Q.2 a. (i) What is duality in electrical networks? Explain the steps involved in construction of a dual of a network.
(ii) Draw the dual of the network shown in Fig.1. (6+2)

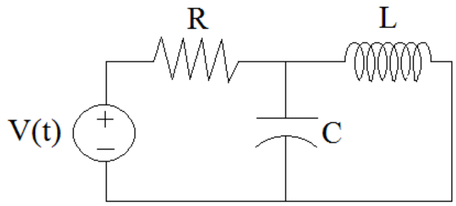


Fig.1

Answer:

(a)

(i) Two circuits are said to be duals if the mesh equations that characterize one of them have the same mathematical equation (or) form as the nodal equations that characterize the other.

(ii) Duality is a transformation in which currents and voltages are interchanged.

(iii) The two electrical networks that are governed by the same types of equations are called dual network.

(iv) The principle application of the duality is that once you have the solution to the network, you can also have the solution to the dual network.

(v) Mutual inductance has no dual property since the circuit with coupled coils have no dual.

Steps to be followed to draw the dual of any network.

(1) In each loop of a network place a node (or) dot, and place an extra node (or) dot called 'Datum node' (or) reference node outside the network.

(2) Draw the lines connecting adjacent nodes passing through each element and also to the reference node by placing the dual of each element in the line passing through original elements.

In an electrical circuits itself there are pairs of terms which can be interchange get new circuits. Such pair of dual terms are given below.

Current	—	Voltage
Open	—	Short
L	—	C
R	—	G
Series	—	Parallel
KCL	—	KVL
voltage Source	—	Current Source.

(ii)

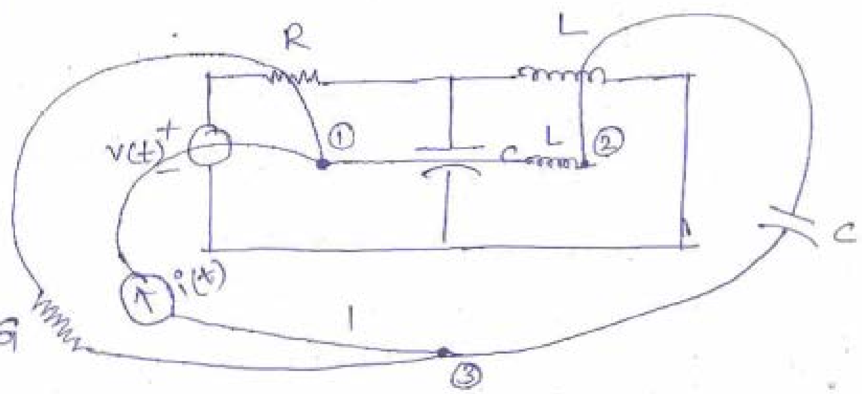
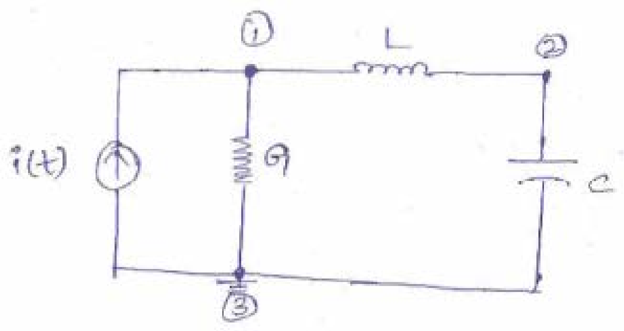


Fig.1

Dual of the network shown in Fig.1



b. Explain the Dot convention rules in magnetically coupled coils. (8)

Answer:

Dot Convention :

Dot convention is used to establish the choice of correct sign for the mutually induced voltages in coupled circuits.

Circular dot marks (or) special symbols are placed at one end of each of two coils which are mutually coupled to simplify the diagrammatic representation of the windings.

Let us consider the fig.1(b) which shows a pair of linear, time invariant, coupled inductors with self inductances L_1 and L_2 and a mutual inductance M. The currents i_1 and i_2 as shown in fig.1(b), each arbitrarily assumed entering at the dotted terminals, and voltages V_1 and V_2 are developed across the inductors.

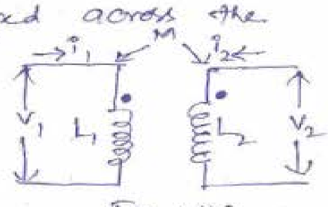


Fig. 1(b)

The voltage across L_1 is given by

$$V_1(t) = L_1 \frac{di_1(t)}{dt} \pm M \frac{di_2(t)}{dt}$$

The first term on the RHS of the above equation is the self induced voltage due to i_1, and the second term represents the mutually induced voltage due to i_2.

