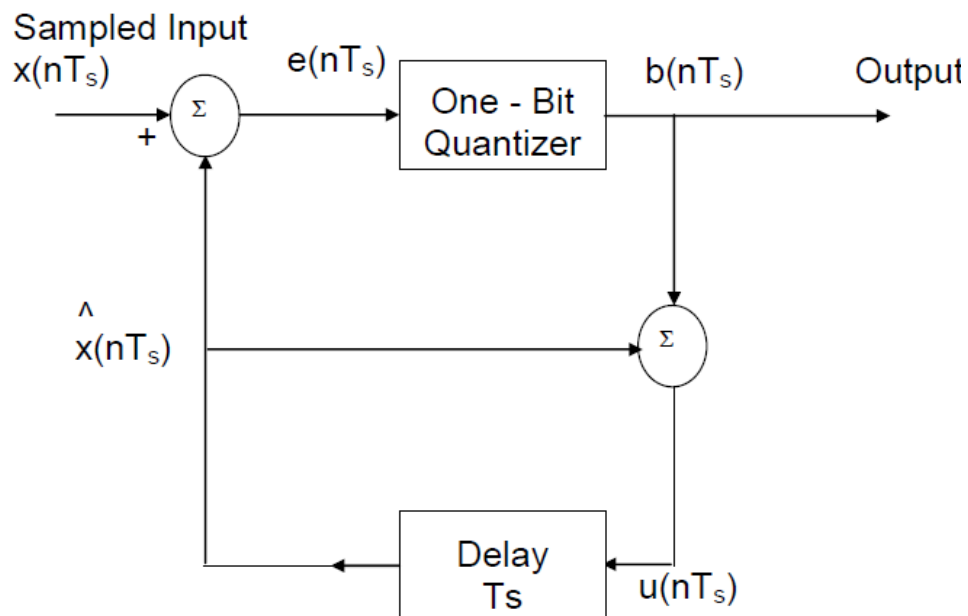
**Answer:**

Delta Modulation is a special case of DPCM. In DPCM scheme if the base band signal is sampled at a rate much higher than the Nyquist rate purposely to increase the correlation between adjacent samples of the signal, so as to permit the use of a simple quantizing strategy for constructing the encoded signal, Delta modulation (DM) is precisely such a scheme. Delta Modulation is the one-bit (or two-level) versions of DPCM. DM provides a staircase approximation to the over sampled version of an input base band signal. The difference between the input and the approximation is quantized into only two levels, namely,  $\pm\delta$  corresponding to positive and negative differences, respectively. Thus, if the approximation falls below the signal at any sampling epoch, it is increased by  $\delta$ . Provided that the signal does not change too rapidly from sample to sample, we find that the stair case approximation remains within  $\pm\delta$  of the input signal. The symbol  $\delta$  denotes the absolute value of the two representation levels of the one-bit quantizer used in the DM.



*Input-Output characteristics of the delta modulator.*

Let the input signal be  $x(t)$  and the staircase approximation to it is  $u(t)$ .

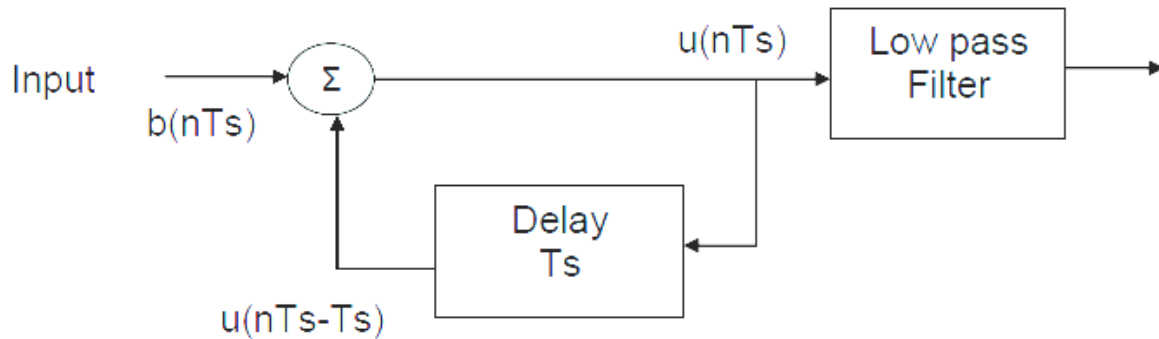


*Block diagram for Transmitter of a DM system*

In the receiver the stair case approximation  $u(t)$  is reconstructed by passing the incoming sequence of positive and negative pulses through an accumulator in a manner similar to that used in the transmitter. The out-of-band quantization noise in the high frequency staircase waveform  $u(t)$  is rejected by passing it through a low-pass filter with a band-width equal to the original signal bandwidth. Delta modulation offers two unique features:

1. No need for Word Framing because of one-bit code word.
2. Simple design for both Transmitter and Receiver





*Block diagram for Receiver of a DM system*

- b. A PCM signal uses a uniform Quantizer followed by a 7 bit binary encoder. The bit rate of the system is equal to  $100 \times 10^6$  bits/second.
- (i) What is the maximum message bandwidth for which system operates satisfactory?
- (ii) Calculate the output signal to quantization noise ratio when the full load sinusoidal modulating wave of frequency 2 MHz is applied to the input. (8)

Answer:

- (i) Let us assume the message bandwidth is F. Sampling frequency is equal to

$$f_s \geq 2F$$

The signaling rate is given as

$$\begin{aligned} r &\geq n \cdot f_s \\ &7 \times 2F \\ r &= 100 \times 10^6 = 14 \text{ W} \\ W &\leq 7.14 \text{ MHz} \end{aligned}$$

So, the maximum bandwidth is 7.14 MHz

- (ii) The output S/N ratio is equal to

$$\begin{aligned} (S/N)_{dB} &= 1.8 + 6n \\ &= 1.8 + 6 \times 7 \\ &= 43.8 \text{ dB} \end{aligned}$$

**Q.5 a. Explain Inter symbol interference.**

(8)

Answer:

### 6.3 INTERSYMBOL INTERFERENCE

Consider Fig. 6.5, which depicts the basic elements of a *baseband binary PAM system*. The input signal consists of a binary data sequence  $\{b_k\}$  with a bit duration of  $T_b$  seconds. This sequence is applied to a pulse generator, producing the discrete PAM signal

$$x(t) = \sum_{k=-\infty}^{\infty} a_k v(t - kT_b) \quad (6.18)$$

where  $v(t)$  denotes the basic pulse, normalized such that  $v(0) = 1$ , as in Eq. 6.2. The coefficient  $a_k$  depends on the input data and the type of format used. The waveform  $x(t)$  represents one realization of the random process  $X(t)$  considered in Section 6.2. Likewise,  $a_k$  is a sample value of the random variable  $A_k$ .

The PAM signal  $x(t)$  passes through a transmitting filter of transfer function  $H_T(f)$ . The resulting filter output defines the transmitted signal, which is modified as a result of transmission through the channel of transfer function  $H_C(f)$ . The channel may represent a coaxial cable or optical fiber, where the major source of system degradation is *dispersion* in the channel. In any event, for the present we assume that the channel is *noiseless* but dispersive. The channel output is passed through a receiving filter of transfer function  $H_R(f)$ . This filter output is sampled synchronously with the transmitter, with the sampling instants being determined by a clock or timing signal that is usually extracted from the receiving filter output. Finally, the sequence of samples thus obtained is used to reconstruct the original data sequence by means of a decision device. Each sample is compared to a threshold. We assume that symbols 1 and 0 are equally likely, and the threshold is set half way between their representation levels. If the threshold is exceeded, a decision is made in favor of symbol 1. If, on the other hand, the threshold is not exceeded, a decision is made in favor of symbol 0. If the sample value equals the threshold exactly, the flip of a fair coin will determine which symbol was transmitted.

The receiving filter output may be written as\*

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k p(t - kT_b) \quad (6.19)$$

where  $\mu$  is a scaling factor and the pulse  $p(t)$  is *normalized* such that

$$p(0) = 1 \quad (6.20)$$

\* To be precise, an arbitrary time delay  $t_0$  should be included in the argument of the pulse  $p(t - kT_b)$  in Eq. 6.19 to represent the effect of transmission delay through the system. For convenience, we have put this delay equal to zero in Eq. 6.19.

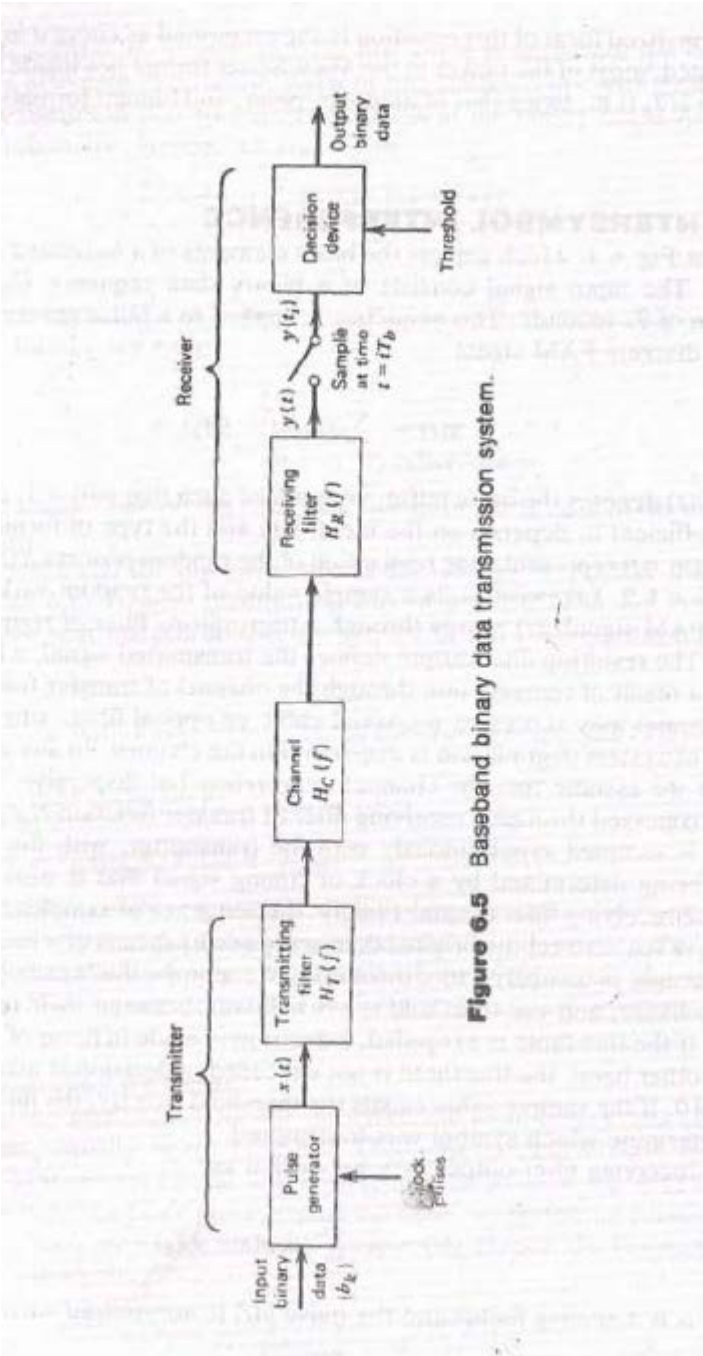


Figure 6.5 Baseband binary data transmission system.