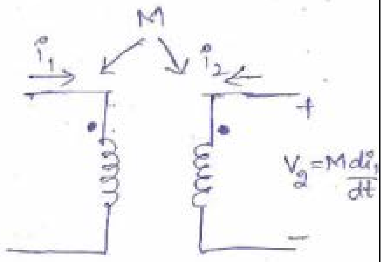
Similarly, the voltage across L_2 is given by

$$V_2(t) = L_2 \frac{di_2(t)}{dt} \pm M \frac{di_1(t)}{dt}$$

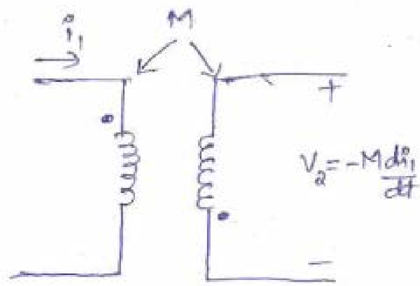
The self-induced voltages are designated with positive sign, mutually induced voltages can be either positive (or) negative depending on the direction of the winding of the coil and the dots placed at one end of each of the two coils.

The dot convention is stated as follows.

(1) If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil



(2) If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.



Q.3 a Find the Laplace Transform of the following signals: **(6+2)**

(i) $f(t) = \cos\omega t$

(ii) $f(t) = U(t-3)$

Answer:

(i) $f(t) = \cos\omega t\, u(t)$

We know that $\cos\omega t = \dfrac{e^{j\omega t} + e^{-j\omega t}}{2}$

$f(t) = \cos\omega t\, u(t)$

Applying Laplace transform on both sides

$L\{f(t)\} = L\{\cos\omega t\, u(t)\}$

$F(s) = L\left\{\left(\dfrac{e^{j\omega t} + e^{-j\omega t}}{2}\right) u(t)\right\}$

$F(s) = \dfrac{1}{2} L\{e^{j\omega t} u(t)\} + \dfrac{1}{2} L\{e^{-j\omega t} u(t)\}$ —(1)

Using Frequency shifting property, we know that

$e^{j\omega t} u(t) \xleftrightarrow{LT} \dfrac{1}{s - j\omega}$

$$e^{-j\omega t} u(t) \xleftrightarrow{LT} \frac{1}{s+j\omega}$$

By applying the frequency Shift Property to eqn ①,
We get

$$F(s) = \frac{1}{2}\left[\frac{1}{s-j\omega}\right] + \frac{1}{2}\left[\frac{1}{s+j\omega}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right]$$

$$= \frac{1}{2}\left[\frac{s+j\omega + s-j\omega}{(s-j\omega)(s+j\omega)}\right]$$

$$\boxed{F(s) = \frac{s}{s^2+\omega^2}}$$

(ii) Given $f(t) = u(t-3)$

We know that

$$L\{u(t)\} = \frac{1}{s}$$

Using Time Shifting property, we have

$$x(t-t_0) \xleftrightarrow{LT} e^{-st_0} X(s)$$

$$\therefore L\{u(t-3)\} = e^{-3s} L\{u(t)\}$$

$$\boxed{L\{u(t-3)\} = \frac{e^{-3s}}{s}}$$

b. Find the Initial and Final values for the following: **(4+4)**

$$X(s) = \frac{s+4}{s^2+3s+5}$$

Answer:

Given $X(s) = \dfrac{s+4}{s^2+3s+5}$

Initial Value $x(0) = \underset{s\to\infty}{Lt}\ s\,X(s)$

$$x(0) = \underset{s\to\infty}{Lt}\ s\left(\frac{s+4}{s^2+3s+5}\right).$$

$$x(0) = \underset{s\to\infty}{Lt}\ \frac{s^2+4s}{s^2+3s+5}$$

$$x(0) = \underset{s\to\infty}{Lt}\ \frac{s^2\left[1+4/s\right]}{s^2\left[1+3/s+5/s^2\right]}$$

$$x(0) = \underset{s\to\infty}{Lt}\ \frac{1+4/s}{1+3/s+5/s^2} = \frac{1}{1} = 1$$

$$\left(\because \text{ As } s\to\infty,\ \frac{1}{s}\to 0\right).$$

Final Value $x(\infty) = \underset{s\to 0}{Lt}\ s\,X(s)$

$$x(\infty) = \underset{s\to 0}{Lt}\ s.\left(\frac{s+4}{s^2+3s+5}\right).$$

$$= \underset{s\to 0}{Lt}\ \frac{s^2+4s}{s^2+3s+5}$$

$$x(\infty) = \frac{0+0}{0+0+5} = 0.$$

Q.4 a. Find the Power dissipated in 10Ω resistor in the circuit shown in Fig.2 using Norton's Theorem. (8)

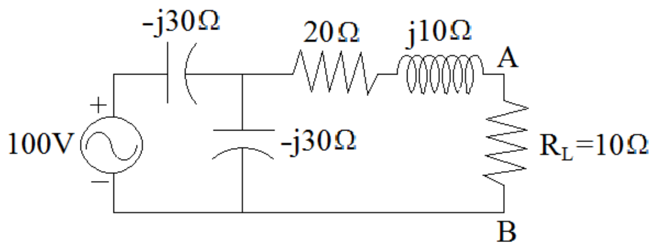
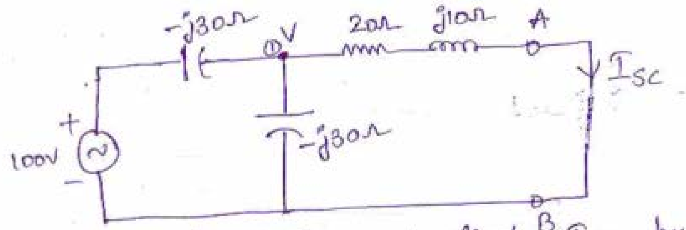


Fig.2

Answer:

4A). (a)



Step ①:- Remove the load R_L and find I_{SC} by putting a short circuit

Applying KCL at node ①

$$\frac{V-100}{-j30} + \frac{V}{-j30} + \frac{V}{20+j10} = 0$$

$$= V\left[\frac{-1}{j30} - \frac{1}{j30} + \frac{1}{20+j10}\right] = \frac{100}{-j30}$$

$$= V\left[j0.033 + j0.033 + 0.04 - j0.02\right] = \frac{100}{30\angle -90} = 3.33\angle 90$$

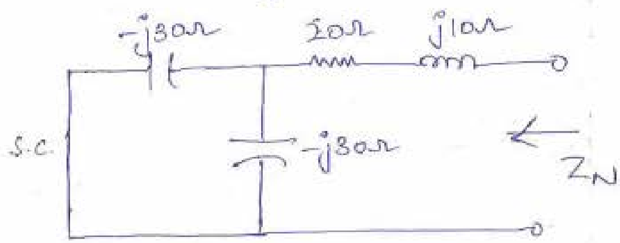
$$= V\left[0.04 + j0.046\right] = 3.33\angle 90$$

$$= V = \frac{3.33\angle 90}{0.06\angle 48.99} = 55.5\angle 41.01\ V$$

$$= I_{sc} = \frac{V}{20+j10} = \frac{55.5\angle 41.01}{22.36\angle 26.56}$$

$$\boxed{I_{sc} = 2.48\angle 14.45\ A}$$

Step ② :- Find Z_N by replacing the voltage and current sources by their internal resistances.



$$Z_N = (20+j10) + (-j30 \parallel -j30)$$

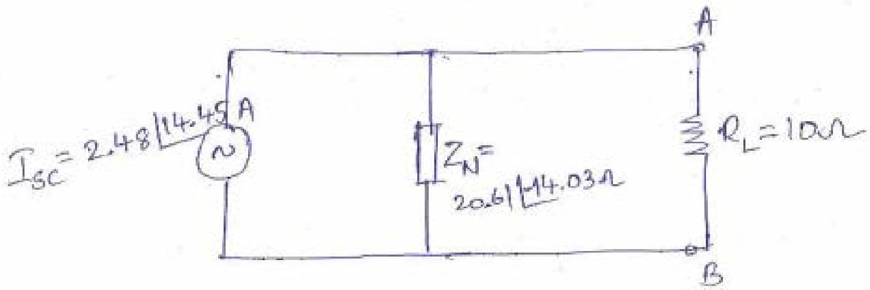
$$Z_N = (20+j10) + \left(\frac{-j30 \times -j30}{-j30 - j30}\right)$$

$$Z_N = (20+j10) + \frac{900}{j60}$$

$$Z_N = 20 + j10 - j15 = 20 - j5$$

$$\boxed{Z_N = 20.61\angle -14.03\ \Omega}$$

Norton's Equivalent Circuit



By using current division formula.

$$I_L = I_{SC} \times \frac{Z_N}{Z_N + R_L}$$

$$I_L = 2.48\angle 14.45 \times \frac{20.61\angle -14.03}{20.61\angle -14.03 + 10}$$

$$I_L = 1.68\angle 9.87 \text{ A}$$

Power dissipated in 10Ω resistor is given by

$$P_L = P_{10\Omega} = I_L^2 R_L$$

$$P_L = P_{10\Omega} = (1.68)^2 \times 10$$

$$\boxed{P_L = P_{10\Omega} = 28.22 \text{ Watts}}$$

b. Using Milliman's theorem, find out the voltage across the load resistance for the circuit shown in Fig.3. (8)

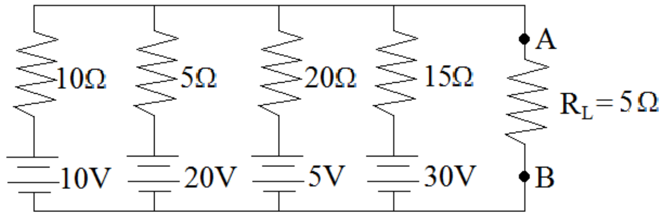
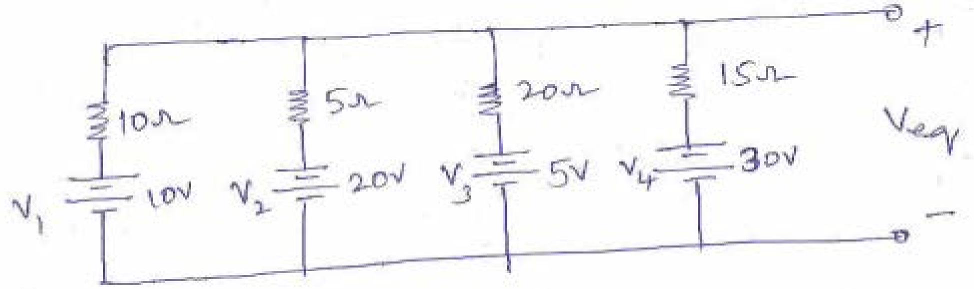