The output  $y(t)$  is produced in response to the binary data waveform applied to the input of the transmitting filter. Specifically, the pulse  $\mu p(t)$  is the response of the cascade connection of the transmitting filter, the channel, and the receiving filter, which is produced by the pulse  $v(t)$  applied to the input of this cascade connection. Therefore, we may relate  $p(t)$  to  $v(t)$  in the frequency domain by writing

$$\mu P(f) = V(f)H_T(f)H_C(f)H_R(f) \quad (6.21)$$

where  $P(f)$  and  $V(f)$  are the Fourier transforms of  $p(t)$  and  $v(t)$ , respectively. Note that the normalization of  $p(t)$  as in Eq. 6.20 means that the total area under the curve of  $P(f)$  equals unity.

The receiving filter output  $y(t)$  is sampled at time  $t_i = iT_b$  (with  $i$  taking on integer values), yielding

$$\begin{aligned} y(t_i) &= \mu \sum_{k=-\infty}^{\infty} a_k p(iT_b - kT_b) \\ &= \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p(iT_b - kT_b) \end{aligned} \quad (6.22)$$

In Eq. 6.22, the first term  $\mu a_i$  is produced by the  $i$ th transmitted bit. The second term represents the residual effect of all other transmitted bits on the decoding of the  $i$ th bit; this residual effect is called *intersymbol interference* (ISI).

In physical terms, ISI arises because of imperfections in the overall frequency response of the system. When a short pulse of duration  $T_b$  seconds is transmitted through a band-limited system, the frequency components constituting the input pulse are differentially attenuated and, more significantly, differentially delayed by the system. Consequently, the pulse appearing at the output of the system is *dispersed* over an interval longer than  $T_b$  seconds. Thus, when a sequence of short pulses (representing binary 1s and 0s) are transmitted through the system, one pulse every  $T_b$  seconds, the dispersed responses originating from different symbol intervals will interfere with each other, thereby resulting in intersymbol interference.

In the absence of ISI, we observe from Eq. 6.22 that

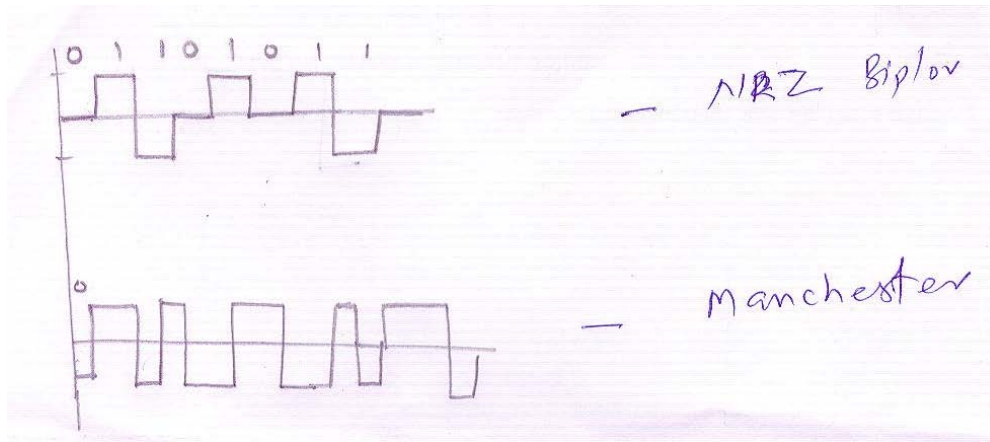
$$y(t_i) = \mu a_i$$

which shows that, under these conditions, the  $i$ th transmitted bit can be decoded correctly. The presence of ISI in the system, however, introduces errors in the decision device at the receiver output. Therefore, in the design of the transmitting and receiving filters, the objective is to minimize the effects of ISI, and thereby deliver the digital data to its destination with the smallest error rate possible.

- b. Construct the NRZ bipolar and Manchester format for the binary sequence 011010110. (4)

Answer:





c. Explain generalized form of Correlative Coding.

(4)

Answer:

### (3) Generalized Form of Correlative Coding

The duobinary and modified duobinary techniques have correlation spans of 1 binary digit and 2 binary digits, respectively. It is a straightforward matter to generalize these two techniques to other schemes, which are known collectively as *correlative coding schemes*. This generalization is shown in Fig. 6.17, where  $H_C(f)$  is defined in Eq. 6.40. It involves the use of a tapped-delay-line filter with tap weights  $w_0, w_1, \dots, w_{N-1}$ . Specifically, a correlative sample  $c_k$  is obtained from a superposition of  $N$  successive input sample values  $b_k$ , as shown by

$$c_k = \sum_{n=0}^{N-1} w_n b_{k-n} \quad (6.56)$$

Thus, by choosing various combinations of integer values for the  $w_n$ , we obtain different forms of correlative coding schemes to suit individual applications. For example, in the duobinary case, we have

$$w_0 = +1$$

$$w_1 = +1$$

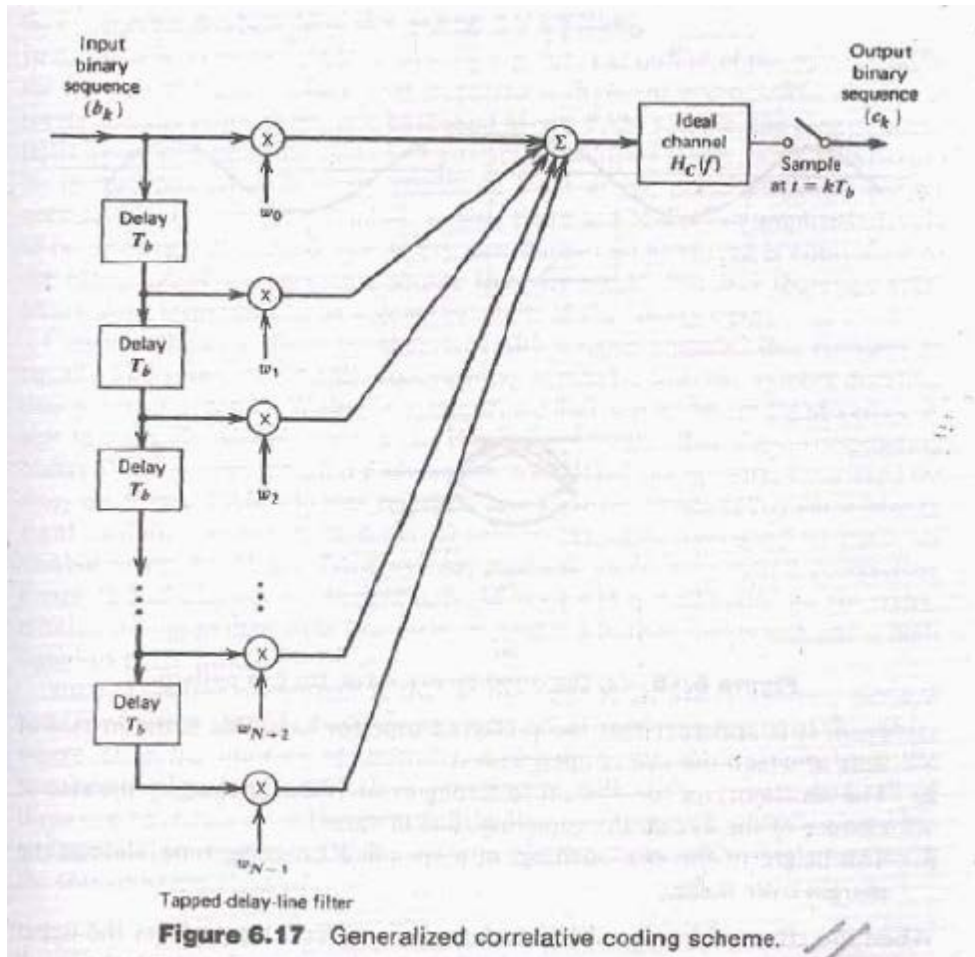
and  $w_n = 0$  for  $n \geq 2$ . In the modified duobinary case, we have

$$w_0 = +1$$

$$w_1 = 0$$

$$w_2 = -1$$

and  $w_n = 0$  for  $n \geq 3$ .