Fig.3

Answer:



To find Veq :-

$$V_{eq} = \frac{V_1 G_1 + V_2 G_2 + \cdots + V_n G_n}{G_1 + G_2 + \cdots + G_n}$$

$$V_{eq} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_4}{R_4}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} \qquad \boxed{\because G = \frac{1}{R}}$$

Since, from the Fig., $V_1 = 10V$ $V_2 = 20V$
$V_3 = 5V$ $V_4 = 30V$

$R_1 = 10\Omega$, $R_2 = 5\Omega$, $R_3 = 20\Omega$, $R_4 = 15\Omega$

$$\therefore V_{eq} = \frac{\frac{10}{10} + \frac{20}{5} + \frac{5}{20} + \frac{30}{15}}{\frac{1}{10} + \frac{1}{5} + \frac{1}{20} + \frac{1}{15}}$$

$$V_{eq} = \frac{1 + 4 + 1/4 + 2}{25/60} = \frac{\frac{4+16+1+8}{4}}{\frac{25}{60}}$$

$$V_{eq} = \frac{87}{5} = 17.4\,V$$

$$\boxed{V_{eq} = 17.4V}$$

To find R_{eq}.

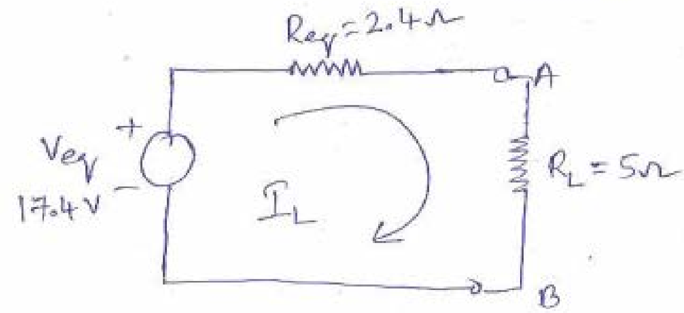
$$R_{eq} = \frac{1}{G_1 + G_2 + \dots + G_n} \qquad \boxed{\because G = \frac{1}{R}}$$

$$R_{eq} = \frac{1}{\frac{1}{10} + \frac{1}{5} + \frac{1}{20} + \frac{1}{15}} = \frac{1}{25/60}$$

$$R_{eq} = \frac{60}{25} = 2.4\Omega$$

$$\boxed{R_{eq} = 2.4\Omega}$$

Milliman's Equivalent Circuit



$$I_L = \frac{V_{eq}}{R_{eq} + R_L} = \frac{17.4}{2.4+5}$$

$$I_L = \frac{17.4}{7.4} = 2.35\,A$$

$$\boxed{I_L = 2.35A}$$

Voltage across the load resistance $R_L = 5\Omega$ is

$$V_L = I_L R_L$$

$$V_{5\Omega} = V_L = 2.35 \times 5$$

$$\boxed{V_L = 11.75\,V}$$

Q.5 a. The Z-Parameters of a two-port network are Z_{11}=30Ω, Z_{22}=40Ω, Z_{12}= Z_{21}=20Ω. Find the equivalent T-network. (8)

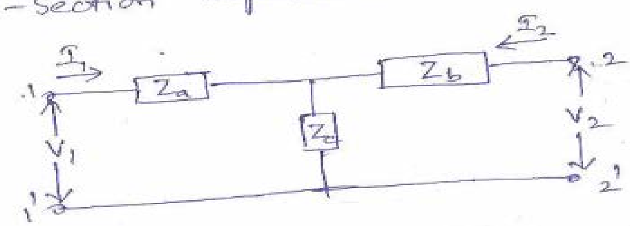
Answer:

(a) Given

$Z_{11} = 30\Omega \qquad Z_{22} = 40\Omega$

$Z_{12} = Z_{21} = 20\Omega$

The T-Section representation of a two-port network is:



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_a + Z_b \quad \text{— (1)}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_c \quad \text{— (2)}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_c \quad \text{— (3)}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_b + Z_c \quad \text{— (4)}$$

from (2) & (3), $\boxed{Z_c = Z_{12} = Z_{21} = 20\Omega}$

from eqn (4), $Z_b = Z_{22} - Z_c = Z_{22} - Z_{21}$

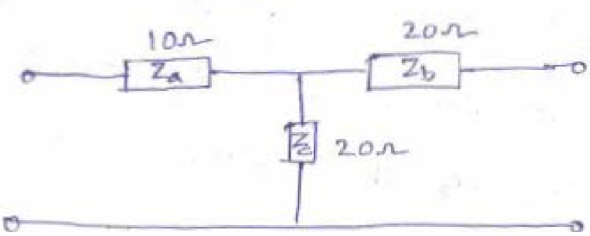
$$\boxed{Z_b = 40 - 20 = 20\Omega}$$

from eqn (1), $Z_a = Z_{11} - Z_b$

$Z_a = Z_{11} - 20$

$Z_a = 30 - 20 = 10\Omega$

$$\boxed{Z_a = 10\Omega}$$



b. The Impedance parameters of a two-port network are $Z_{11}=6\Omega$, $Z_{22}=4\Omega$, $Z_{12}=Z_{21}=3\Omega$. Compute the Y-Parameters and write the equations. (8)

Answer:

(b) Given Impedance parameters of a two-port network are

$Z_{11}=6\Omega$, $Z_{22}=4\Omega$, $Z_{12}=Z_{21}=3\Omega$

The Y-Parameters are given by

(1) $$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22}-Z_{12}Z_{21}} = \frac{4}{(6\times4)-(3\times3)} = \frac{4}{15}\,\mho$$

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$$\boxed{Y_{11} = \frac{4}{15}\ \mho}$$

(2) $Y_{12} = \dfrac{-Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \dfrac{-3}{(6\times4)-(3\times3)} = -\dfrac{\cancel{3}^{1}}{\cancel{15}_{5}}$

$$\boxed{Y_{12} = -\frac{1}{5}\ \mho}$$

(3) $Y_{21} = Y_{12} = -\dfrac{Z_{12}}{\Delta Z} = -\dfrac{\cancel{3}^{1}}{\cancel{15}_{5}}$

$$\boxed{Y_{12} = Y_{21} = -\frac{1}{5}}$$

(4) $Y_{22} = \dfrac{Z_{11}}{\Delta Z} = \dfrac{4}{24-9} = \dfrac{\cancel{6}^{2}}{\cancel{15}_{5}}\ \mho$

$$\boxed{Y_{22} = \frac{2}{5}\ \mho}$$

The equations of Y-Parameters $I_1 = \frac{4}{15}V_1 - \frac{1}{5}V_2$

$I_2 = -\frac{1}{5}V_1 + \frac{2}{5}V_2$

Q.6 a. Define Selectivity and Bandwidth of R-L-C circuit. (8)

Answer:

22.6. Selectivity and Bandwidth

A series RLC circuit gives unequal response to voltages of different frequencies. At the frequency of resonance the impedance is minimum

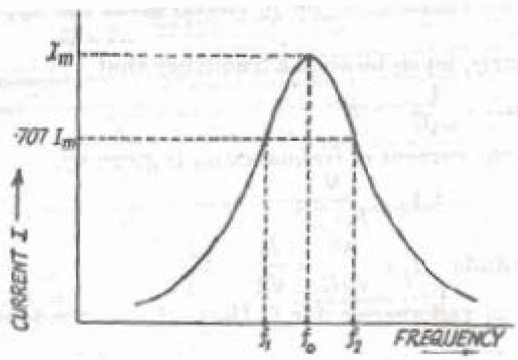


Fig. 22.5. Current *versus* frequency curve of a series RLC circuit.

and the current is maximum. As the frequency of the applied voltage is either reduced or increased from this resonance frequency, the impedance increases and the current fall. Fig. 22.5 shows the variation of current I with frequency. Thus a series RLC circuit possesses frequency selectivity.

Fig. 22.5 shows current *versus* frequency curves of a series RLC circuit for small value of R. The frequencies f_1 and f_2 at which current I falls to $\frac{1}{\sqrt{2}}$ (or 0.707) of its maximum value $I_0 \left(= \frac{V}{R}\right)$ are called *half-power frequencies* or *3 dB frequencies*. The bandwidth $(f_2 - f_1)$ is called the *half-power bandwidth* or *3 dB bandwidth* or simply the *bandwidth* of the circuit.

Selectivity of a resonant circuit is defined as the ratio of resonant frequency f_0 to the 3-dB bandwidth. Thus selectivity

$$= \frac{\text{Resonance frequency}}{\text{3 dB bandwidth}} = \frac{f_0}{(f_2 - f_1)} \qquad ...(22.16)$$

The current in the series RLC circuit is given by,

$$\mathbf{I} = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Let ω_2 be such a frequency

$$\omega_2 L - \frac{1}{\omega_2 C} = R \qquad ...(22.17)$$

Then at frequency ω_2,

$$\mathbf{I}_2 = \frac{V}{R + jR}$$

Magnitude $I_2 = \frac{V}{\sqrt{2}R} = \frac{I_0}{\sqrt{2}}$

Thus ω_2 radians/sec. (or f_2 Hertz) gives the upper half-power frequency.

Similarly, let ω_1 be such a frequency that,

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

Then the current at frequency ω_1 is given by,

$$\mathbf{I}_2 = \frac{V}{R - jR}$$

Magnitude $\frac{V}{\cdot} \quad \frac{I_0}{\cdot}$

Thus ω_1 radians/sec. (or f_1 Hertz) forms the lower half —— frequency.

This current ratio expressed in decibels + given by

$$\text{Ratio in decibels} = 20 \log_{10} \frac{I_1}{I}$$

$$= 20 \log_{10} \frac{\frac{I_0}{\sqrt{2}}}{}$$

$$= -3.$$

Thus I_1 is 3 dB lower than I_0. Similarly I_2 is 3 dB lower than I_0. Hence f_1 and f_2 are called 3 dB frequencies.