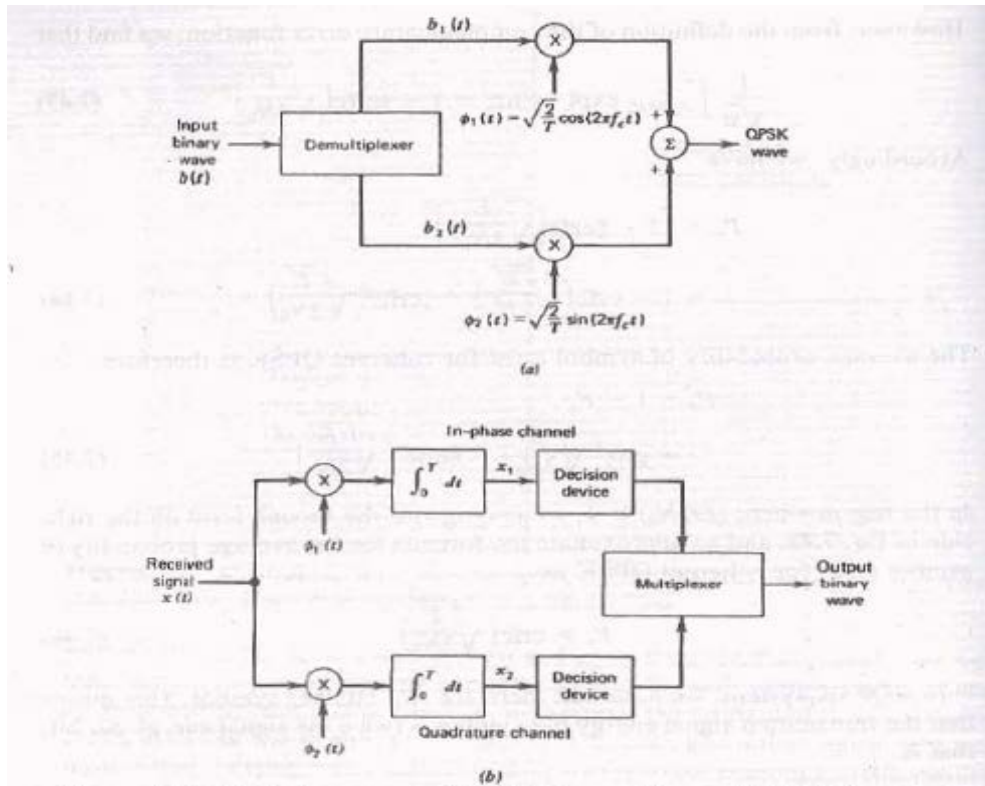


Q.6 a. Draw and Explain block diagram of QPSK transmitter and receiver. (8)

Answer:

Consider next the generation and demodulation of QPSK. Figure 7.9a shows the block diagram of a typical QPSK transmitter. The input binary sequence  $b(t)$  is represented in polar form, with symbols 1 and 0 represented by  $+\sqrt{E_b}$  and  $-\sqrt{E_b}$  volts, respectively. This binary wave is divided by means of a demultiplexer into two separate binary waves consisting of the odd- and even-numbered input bits. These two binary waves are denoted by  $b_1(t)$  and  $b_2(t)$ . We note that in any signaling interval, the amplitudes of  $b_1(t)$  and  $b_2(t)$  equal  $s_{i1}$  and  $s_{i2}$ , respectively, depending on the particular dibit that is being transmitted. The two binary waves  $b_1(t)$  and  $b_2(t)$  are used to modulate a pair of quadrature carriers or orthonormal basis functions:  $\phi_1(t)$  equal to  $\sqrt{2/T} \cos(2\pi f_c t)$  and  $\phi_2(t)$  equal to  $\sqrt{2/T} \sin(2\pi f_c t)$ . The result is a pair of binary PSK waves, which may be detected independently due to the orthogonality of  $\phi_1(t)$  and  $\phi_2(t)$ . Finally,





**Figure 7.9** Block diagrams for (a) QPSK transmitter, and (b) QPSK receiver.

the two binary PSK waves are added to produce the desired QPSK wave. Note that the symbol duration,  $T$ , of a QPSK wave is twice as long as the bit duration,  $T_b$ , of the input binary wave. That is, for a given bit rate  $1/T_b$ , a QPSK wave requires half the transmission bandwidth of the corresponding binary PSK wave. Equivalently, for a given transmission bandwidth, a QPSK wave carries twice as many bits of information as the corresponding binary PSK wave.

The QPSK receiver consists of a pair of correlators with a common input and supplied with a locally generated pair of coherent reference signals  $\phi_1(t)$  and  $\phi_2(t)$ , as in Fig. 7.9b. The correlator outputs,  $x_1$  and  $x_2$ , are each compared with a threshold of zero volts. If  $x_1 > 0$ , a decision is made in favor of symbol 1 for the upper or in-phase channel output, but if  $x_1 < 0$  a decision is made in favor of symbol 0. Similarly, if  $x_2 > 0$ , a decision is made in favor of symbol 1 for the lower or quadrature channel output, but if  $x_2 < 0$ , a decision is made in favor of symbol 0. Finally, these two binary sequences at the in-phase and quadrature channel outputs are combined in a *multiplexer* to reproduce the original binary sequence at the transmitter input with the minimum probability of symbol error.

**b. Explain the concept of carrier synchronization in PSK.**

(8)

Answer:

**(1) Carrier Synchronization**

The most straightforward method of carrier synchronization is to modulate the data-bearing signal onto a carrier in such a way that the power spectrum of the modulated signal contains a discrete component at the carrier frequency. Then a narrow-band *phase-locked loop* (PLL) can be used to track this component, thereby providing the desired reference signal at the receiver; a phase-locked loop consists of a voltage-controlled oscillator (VCO), a loop filter, and a multiplier that are connected together in the form of a negative feedback system. The disadvantage of such an approach is that since the residual component

\* For a statistical analysis of the synchronization problems, and descriptions of carrier and clock recovery circuits, see Stiffler (1971), Lindsey (1972), and Lindsey and Simon (1973, Chapters 2 and 9).

does not convey any information other than the frequency and phase of the carrier, its transmission represents a waste of power.

Accordingly, modulation techniques that conserve power are always of interest in practice. In particular, the modulation employed is often of such a form that, in the absence of a dc component in the power spectrum of the data-bearing signal, the receiver requires the use of a *suppressed carrier-tracking loop* for providing a coherent secondary carrier (subcarrier) reference. For example, Fig. 7.39 shows the block diagram of a carrier-recovery circuit for  $M$ -ary PSK. This circuit is called the  *$M$ th-power loop*. For the special case of  $M = 2$ , the circuit is called a *squaring loop*. However, when the squaring loop or its generalization is used for carrier recovery, we encounter a *phase ambiguity* problem. Consider, for example, the simple case of binary PSK. Since a squaring loop contains a squaring device at its input end, it is clear that changing the sign of the input signal leaves the sign of the recovered carrier unaltered. In other words, the squaring loop exhibits a  $180^\circ$  phase ambiguity. Correspondingly, the generalization of the squaring loop for  $M$ -ary PSK exhibits  $M$  phase ambiguities in the interval  $(0, 2\pi)$ .

Another method for carrier recovery involves the use of a *Costas loop*. Figure 7.40 shows a Costas loop for binary PSK. The loop consists of two paths, one referred to as *in-phase* and the other referred to as *quadrature*, that are coupled together via a common voltage-controlled oscillator (VCO) to form a negative feedback system. When synchronization is attained, the demodulated data waveform appears at the output of the in-phase path, and the corre-

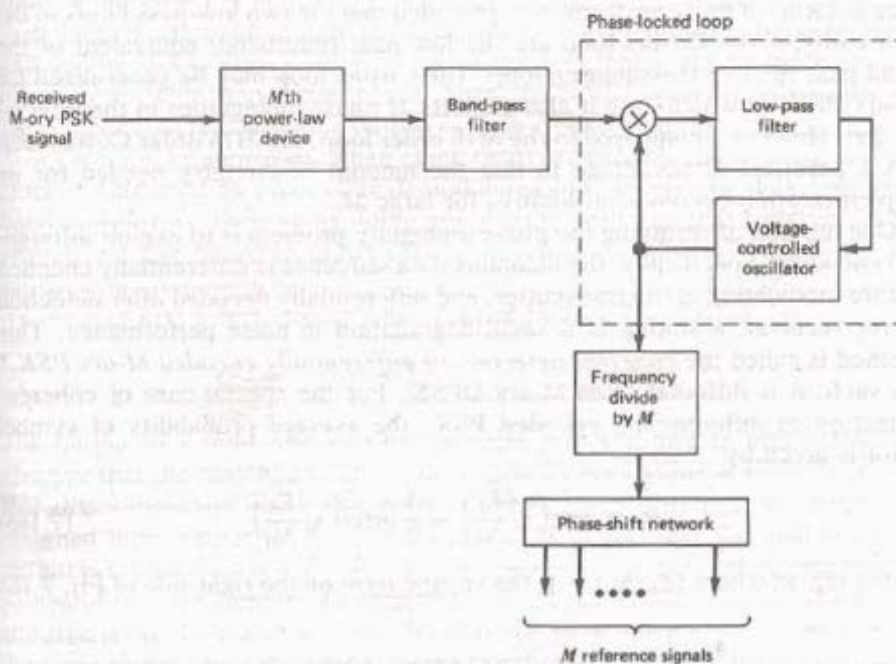


Figure 7.39  $M$ th power loop.