Frequencies f_1 and f_2 are also called *half-power frequencies* because the power dissipation in the circuit at these frequencies is half of the power dissipation at the resonant frequency f_0, may be seen as below:

P_0 = Power dissipation at $f_0 = I_0^2 R$.

P_1 = Power dissipation at $f_1 = I_1^2 R = \frac{I_0^2 R}{2} = \frac{P_0}{2}$.

P_2 = Power dissipation at $f_2 = I_2^2 R = \frac{I_0^2 R}{2} = \frac{P_0}{2}$

Expression for Quality factor of an Inductor (or) a coil.

In an Inductor, the maximum energy stored is given by $= \frac{1}{2} L I_m^2$

Energy dissipated in 'R'/cycle $= I_{rms}^2 R \times T$

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 \times R \times \frac{1}{f}. \quad \left[\because I_{rms} = \frac{I_m}{\sqrt{2}},\ T = \frac{1}{f}\right]$$

$$= \frac{I_m^2 R}{2f}$$

Quality factor of the coil $Q = 2\pi \times \dfrac{\frac{1}{2} L I_m^2}{\frac{I_m^2 R}{2f}}$

$$Q = 2\pi \times \frac{1}{\cancel{2}} L \cancel{I_m^2} \times \frac{\cancel{2}f}{\cancel{I_m^2} R}$$

$$Q = \frac{2\pi f L}{R} = \frac{\omega L}{R}.$$

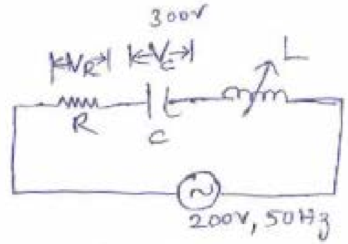
$$\boxed{\therefore Q = \frac{\omega L}{R}}$$

b A resistor and a capacitor are in series with a variable Inductor. When the circuit is connected to a 200V, 50Hz supply, the maximum current obtainable by varying the Inductance is 0.314A, the voltage across the capacitor is then 300V. Find the circuit elements. (8)

Answer:

(b)

Given data $V_C = 300\,V$



$f = 50\,Hz$

$I_o = 0.314\,A$ (at resonance)

$V = 200\,V$.

(1) At resonance, $I_o = \frac{V}{R}$

$$R = \frac{V}{I_0}$$

$$R = \frac{200}{0.314} = 637\,\Omega$$

$$\boxed{R = 637\,\Omega}$$

At resonance, the Voltage across the Inductor is equal to the voltage across the capacitor ($V_L = V_C$)

(2) $V_c = I_0 X_c$

$V_c = I_0 / 2\pi f C \quad \left(\because X_c = \frac{1}{\omega C} = \frac{1}{2\pi f c}\right)$

$$C = \frac{I_0}{2\pi f V_c} = \frac{0.314}{2\pi \times 50 \times 300}$$

$$C = 3.33 \times 10^{-6}\,F$$

$$\boxed{C = 3.33\,\mu F}$$

(3) $V_L = I_0 X_L$

$I_0 = \frac{V_L}{X_L} = \frac{V_L}{2\pi f L} \quad (\because X_L = 2\pi f L)$

$$L = \frac{V_L}{2\pi f I_0} = \frac{300}{2\pi \times 50 \times 0.314}$$

$$\boxed{L = 3.0377}$$

Q.7 a. Explain how $\frac{R}{L} = \frac{G}{C}$ is a distortionless condition of a transmission line. (8)

Answer:

(a) Distortionless condition can help in designing new lines (or) modifying old ones to minimize distortion. The line parameters that allows propagation without distortion and such a condition is termed as distortionless line.

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The distortionless Condition can be derived by the expression for the propagation Constant P as given

$$P = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$P = \sqrt{L\left(\frac{R}{L}+j\omega\right)C\left(\frac{G}{C}+j\omega\right)}$$

$$P = \sqrt{LC}\sqrt{\left(\frac{R}{L}+j\omega\right)\left(\frac{G}{C}+j\omega\right)}$$

If $\frac{R}{L} = \frac{G}{C}$, then ——— (1)

$$P = \sqrt{LC}\sqrt{\left(\frac{R}{L}+j\omega\right)\left(\frac{R}{L}+j\omega\right)} = \sqrt{LC}\left(\frac{R}{L}+j\omega\right)$$

(or)

$$P = \sqrt{LC}\sqrt{\left(\frac{G}{C}+j\omega\right)\left(\frac{G}{C}+j\omega\right)} = \sqrt{LC}\left(\frac{G}{C}+j\omega\right)$$

$$P = R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC} \quad (\text{or}) \quad G\sqrt{\frac{L}{C}} + j\omega\sqrt{LC}.$$

But Propagation Constant is a Complex quantity, it is given as $P = \alpha + j\beta$

$$\therefore P = \alpha + j\beta = R\sqrt{\frac{C}{L}} + j\omega \quad (\text{or}) \; G\sqrt{\frac{L}{C}} + j\omega\sqrt{LC} \quad —(2)$$

By equating real and imaginary parts.

The above eqn (2) shows that

Real part α of the Propagation constant is $= R\sqrt{\frac{C}{L}}$ (or) $G\sqrt{\frac{L}{C}}$.

which is Independent of frequency and the Imaginary part β is $\omega\sqrt{LC}$ is a Constant multipled by "ω".

$$\therefore \boxed{\begin{array}{l} \alpha = R\sqrt{\frac{C}{L}} \; (\text{or}) \; G\sqrt{\frac{L}{C}} \\ \beta = \omega\sqrt{LC} \end{array}} \leftarrow (3)$$

When the line parameters are connected by the equation (1) neither frequency nor delay distortion exists, and the line is said to be distortionless. The condition expressed by equation (1) is said to be distortionless condition.

We know that Phase velocity $V_p = \frac{\omega}{\beta}$

$$V_p = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$\boxed{V_p = \frac{1}{\sqrt{LC}}} \quad -(5)$$

The characteristic Impedance Z_0 can be expressed

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L\left(\frac{R}{L}+j\omega\right)}{C\left(\frac{G}{C}+j\omega\right)}}$$

Since $\frac{R}{L} = \frac{G}{C}$ is a distortionless condition, then

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}} \quad -(6)$$ — which is a purely real.

∴ In a distortionless line, all frequency components have the same attenuation and phase velocity.

∴ Since α is Independent of 'ω' and 'β' is multiplied by 'ω', also satisfies distortionless condition.

b. Derive the expressions for Attenuation Constant(α) and Phase constant (β) of a transmission line in terms of R, L, C and G. (8)

Answer:

We know that Propagation constant P is a Complex quantity and can be expressed as $P = \alpha + j\beta$.

$$\therefore \quad P = \alpha + j\beta = \sqrt{ZY}$$

$$P = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad \text{— (1)}$$

Squaring on both sides.

$$(\alpha+j\beta)^2 = \left(\sqrt{(R+j\omega L)(G+j\omega C)}\right)^2$$

$$\alpha^2+2j\alpha\beta-\beta^2 = (R+j\omega L)(G+j\omega C)$$

$$\alpha^2-\beta^2+j2\alpha\beta = RG-\omega^2LC+j(R\omega C+G\omega L)$$

Equating the real parts, we get

$$\alpha^2-\beta^2 = RG-\omega^2LC \quad \text{--- (2)}$$

Again from eqn (1)

$$\alpha+j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

Taking modulus on both sides

$$|\alpha+j\beta| = \left|\sqrt{(R+j\omega L)(G+j\omega C)}\right|$$

$$\alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \quad \text{--- (3)}$$

Adding eqn (2) & (3)

$$\alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\alpha^2-\beta^2 = RG-\omega^2LC$$

$$2\alpha^2 = (RG-\omega^2LC)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\boxed{\alpha = \sqrt{\frac{1}{2}\left[(RG-\omega^2LC)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}}$$

Subtracting eqn (2) & (3)

$$\alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\alpha^2-\beta^2 = RG-\omega^2LC$$

$$2\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}-(RG-\omega^2LC)$$

$$\boxed{\beta = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}-(RG-\omega^2LC)\right]}}$$

Q.8 a. Explain the Phenomenon of Standing Wave ratio (SWR) and derive the relationship between SWR and Reflection Coefficient. (2+6)

Answer:

(a) When a H.F. transmission line is not correctly terminated, the travelling electromagnetic wave from generator at the sending end is reflected completely (or) partially at the termination.

The combination of Incident and reflected wave give rise to standing waves of current and voltage along the line.

Defination: The ratio of the maximum to minimum magnitudes of current (or) voltage on a line having standing waves is called the standing wave ratio. It is abbreviated as SWR and is denoted by S.

$$\therefore \ VSWR = \frac{V_{max}}{V_{min}}$$

Range of VSWR is always greater than 1 i.e.

$1 \leq VSWR < \infty$

Relation between SWR and Reflection Coefficient

The voltage maximum are where the incident and reflected voltage are in phase and add up gives

$$V_{max} = |V_i| + |V_r| \quad \text{--- (1)}$$

where V_i is the r.m.s. value of incident voltage
V_r is the r.m.s. value of reflected voltage

$$V_{min} = |V_i| - |V_r| \quad \text{--- (2)}$$

Hly

Equation (2) represents the voltage minimum are those where the incident and reflected voltage are out of phase and will have opposite sign.

$$\therefore \quad VSWR = \left| \frac{V_{max}}{V_{min}} \right|$$

$$VSWR = \frac{|V_i| + |V_r|}{|V_i| - |V_r|}$$

Dividing numerator and denominator by V_i, we have

$$VSWR = \frac{1 + \frac{|V_r|}{|V_i|}}{1 - \frac{|V_r|}{|V_i|}}$$

But the reflection coefficient is given by,

$$K = \frac{V_r}{V_i}$$

$$\therefore \quad VSWR = \frac{1 + |K|}{1 - |K|} \quad (or) \quad \boxed{S = \frac{1 + |K|}{1 - |K|}}$$

$$\boxed{|K| = \frac{S - 1}{S + 1}}$$

b. A certian lossless line has a characteristic impedance of 400Ω. Determine the SWR with the following receiving end Impedance. (4+4)