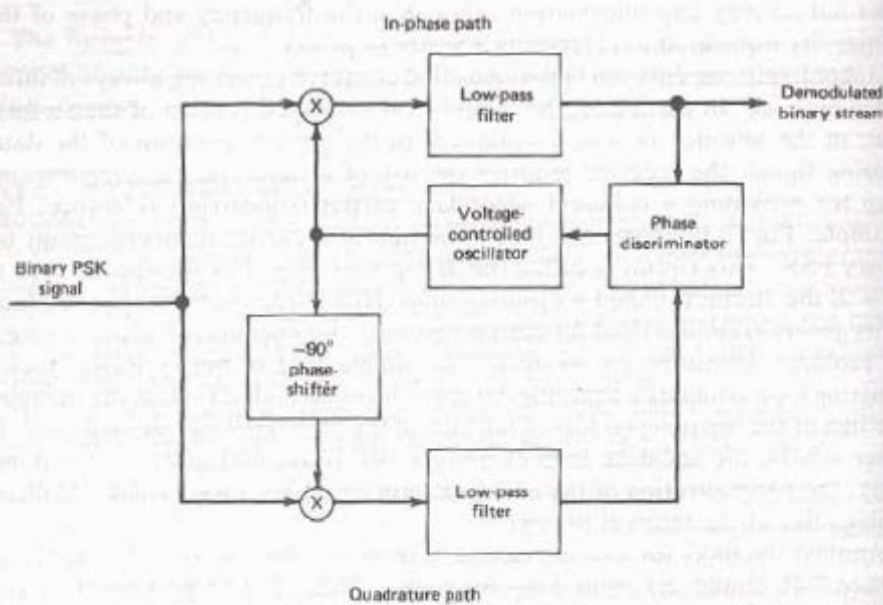


Figure 7.40 Costas loop.

sponding output of the quadrature path is zero under ideal conditions. Analysis of the Costas loop shows that it too exhibits the same phase ambiguity problem as the squaring loop. Moreover, the Costas loop is equivalent to the squaring loop in terms of noise performance, provided that the two low-pass filters in the two paths of the Costas loop are the low-pass (baseband) equivalent of the band-pass filter in the squaring loop. The Costas loop may be generalized for  $M$ -ary PSK, in which case it also exhibits  $M$  phase ambiguities in the interval  $(0, 2\pi)$ . However, compared to the  $M$ th order loop, the  $M$ th order Costas loop has a practical disadvantage in that the amount of circuitry needed for its implementation becomes prohibitive for large  $M$ .

One method of resolving the phase ambiguity problem is to exploit differential encoding. Specifically, the incoming data sequence is differentially encoded before modulation at the transmitter, and differentially decoded after detection at the receiver, resulting in a small degradation in noise performance. This method is called the *coherent detection of differentially encoded  $M$ -ary PSK*.<sup>\*</sup> As such, it is different from  $M$ -ary DPSK. For the special case of coherent detection of differentially encoded PSK, the average probability of symbol error is given by

$$P_e = \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) - \frac{1}{2} \operatorname{erfc}^2\left(\sqrt{\frac{E_b}{N_0}}\right) \quad (7.165)$$

In the region where  $(E_b/N_0) \gg 1$ , the second term on the right side of Eq. 7.165

<sup>\*</sup> For the noise analysis of coherent detection of differentially encoded  $M$ -ary PSK, see Lindsey and Simon (1973, pp. 242–246).



has a negligible effect; hence, this modulation scheme has an average probability of symbol error practically the same as that for coherent QPSK or MSK. (The formula of 7.165 is included in the summary presented in Table 7.4.) For the coherent detection of differentially encoded QPSK, the average probability of symbol error is given by

$$P_e = 2\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) - 2\text{erfc}^2\left(\sqrt{\frac{E_b}{N_0}}\right) + \text{erfc}^3\left(\sqrt{\frac{E_b}{N_0}}\right) - \frac{1}{4}\text{erfc}^4\left(\sqrt{\frac{E_b}{N_0}}\right) \quad (7.166)$$

For large  $E_b/N_0$ , this average probability of symbol error is approximately twice that of coherent QPSK.

**Q.7 a. Explain Gram-Schmidt orthogonalization procedure.**

(8)

**Answer:**

### 3.2 GRAM-SCHMIDT ORTHOGONALIZATION PROCEDURE

According to the model of Fig. 3.1, the task of transforming an incoming message  $m_i$ ,  $i = 1, 2, \dots, M$ , into a modulated wave  $s_i(t)$  may be divided into separate discrete-time and continuous-time operations. The justification for

\* For detailed treatments of the geometric representation of signals, see the following references: Wozencraft and Jacobs (1965, pp. 211–284), Sakrison (1968, pp. 219–271), Franks (1969, pp. 1–65); and Viterbi and Omura (1979, pp. 47–127).

this separation lies in the *Gram–Schmidt orthogonalization procedure*, which permits the representation of any set of  $M$  energy signals,  $\{s_i(t)\}$ , as linear combinations of  $N$  *orthonormal basis functions*, where  $N \leq M$ . That is to say, we may represent the given set of real-valued energy signals  $s_1(t), s_2(t), \dots, s_M(t)$ , each of duration  $T$  seconds, in the form

$$s_i(t) = \sum_{j=1}^N s_{ij} \phi_j(t) \quad \begin{array}{l} 0 \leq t \leq T \\ i = 1, 2, \dots, M \end{array} \quad (3.6)$$

where the coefficients of the expansion are defined by

$$s_{ij} = \int_0^T s_i(t) \phi_j(t) dt \quad \begin{array}{l} i = 1, 2, \dots, M \\ j = 1, 2, \dots, N \end{array} \quad (3.7)$$

The real-valued basis functions  $\phi_1(t), \phi_2(t), \dots, \phi_N(t)$  are *orthonormal*, by which we mean

$$\int_0^T \phi_i(t) \phi_j(t) dt = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad (3.8)$$

The first condition of Eq. 3.8 states that each basis function is *normalized* to have unit energy. The second condition states that the basis functions  $\phi_1(t), \phi_2(t), \dots, \phi_N(t)$  are *orthogonal* with respect to each other over the interval  $0 \leq t \leq T$ .

Given the set of coefficients  $\{s_{ij}\}, j = 1, 2, \dots, N$ , operating as input, we may use the scheme shown in Fig. 3.3a to generate the signal  $s_i(t), i = 1, 2, \dots, M$ , which follows directly from Eq. 3.6. It consists of a bank of  $N$  multipliers, with each multiplier supplied with its own basis function, followed by a summer. This scheme may be viewed as performing a similar role to that of the second stage or modulator in the transmitter of Fig. 3.1. Conversely, given the set of signals  $\{s_i(t)\}, i = 1, 2, \dots, M$ , operating as input, we may use the scheme shown in Fig. 3.3b to calculate the set of coefficients  $\{s_{ij}\}, j = 1, 2, \dots, N$ , which follows directly from Eq. 3.7. This second scheme consists of a bank of  $N$  *product-integrators* or *correlators* with a common input, and with each one supplied with its own basis function. As will be shown later, such a bank of correlators may be used as the first stage or detector in the receiver of Fig. 3.1.

To prove the Gram–Schmidt orthogonalization procedure, we proceed in two stages, as indicated next.

### (1) Stage 1

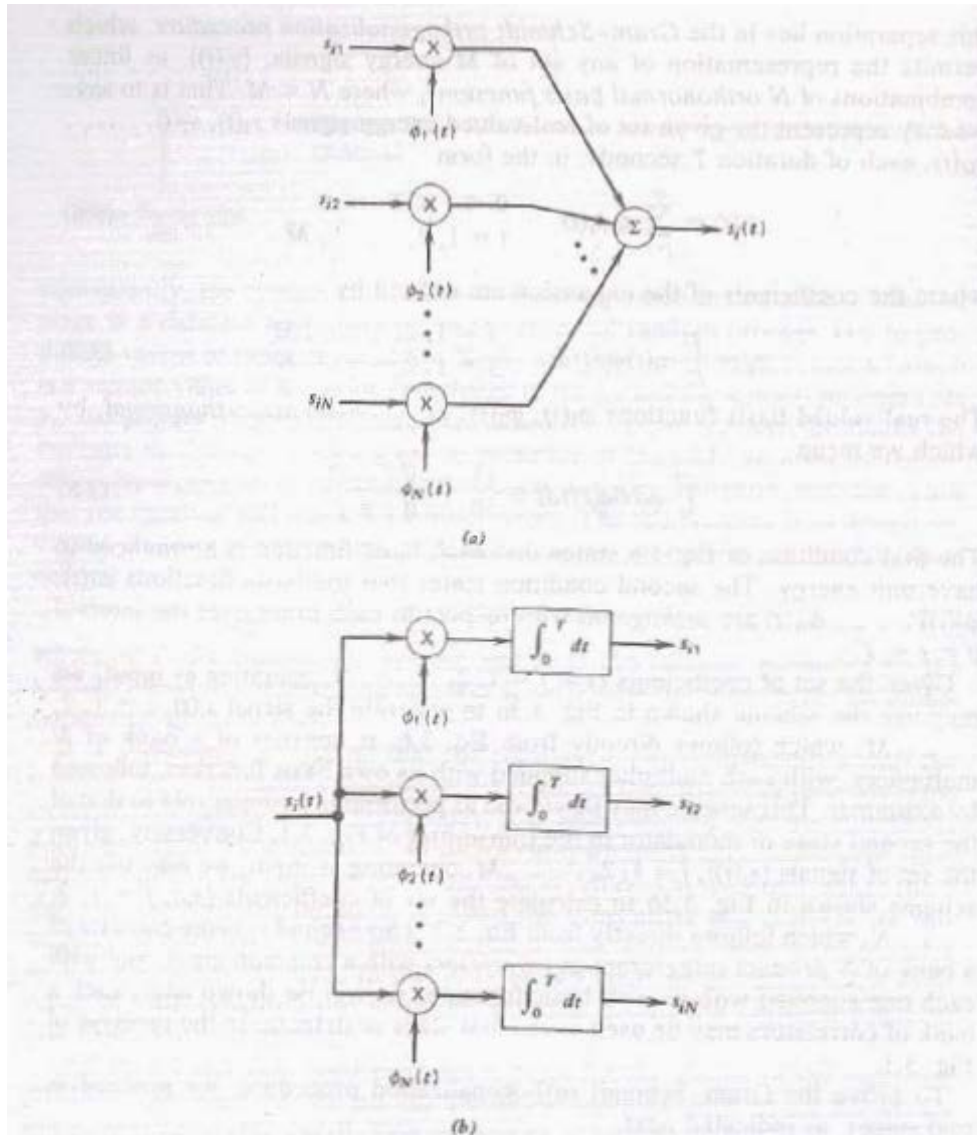
First, we have to establish whether or not the given set of signals  $s_1(t), s_2(t), \dots, s_M(t)$  is *linearly independent*. If not, then (by definition) there exists a set of coefficients  $a_1, a_2, \dots, a_M$ , not all equal to zero, such that we may write

$$a_1 s_1(t) + a_2 s_2(t) + \dots + a_M s_M(t) = 0 \quad 0 \leq t \leq T \quad (3.9)$$

Suppose, in particular, that  $a_M \neq 0$ . Then, we may express the corresponding signal  $s_M(t)$  as

$$s_M(t) = - \left[ \frac{a_1}{a_M} s_1(t) + \frac{a_2}{a_M} s_2(t) + \dots + \frac{a_{M-1}}{a_M} s_{M-1}(t) \right] \quad (3.10)$$





**Figure 3.3** (a) Scheme for generating the signal  $s_i(t)$ . (b) Scheme for generating the set of coefficients  $\{s_j\}$ .

which implies that  $s_M(t)$  may be expressed in terms of the remaining  $(M - 1)$  signals.

Consider, next, the set of signals  $s_1(t), s_2(t), \dots, s_{M-1}(t)$ . Either this set of signals is linearly independent, or it is not. If not, then there exists a set of numbers  $b_1, b_2, \dots, b_{M-1}$ , not all equal to zero, such that

$$b_1 s_1(t) + b_2 s_2(t) + \dots + b_{M-1} s_{M-1}(t) = 0 \quad 0 \leq t \leq T \quad (3.11)$$

Suppose that  $b_{M-1} \neq 0$ . Then, we may express  $s_{M-1}(t)$  as a linear combination of the remaining  $M - 2$  signals, as shown by

$$s_{M-1}(t) = - \left[ \frac{b_1}{b_{M-1}} s_1(t) + \frac{b_2}{b_{M-1}} s_2(t) + \dots + \frac{b_{M-2}}{b_{M-1}} s_{M-2}(t) \right] \quad (3.12)$$

Now, testing the set of signals  $s_1(t), s_2(t), \dots, s_{M-2}(t)$  for linear independence, and continuing in this fashion, it is clear that we will eventually end up with a linearly independent subset of the original set of signals. Let  $s_1(t), s_2(t), \dots, s_N(t)$  denote this subset of linearly independent signals, where  $N \leq M$ . The important point to note is that each member of the original set of signals  $s_1(t), s_2(t), \dots, s_M(t)$  may be expressed as a linear combination of this subset of  $N$  signals.

## (2) Stage 2

Next, we wish to show that it is possible to construct a set of  $N$  orthonormal basis functions  $\phi_1(t), \phi_2(t), \dots, \phi_N(t)$  from the linearly independent signals  $s_1(t), s_2(t), \dots, s_N(t)$ . As a starting point, define the first basis function as

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} \quad (3.13)$$

where  $E_1$  is the energy of the signal  $s_1(t)$ . Then, clearly, we have

$$\begin{aligned} s_1(t) &= \sqrt{E_1} \phi_1(t) \\ &= s_{11} \phi_1(t) \end{aligned} \quad (3.14)$$

where the coefficient  $s_{11} = \sqrt{E_1}$  and  $\phi_1(t)$  has unit energy, as required.

Next, using the signal  $s_2(t)$ , we define the coefficient  $s_{21}$  as

$$s_{21} = \int_0^T s_2(t) \phi_1(t) dt \quad (3.15)$$

We may thus define a new intermediate function

$$g_2(t) = s_2(t) - s_{21} \phi_1(t) \quad (3.16)$$

which is orthogonal to  $\phi_1(t)$  over the interval  $0 \leq t \leq T$ . Now, we are ready to define the second basis function as

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t) dt}} \quad (3.17)$$

Substituting Eq. 3.16 in 3.17, and simplifying, we get the desired result

$$\phi_2(t) = \frac{s_2(t) - s_{21} \phi_1(t)}{\sqrt{E_2 - s_{21}^2}} \quad (3.18)$$

where  $E_2$  is the energy of the signal  $s_2(t)$ . It is clear from Eq. 3.17 that,

$$\int_0^T \phi_2^2(t) dt = 1$$

and from Eq. 3.18 that

$$\int_0^T \phi_1(t) \phi_2(t) dt = 0$$

That is to say,  $\phi_1(t)$  and  $\phi_2(t)$  form an orthonormal set.

Continuing in this fashion, we may define

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij} \phi_j(t) \quad (3.19)$$

where the coefficients  $s_{ij}, j = 1, 2, \dots, i-1$ , are themselves defined by

$$s_{ij} = \int_0^T s_i(t) \phi_j(t) dt \quad (3.20)$$

Then it follows readily that the set of functions

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t) dt}} \quad i = 1, 2, \dots, N \quad (3.21)$$

forms an orthonormal set.

Since we have shown that each one of the derived subset of linearly independent signals  $s_1(t), s_2(t), \dots, s_N(t)$  may be expressed as a linear combination of the orthonormal basis functions  $\phi_1(t), \phi_2(t), \dots, \phi_N(t)$ , it follows that each one of the original set of signals  $s_1(t), s_2(t), \dots, s_M(t)$  may be expressed as a linear combination of this set of basis functions, as described in Eq. 3.6. This completes the proof of the Gram–Schmidt orthogonalization procedure.

Note that the conventional Fourier series expansion of a periodic signal is an example of a particular expansion of this type. There are, however, two important distinctions that should be made:

1. The form of the basis functions  $\phi_1(t), \phi_2(t), \dots, \phi_N(t)$  has not been specified. That is to say, unlike the Fourier series expansion of a periodic signal, we have not restricted the Gram–Schmidt orthogonalization procedure to be in terms of sinusoidal functions.
2. The expansion of the signal  $s_i(t)$  into a finite number of terms is not an approximation wherein only the first  $k$  terms are significant but rather an exact expression where  $N$  and only  $N$  terms are significant.

**b. Explain properties of Matched filters.**

**(8)**

**Answer:**



## (2) Properties of Matched Filters

We note that a filter, which is matched to a known signal  $\phi(t)$  of duration  $T$  seconds, is characterized by an impulse response that is a time-reversed and delayed version of the input  $\phi(t)$ , as shown by

$$h_{opt}(t) = \phi(T - t) \quad (3.98)$$

In the frequency domain, the matched filter is characterized by a transfer function that is, except for a delay factor, the complex conjugate of the Fourier transform of the input  $\phi(t)$ , as shown by

$$H_{opt}(f) = \Phi^*(f)\exp(-j2\pi fT) \quad (3.99)$$

Based on this fundamental pair of relations, we may derive some important properties of matched filters, which should help the reader develop an intuitive grasp of how a matched filter operates.

### PROPERTY 1

*The spectrum of the output signal of a matched filter with the matched signal as input is, except for a time delay factor, proportional to the energy spectral density of the input signal.*

Let  $\Phi_o(f)$  denote the Fourier transform of the filter output  $\phi_o(t)$ . Then

$$\begin{aligned} \Phi_o(f) &= H_{opt}(f)\Phi(f) \\ &= \Phi^*(f)\Phi(f)\exp(-j2\pi fT) \\ &= |\Phi(f)|^2 \exp(-j2\pi fT) \end{aligned} \quad (3.100)$$

which is the desired result.

### PROPERTY 2

*The output signal of a matched filter is proportional to a shifted version of the autocorrelation function of the input signal to which the filter is matched.*

This property follows directly from Property 1, recognizing that the autocorrelation function and energy spectral density of a signal form a Fourier transform pair. Thus, taking the inverse Fourier transform of Eq. 3.100, we may express the matched-filter output as

$$\phi_o(t) = R_\phi(t - T) \quad (3.101)$$

where  $R_\phi(\tau)$  is the autocorrelation function of the input  $\phi(t)$  for lag  $\tau$ . Equation 3.101 is the desired result. Note that at time  $t = T$ , we have

$$\phi_o(T) = R_\phi(0) = E \quad (3.102)$$

where  $E$  is the signal energy. That is, in the absence of noise, the maximum value of the matched-filter output, attained at time  $t = T$ , is proportional to the signal energy.

**PROPERTY 3**

The output signal-to-noise ratio of a matched filter depends only on the ratio of the signal energy to the power spectral density of the white noise at the filter input.

To demonstrate this property, consider a filter matched to an input signal  $\phi(t)$ . From Property 2, the maximum value of the filter output, at time  $t = T$ , is proportional to the signal energy  $E$ . Substituting Eq. 3.92 in Eq. 3.87 gives the average output noise power as

$$E[n^2(t)] = \frac{N_0}{2} \int_{-\infty}^{\infty} |\Phi(f)|^2 df = \frac{N_0}{2} E \quad (3.103)$$

where we have made use of Rayleigh's energy theorem. Therefore, the output signal-to-noise ratio has the maximum value

$$(\text{SNR})_{o,\max} = \frac{E^2}{N_0 E/2} = \frac{2E}{N_0} \quad (3.104)$$

This result is perhaps the most important parameter in the calculation of the performance of signal processing systems using matched filters. From Eq.

\* The use of a matched-filter pair forms a basis of two important applications: (1) *spread-spectrum modulation*, discussed in Chapter 9, and (2) *pulse-compression radar*, discussed in Rihaczek (1969).

3.104 we see that dependence on the waveform of the input  $\phi(t)$  has been completely removed by the matched filter. Accordingly, in evaluating the ability of a matched-filter receiver to combat additive white Gaussian noise, we find that all signals that have the same energy are equally effective. Note that the signal energy  $E$  is in joules and the noise spectral density  $N_0/2$  is in watts per hertz, so that the ratio  $2E/N_0$  is dimensionless; however, the two quantities have different physical meaning. We refer to  $E/N_0$  as the *signal energy-to-noise density ratio*.

#### PROPERTY 4

The matched-filtering operation may be separated into two matching conditions; namely, spectral phase matching that produces the desired output peak at time  $T$ , and spectral amplitude matching that gives this peak value its optimum signal-to-noise density ratio.