(i) $Z_R = 70 + j0\Omega$

(ii) $Z_R = 450 + j50\Omega$

Answer:

Given data

$Z_0 = 400\Omega$ $\quad S = ?$

Case ①:- $Z_R = 70 + j0\,\Omega$

Reflection Coefficient $K = \frac{Z_R - Z_0}{Z_R + Z_0}$

$K = \frac{(70+j0) - 400}{(70+j0) + 400} = -\frac{33}{47}$

$|K| = |-33/47| = 33/47$

SWR $S = \frac{1+|K|}{1-|K|} = \frac{1+33/47}{1-33/47} = 5.71$

$\boxed{S = 5.71}$

Case ②:- $Z_R = 450 + j50$ $\quad Z_0 = 400\Omega$

$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{450+j50-400}{450+j50+400} = \frac{50+j50}{950+j50}$

$K = 0.074\angle 41.99$

$|K| = 0.074$

SWR $S = \frac{1+|K|}{1-|K|} = \frac{1+0.074}{1-0.074} = \frac{1.074}{0.926}$

$\boxed{S = 1.159}$

Q.9 a. Define Decibel and Neper? Derive the numerical relationship between the Decibel and Neper. (2+2+4)

Answer:

Decibel :

The attenuation of a wave filter. Can be expressed in decibels (or) Nepers.

A decibel is defined as ten times the logarithms of the ratio of the Input power to the output power.

$$\therefore \text{Decibel } D = 10\log_{10}\left(\frac{P_1}{P_2}\right).$$

In terms of Voltage (or) current ratio, ~~the~~ decibel Can be defined as

(18)

$$D = 20\log_{10}\left(\frac{V_1}{V_2}\right) \text{ (or) } 20\log_{10}\left(\frac{I_1}{I_2}\right)$$

Neper :-

It is defined as the natural logarithm of the ratio of Input voltage (or current) to the output voltage (or current), It is denoted by 'N'

$$N = \log_e\left|\frac{V_1}{V_2}\right| \text{ (or) } \log_e\left|\frac{I_1}{I_2}\right|$$

In terms of P a m ?" do, Neper can be defined as

$$N = \frac{1}{2}\log_e\left|\frac{P_1}{P_2}\right| \text{ (or) } \frac{1}{2}\log_e\left(\frac{P_1}{P_2}\right)$$

Relationship between Decibel and Neper

Neper :- We know that $\therefore e^N = \left|\frac{I_1}{I_2}\right| = \left|\frac{V_1}{V_2}\right|$ In terms of voltage and current

In terms of Power, $\frac{P_1}{P_2} = e^{2N}$ — ①

Decibel :- Attenuation in $dB = 10\log_{10}(P_1/P_2)$ In terms of Power ratio

$$\therefore \frac{dB}{10} = \log_{10}(P_1/P_2)$$

$$10^{dB/10} = \frac{P_1}{P_2} \quad — ②$$

Equating equation ① & ②, we get

$$e^{2N} = 10^{dB/10}$$

Taking logarithm on both sides

$$\log_{10} e^{2N} = \log_{10} 10^{dB/10}$$

$$2N\log_{10} e = \frac{dB}{10}\log_{10} 10$$

$$\frac{dB}{10} = 2N \times 0.432 \quad \left[\because \log_{10} e = 0.432\right]$$

$$\boxed{dB = N \times 8.684}$$

(or)

$$\boxed{N = 0.115 \times dB}$$

b. Design a Constant-K Low pass filter to have a cut-off frequency of 2 kHz and terminating impedance of 600Ω. Design for both T and π sections. (8)

Answer:

(b) Given data

$f_c = 2KHz$

R_o (or) $R_K = 600\Omega$ which is a design (or) terminating Impedance

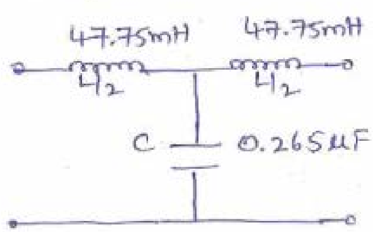
To find Inductance (L)

$$L = \frac{R_K}{\pi f_c} = \frac{600}{\pi \times 2 \times 10^3} = 0.095\,H$$

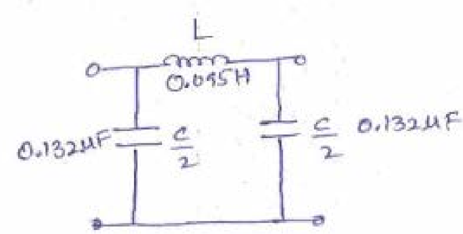
To find Capacitance (C)

$$C = \frac{1}{\pi R_K f_c} = \frac{1}{\pi \times 600 \times 2 \times 10^3} = 0.265\,\mu F$$

∴ The Designed T and π Section LPF is shown in figure.



T-Section



π-Section

TEXT BOOK

I. Network Analysis; G. K. Mittal; 14th Edition (2007) Khanna Publications; New Delhi

II. Transmission Lines and Networks; Umesh Sinha, 8th Edition (2003); Satya Prakashan, Incorporating Tech India Publications, New Delhi