In polar form, the spectrum of the signal  $\phi(t)$  being matched may be expressed as

$$\Phi(f) = |\Phi(f)|\exp[j\theta(f)]$$

where  $|\Phi(f)|$  is the amplitude spectrum and  $\theta(f)$  is the phase spectrum of the signal. The filter is said to be *spectral phase matched* to the signal  $\phi(t)$  if the transfer function of the filter is defined by\*

$$H(f) = |H(f)|\exp[-j\theta(f) - j2\pi fT]$$

where  $|H(f)|$  is real and nonnegative and  $T$  is a positive constant. The output of such a filter is

$$\begin{aligned}\phi'_o(t) &= \int_{-\infty}^{\infty} H(f)\Phi(f)\exp(j2\pi ft)df \\ &= \int_{-\infty}^{\infty} |H(f)||\Phi(f)|\exp[j2\pi f(t - T)]df\end{aligned}$$

where the product  $|H(f)||\Phi(f)|$  is real and nonnegative. The spectral phase matching ensures that all the spectral components of the output  $\phi'_o(t)$  add constructively at time  $t = T$ , thereby causing the output to attain its maximum value, as shown by

$$\phi'_o(t) \leq \phi'_o(T) = \int_{-\infty}^{\infty} |\Phi(f)||H(f)|df$$

For *spectral amplitude matching*, we choose the amplitude response  $|H(f)|$  of the filter to shape the output for best signal-to-noise ratio at  $t = T$  by using

$$|H(f)| = |\Phi(f)|$$

and the standard matched filter is the result. //

**Q.8 a. Define Frequency Hop Spread Spectrum. Describe slow frequency hopping. (8)**  
**Answer:**



## 9.6 FREQUENCY-HOP SPREAD SPECTRUM

In the type of spread-spectrum systems discussed previously, the use of a PN sequence to modulate a phase-shift-keyed signal achieves *instantaneous* spreading of the transmission bandwidth. The ability of such a system to combat the effects of jammers is determined by the processing gain of the system, which is a function of the PN sequence length. The processing gain can be made larger by employing a PN sequence with narrow chip duration, which, in turn, permits a greater transmission bandwidth and more chips per bit. However, the capabilities of physical devices used to generate the PN spread-spectrum signals impose a practical limit on the attainable processing gain. Indeed, it may turn out that the processing gain so attained is still not large enough to overcome the effects of some jammers of concern, in which case we have to resort to other methods. One such alternative method is to force the jammer to cover a wider spectrum by *randomly hopping* the data-modulated carrier from one frequency to the next. In effect, the spectrum of the transmitted signal is spread *sequentially* rather than instantaneously; the term "sequentially" refers to the pseudo-random-ordered sequence of frequency hops.

The type of spread spectrum in which the carrier hops randomly from one frequency to another is called *frequency-hop (FH) spread spectrum*. A common modulation format for FH systems is that of *M-ary frequency-shift keying (MFSK)*. The combination is referred to simply as FH/MFSK. (A description of M-ary FSK was presented in Section 7.6).

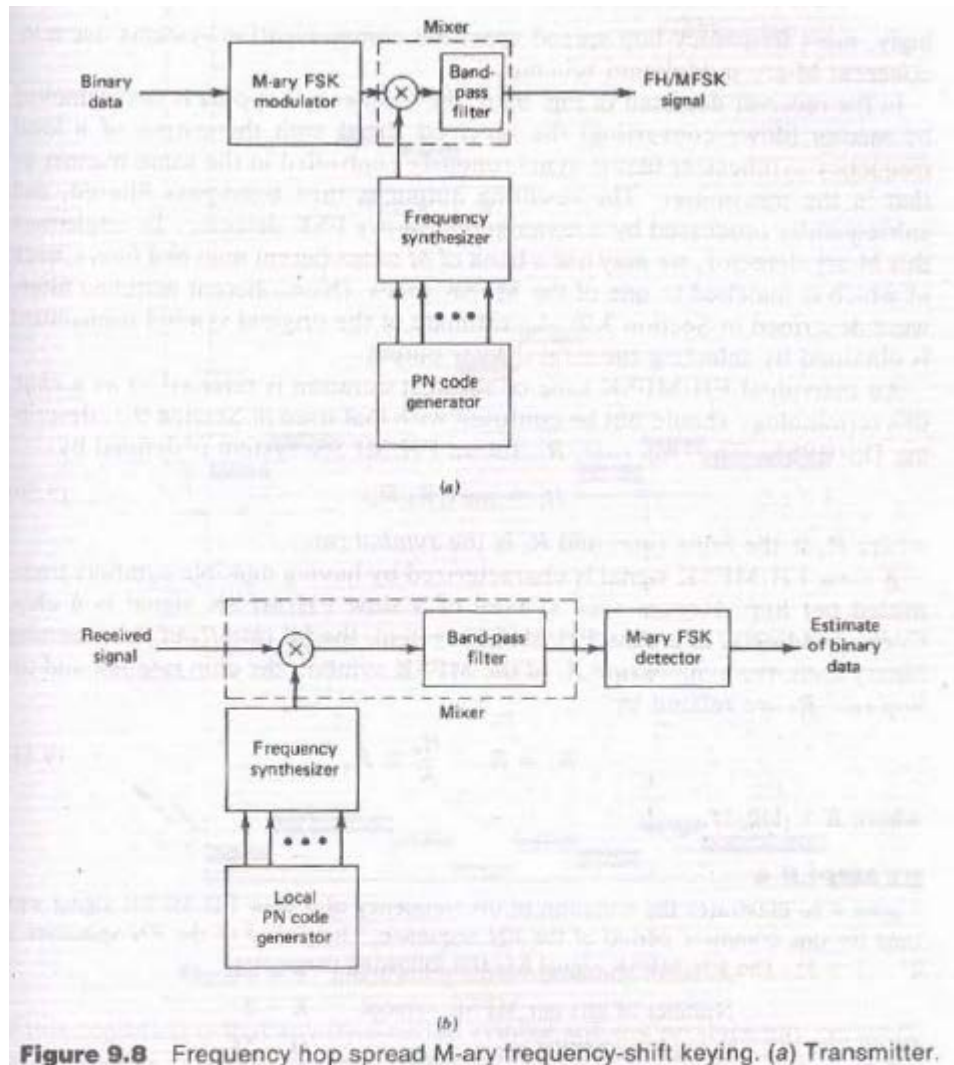
Since frequency hopping does not cover the entire spread spectrum instantaneously, we are led to consider the rate at which the hops occur. In this context, we may identify two basic (technology-independent) characterizations of frequency hopping:

1. *Slow-frequency hopping*, in which the symbol rate  $R_s$  of the MFSK signal is an integer multiple of the hop rate  $R_h$ . That is, several symbols are transmitted on each frequency hop.
2. *Fast-frequency hopping*, in which the hop rate  $R_h$  is an integer multiple of the MFSK symbol rate  $R_s$ . That is, the carrier frequency will change or hop several times during the transmission of one symbol.

Obviously, slow-frequency hopping and fast-frequency hopping are the converse of one another. In the sequel, these two characterizations of frequency hopping are considered in turn.

### (1) Slow-frequency Hopping

Figure 9.8a shows the block diagram of an FH/MFSK transmitter, which involves *frequency modulation* followed by *mixing*. First, the incoming binary data are applied to an M-ary FSK modulator. The resulting modulated wave and the output from a digital *frequency synthesizer* are then applied to a mixer that consists of a multiplier followed by a filter. The filter is designed to select the sum frequency component resulting from the multiplication process as the transmitted signal. In particular, successive (not necessarily disjoint)  $k$ -bit



**Figure 9.8** Frequency hop spread M-ary frequency-shift keying. (a) Transmitter. (b) Receiver.

segments of a PN sequence drive the frequency synthesizer, which enables the carrier frequency hop over  $2^k$  distinct values. On a single hop, the bandwidth of the transmitted signal is the same as that resulting from the use of a conventional M-ary frequency-shift-keying (MFSK) format with an alphabet of  $M = 2^k$  orthogonal signals. However, for a complete range of  $2^k$ -frequency hops, the transmitted FH/MFSK signal occupies a much larger bandwidth. Indeed, with present-day technology, FH bandwidths of the order of several GHz are attainable, which is an order of magnitude larger than that achievable with direct-sequence spread spectra. An implication of these large FH bandwidths is that coherent detection is possible only within each hop, because frequency synthesizers are unable to maintain phase coherence over successive hops. Accord-

ingly, most frequency-hop spread-spectrum communication systems use non-coherent M-ary modulation schemes.

In the receiver depicted in Fig. 9.8b, the frequency hopping is first removed by *mixing* (down-converting) the received signal with the output of a local frequency synthesizer that is synchronously controlled in the same manner as that in the transmitter. The resulting output is then band-pass filtered, and subsequently processed by a *noncoherent* M-ary FSK detector. To implement this M-ary detector, we may use a bank of  $M$  noncoherent matched filters, each of which is matched to one of the MFSK tones. (Noncoherent matched filters were described in Section 3.9). An estimate of the original symbol transmitted is obtained by selecting the largest filter output.

An individual FH/MFSK tone of shortest duration is referred to as a *chip*; this terminology should not be confused with that used in Section 9.3, describing DS/BPSK. The *chip rate*,  $R_c$ , for an FH/MFSK system is defined by

$$R_c = \max(R_h, R_s) \quad (9.50)$$

where  $R_h$  is the *hop rate*, and  $R_s$  is the *symbol rate*.

A slow FH/MFSK signal is characterized by having multiple symbols transmitted per hop. Hence, each symbol of a slow FH/MFSK signal is a chip. Correspondingly, in a slow FH/MFSK system, the bit rate  $R_b$  of the incoming binary data, the symbol rate  $R_s$  of the MFSK symbol, the chip rate  $R_c$ , and the hop rate  $R_h$  are related by

$$R_c = R_s = \frac{R_b}{K} \geq R_h \quad (9.51)$$

where  $K = \log_2 M$ .

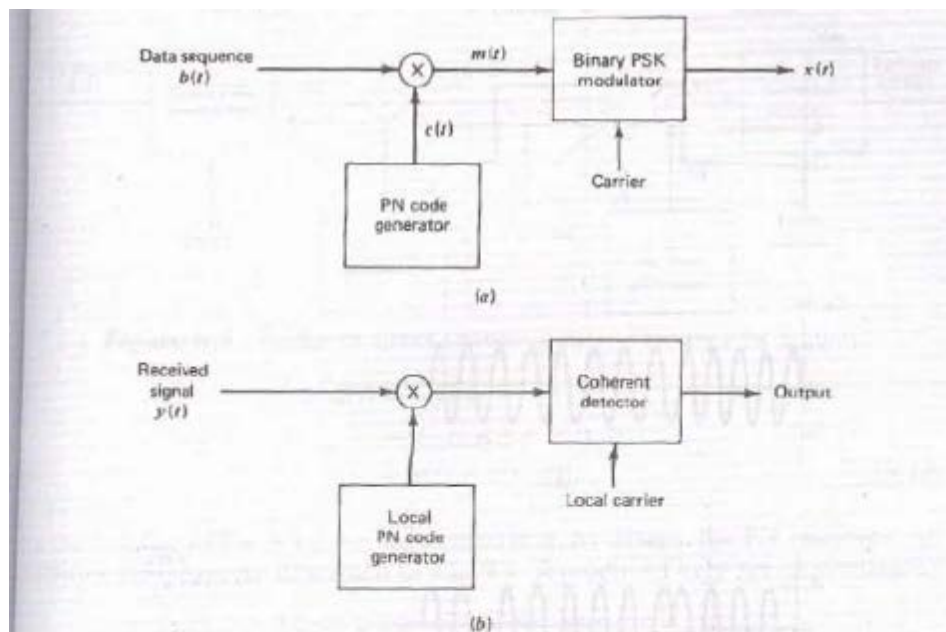
- b. Explain Direct Sequence Spread Coherent Binary Phase Shift Keying system with the help of neat transmitter and receiver block diagrams. (8)

Answer:

### 9.3 DIRECT-SEQUENCE SPREAD COHERENT BINARY PHASE-SHIFT KEYING

The spread-spectrum technique described in the previous section is referred to as *direct-sequence spread spectrum*. The discussion presented there was in the context of baseband transmission. To provide for the use of this technique over a band-pass channel (e.g., satellite channel), we may incorporate *coherent binary phase-shift keying* (PSK) into the transmitter and receiver, as shown in





**Figure 9.5** Direct-sequence spread coherent phase-shift keying. (a) Transmitter. (b) Receiver.

Fig. 9.5. The transmitter of Fig. 9.5a involves two stages of modulation. The first stage consists of a product modulator or multiplier with the data sequence and PN sequence as inputs. The second stage consists of a binary PSK modulator. The transmitted signal  $x(t)$  is thus a *direct-sequence spread binary phase-shift-keyed (DS/BPSK) signal*. The phase modulation  $\theta(t)$  of  $x(t)$  has one of two values, 0 and  $\pi$ , depending on the polarities of the data sequence  $b(t)$  and PN sequence  $c(t)$  at time  $t$  in accordance with the truth table of Table 9.2. The receiver, shown in Fig. 9.5b, consists of two stages of demodulation. The received signal  $y(t)$  and a locally generated replica of the PN sequence are applied to a multiplier. This multiplication represents the first stage of demodulation in the receiver. The second stage of demodulation consists of a coherent detector, the output of which provides an estimate of the original data sequence.

**Q.9** Write short notes on any TWO of the following:-

(8×2)

- (i) Digital Multiplexers
- (ii) Digital Radio
- (iii) Code Division Multiple Access.

**Answer:**

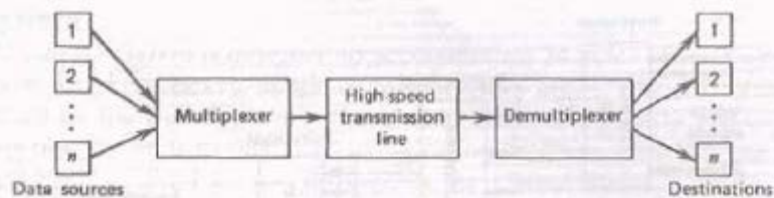
### (1) Digital Multiplexers

In Chapter 4 we introduced the idea of time-division multiplexing whereby a group of analog signals (e.g., voice signals) are sampled sequentially in time at a *common* sampling rate and then multiplexed for transmission over a common line. In this section we consider the multiplexing of digital signals at different bit rates.† This enables us to combine several digital signals, such as computer

\* The use of mean opinion score as a subjective measure of quality is discussed in the following references: Daumer (1982) and Jayant and Noll (1985, pp. 10–12). For a comparison of digital speech coders, based on MOS ratings, see Jayant (1986).

† For additional information on digital multiplexers, see Bell Telephone Laboratories (1970, Chapter 26).





**Figure 5.25** Conceptual diagram of multiplexing–demultiplexing.

outputs, digitized voice signals, digitized facsimile and television signals, into a single data stream (at a considerably higher bit rate than any of the inputs). Figure 5.25 shows a conceptual diagram of the digital multiplexing–demultiplexing operation.

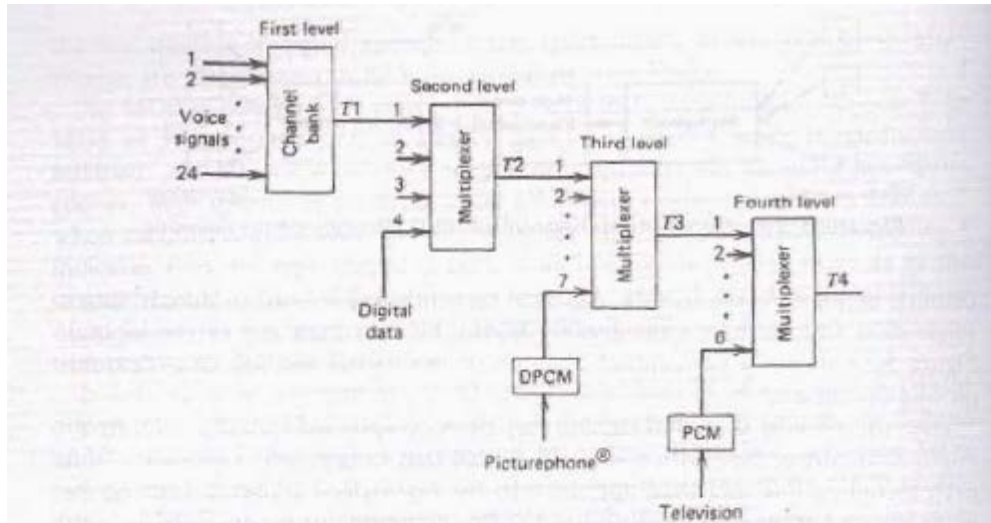
The multiplexing of digital signals may be accomplished by using a *bit-by-bit interleaving procedure* with a selector switch that sequentially takes a bit from each incoming line and then applies it to the high-speed common line. At the receiving end of the system the output of this common line is separated out into its low-speed individual components and then delivered to their respective destinations.

Two major groups of digital multiplexers are used in practice:

1. One group of multiplexers is designed to combine relatively low-speed digital signals, up to a maximum rate of 4800 bits per second, into a higher speed multiplexed signal with a rate of up to 9600 bits per second. These multiplexers are used primarily to transmit data over voice-grade channels of a telephone network. Their implementation requires the use of *modems* in order to convert the digital format into an analog format suitable for transmission over telephone channels. The theory of a modem (*modulator–demodulator*) is covered in Chapter 7.
2. The second group of multiplexers, designed to operate at much higher bit rates, forms part of the data transmission service generally provided by communication carriers. For example, Fig. 5.26 shows a block diagram of the digital hierarchy based on the T1 carrier, which has been developed by the Bell System. The T1 carrier system, described below, is designed to operate at 1.544 megabits per second, the T2 at 6.312 megabits per second, the T3 at 44.736 megabits per second, and the T4 at 274.176 megabits per second. The system is thus made up of various combinations of lower order T-carrier subsystems designed to accommodate the transmission of voice signals, Picturephone® service, and television signals by using PCM, as well as (direct) digital signals from data terminal equipment.

There are some basic problems involved in the design of a digital multiplexer, irrespective of its grouping:

1. Digital signals cannot be directly interleaved into a format that allows for their eventual separation unless their bit rates are locked to a common clock. Accordingly, provision has to be made for *synchronization* of the incoming digital signals, so that they can be properly interleaved.



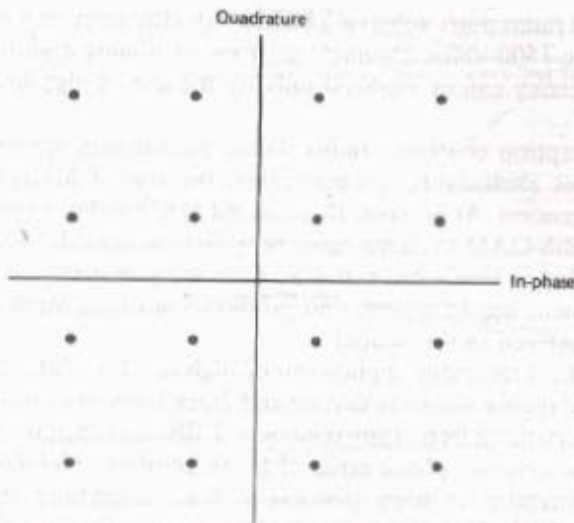
**Figure 5.26** Digital hierarchy, Bell system.

2. The multiplexed signal must include some form of *framing*, so that its individual components can be identified at the receiver.
3. The multiplexer has to handle small variations in the bit rates of the incoming digital signals. For example, a 1000-kilometer coaxial cable carrying  $3 \times 10^8$  pulses per second will have about one million pulses in transit, with each pulse occupying about one meter of the cable. A 0.01 percent variation in the propagation delay, produced by a  $1^\circ\text{F}$  decrease in temperature, will result in 100 fewer pulses in the cable. Clearly, these pulses must be absorbed by the multiplexer.

In order to cater for the requirements of synchronization and rate adjustment to accommodate small variations in the input data rates, we may use a technique known as *bit stuffing*. The idea here is to have the outgoing bit rate of the multiplexer slightly higher than the sum of the maximum expected bit rates of the input channels by stuffing in additional non-information carrying pulses. All incoming digital signals are stuffed with a number of bits sufficient to raise each of their bit rates to equal that of a locally generated clock. To accomplish bit stuffing, each incoming digital signal or bit stream is fed into an *elastic store* at the multiplexer. The elastic store is a device that stores a bit stream in such a manner that the stream may be read out at a rate different from the rate at which it is read in. At the demultiplexer, the stuffed bits must obviously be removed from the multiplexed signal. This requires a method that can be used to identify the stuffed bits. To illustrate one such method, and also show one method of providing frame synchronization, we describe the signal format of the Bell System *M12 multiplexer*, which is designed to combine four T1 bit streams into one T2 bit stream. We begin the description by considering the T1 system first and then the M12 multiplexer.

## (2) Digital Radio

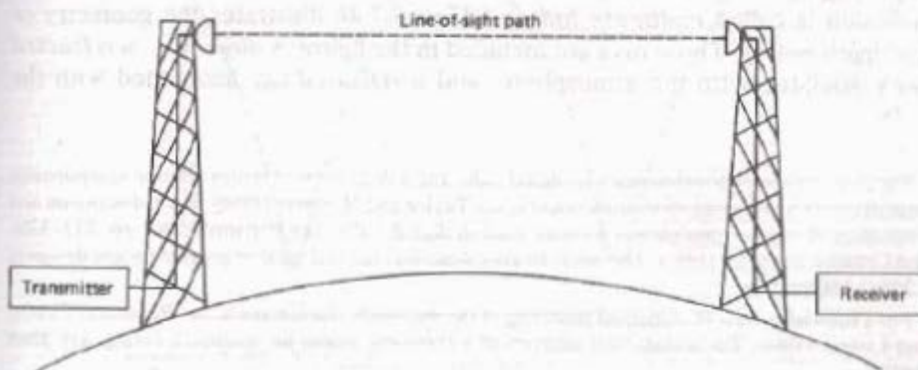
In *digital radio*, information originating from a source is transmitted to its destination by means of digital modulation techniques over an appropriate



**Figure 7.44** Signal constellation of 16-QAM, 9600 b/s modem.

number of microwave radio links, with each link offering a *line-of-sight* propagation path. Such a link is illustrated in Fig. 7.45. Antennas are placed on towers whose heights are large enough to provide a direct, unobstructed path between the transmitter and the receiver.

In radio systems, rigid transmission bandwidths are normally specified so as to establish well-defined, noninterleaving channels. Efficient utilization of these channels usually requires the use of multilevel signaling techniques, which makes it possible to achieve high bit rates despite bandwidth restrictions. This requirement arises because digital radios have to provide essentially the same bandwidth efficiency as analog FM radios that they are in many instances replacing. Consider, for example, the 4-GHz band for which the allocated bandwidth is typically 20 MHz. An analog microwave radio, based on frequency modulation of frequency-division multiplexed (FDM) single-sideband signals, carries 1500 voice channels over this bandwidth. In a digital radio, on the other hand, 64 kb/s PCM is frequently used for each voice channel. Accord-



**Figure 7.45** Line-of-sight radio line.



ingly, the digital radio must achieve a bandwidth efficiency of 4.8 b/s/Hz if it is to accommodate 1500 voice channels as does its analog counterpart. Such a bandwidth efficiency can be realized only by the use of high-level modulation formats.

From the inception of digital radio, linear modulation schemes have been employed almost exclusively. In particular, the use of  $M$ -ary QAM has received much attention. At present, there are 64-QAM radio systems in the field, and prototype 256-QAM systems have been demonstrated.\* However, as the number of amplitude levels increases, so does the sensitivity of the modulated wave to equipment imperfections and propagation impairments. These issues are briefly considered in the sequel.

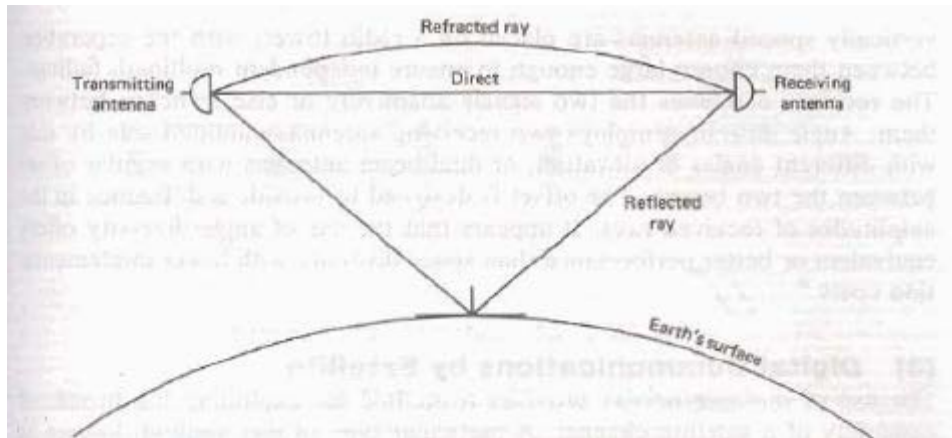
First of all, the successful application of high-level modulation schemes requires the use of highly accurate carrier and clock recovery circuits. For example, a 64-QAM radio suffers approximately 1 dB degradation in error performance for an rms carrier phase error of  $1^\circ$ . In addition, high-level modulation schemes are sensitive to linear distortions (i.e., amplitude and group-delay distortions) resulting from the use of filters in the transmission path. It is therefore of critical importance that all filters in both the transmitter and the receiver be equalized, so that their amplitude responses are essentially symmetric and their group-delay responses are essentially flat inside the bandwidth occupied by the modulated signal. Moreover, high-level modulation schemes are very sensitive to nonlinear distortion, and this sensitivity increases with the number of amplitude levels used. The primary source of nonlinear distortion in a digital radio is the power amplifier at the transmitter output. It is therefore necessary to operate the power amplifier at a large *backoff* from saturation or else provide some means of linearizing the amplifier.

Another issue that has to be considered in the design of digital radio systems is that of propagation impairments. Most of the time, a line-of-sight microwave radio link behaves as a wide-band, low-noise channel capable of providing highly reliable, high-speed data transmission. The channel, however, suffers from anomalous propagation conditions that arise from natural phenomena, which can cause the error performance of a digital radio to be severely degraded. Such anomalies manifest themselves by causing the transmitted signal to propagate along several paths, each of different electrical length. This phenomenon is called *multipath fading*.† Figure 7.46 illustrates the geometry of multipath fading. Three rays are included in the figure: a *direct ray*, a *refracted ray* associated with the atmosphere, and a *reflected ray* associated with the

\* For an overview of developments in digital radio and a discussion of issues relating to equipment imperfections and propagation impairments, see Taylor and Hartman (1986). For a discussion and evaluation of various modulation formats used in digital radio, see Bellamy (1982, pp. 272–320), and Chamberlain et al. (1987). The last reference describes the technical requirements and design of a 256-QAM modem.

† For a tutorial review of statistical modeling of the multipath phenomenon, see Rummel, Coutts, and Liniger (1986). For a statistical analysis of a three-ray model for multipath fading, see Shafi (1987).

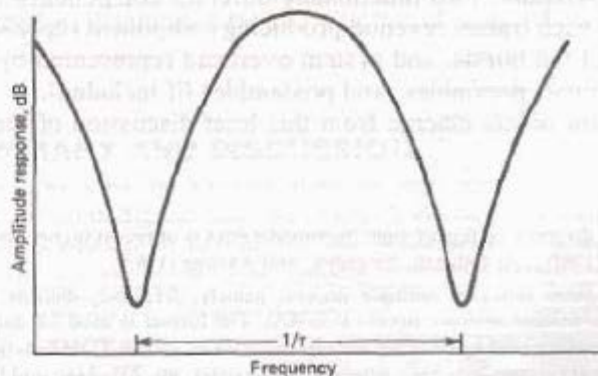




**Figure 7.46** Illustrating direct and indirect paths in a radio environment.

terrain. Figure 7.47 shows a plot of the amplitude response of the channel. When the direct ray is out of phase with the indirect rays, deep notches appear in the amplitude response. The time delay  $\tau$  between the direct and indirect rays determines the frequency separation between the notches, and their relative amplitudes determine the notch depth. In digital radio, this form of distortion leads to intersymbol interference, which in turn causes the error performance of the system to be severely degraded. The degradation is directly proportional to the transmitted bit rate, since increasing the bit rate corresponds to a reduction in bit duration and therefore greater vulnerability to intersymbol interference. To combat the deleterious effects of multipath fading in digital radio, diversity techniques are widely used either by themselves or in combination with adaptive equalizers.

In digital radio, diversity techniques are frequency-, space-, or angle-based. Each of these techniques may be used to provide a second version of the transmitted signal for processing at the receiver. *Frequency diversity* is implemented by switching operation of the system to an alternate channel when multipath fading causes outage in a working channel. In *space diversity*, two



**Figure 7.47** Frequency response due to multipath fading.

vertically spaced antennas are placed on a radio tower, with the separation between them chosen large enough to ensure independent multipath fadings. The receiver combines the two signals adaptively or else switches between them. *Angle diversity* employs two receiving antennas mounted side by side with different angles of elevation, or dual-beam antennas with angular offset between the two beams. The offset is designed to provide a difference in the amplitudes of received rays. It appears that the use of angle diversity offers equivalent or better performance than space diversity with lower implementation costs.\*

### TEXT BOOK

- I. Digital Communications, Wiley Student Edition, Simon Haykin