

Q.2 a. Write the advantages of Laplace transformation.

(4)

Answer:

Advantages of Laplace transformations

- (1) Solution of differential equation is systematic and routine.
- (2) This method gives total solution i.e. complementary function and particular solution in one operation.
- (3) Initial conditions are automatically specified.
- (4) Much less time needed to solve differential equation as compare to classical method.
- (5) It provide direct solution for non homogeneous differential equation.

b. Find the convolution integral when  $f_1(t) = e^{-t}$  and  $f_2(t) = e^{-2t}$

(4)

Answer:

Convolution integral is given by

$$f_1(t) * f_2(t) = \int_0^t f_1(t-\tau) f_2(\tau) d\tau$$

$$= \int_0^t e^{-(t-\tau)} e^{-2\tau} d\tau$$

$$= e^{-t} \int_0^t e^{\tau} \cdot e^{-2\tau} d\tau$$

$$= +e^{-t} \int_0^t e^{-\tau} d\tau$$

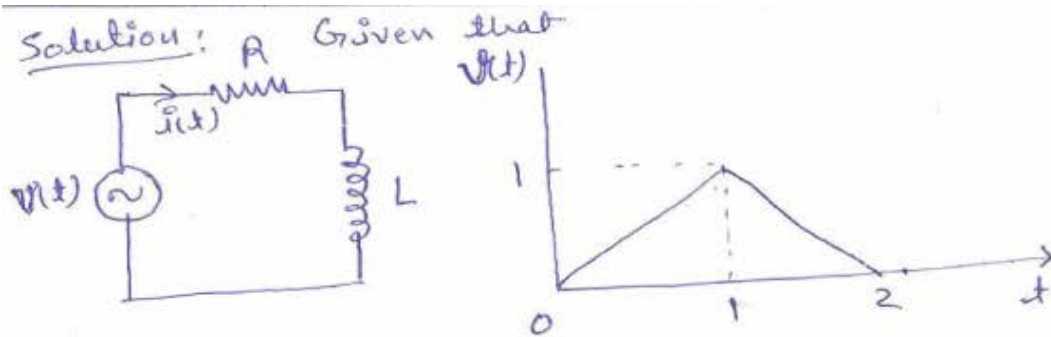
$$= +e^{-t} \left[ -e^{-\tau} \right]_0^t$$

$$= +e^{-t} [-e^{-t} + 1] = +e^{-t} - e^{-2t}$$

$$f_1(t) * f_2(t) = e^{-t} - e^{-2t}$$

- c. Voltage  $V(s) = \frac{1 - 2e^{-s} + e^{-2s}}{s^2}$  is applied as input to a series RL circuit with  $R = 2\Omega$ ,  $L = 2$  H. Calculate  $i(t)$  using Laplace transform through the circuit. (Assume  $i(0^+) = 0$ ) (8)

Answer:



By KVL, we have

$$Ri + L \frac{di}{dt} = v(t)$$

Taking its Laplace transform, we get

$$RI(s) + LsI(s) - Li(0^+) = V(s)$$

Since  $i(0^+) = 0$

$$RI(s) + LsI(s) = V(s)$$

$$I(s)[R + sL] = V(s)$$

$$I(s) = \frac{V(s)}{(R + sL)}$$

Given

$$V(s) = \frac{1 - 2e^{-s} + e^{-2s}}{s^2}$$

Put value of  $R$  and  $L$ , we have

$$I(s) = \frac{1 - 2e^{-s} + e^{-2s}}{s^2(2 + 2s)}$$

$$i(t) = \mathcal{L}^{-1} I(s) = \mathcal{L}^{-1} \left[ \frac{1 - 2e^{-s} + e^{-2s}}{2s^2(1+s)} \right]$$

$$i(t) = \frac{t - e^{-t} - 1}{2} [u(t) - 2u(t-1) + u(t-2)]$$

Q.3 a. State and prove maximum power transfer theorem.

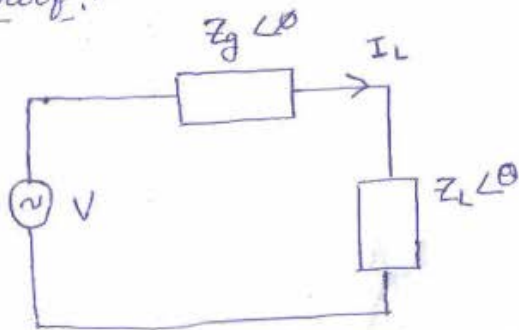
(8)

Answer:

### Maximum Power Transfer Theorem

This theorem states that if maximum power transfer has to take place between the source and load, the resistance of load should be equal to that of the source and the reactance of the load should be equal to that of source in magnitude but opposite in sign. That is, if the source is inductive, the load should be capacitive and vice versa.

Proof:-



Let the generator impedance be  $Z_g \angle \phi$   
 where  $Z_g = R_g + jX_g$   
 and  $\phi = \tan^{-1} \frac{X_g}{R_g}$

The load impedance to assume to be  $Z_L \angle \theta$

where  $Z_L = R_L + jX_L$

and  $\theta = \tan^{-1} \frac{X_L}{R_L}$  as shown in figure.

The power  $P$ , in the load is

$$P = I_L^2 R_L$$

where  $I_L$  is the current flowing in the circuit and is given by

$$I_L = \frac{V}{Z_g + Z_L} = \frac{V}{(R_g + R_L) + j(X_g + X_L)}$$

$$\text{Therefore } P = I_L^2 R_L = \frac{V^2}{(R_g + R_L)^2 + (X_g + X_L)^2} R_L \quad \text{--- (1)}$$

An inspection of equation (1) shows that the power will be maximum when  $X_L = -X_g$  and then power will become

$$P = \frac{V^2 R_L}{R_L + R_g} \quad \text{--- (2)}$$

As far as variation of  $R_L$  is concerned, the value of  $R_L$  which makes  $P$  a maximum will be that which makes  $\frac{dP}{dR_L}$  zero.

Therefore differentiating Eq. (2) w.r.t.  $P$ , gives

$$\frac{dP}{dR_L} = V^2 \left\{ \frac{(R_L + R_g)^2 - 2(R_L + R_g)R_L}{(R_L + R_g)^4} \right\}$$

$$0 = V^2 \left\{ \frac{(R_L + R_g)^2 - 2R_L^2 - 2R_g R_L}{(R_L + R_g)^4} \right\}$$

$$0 = (R_L + R_g)^2 - 2R_L^2 - 2R_g R_L$$

$$0 = R_g^2 - R_L^2 \quad \text{or} \quad \boxed{R_g = R_L}$$

Hence proved.

and value of maximum power

$$P_{\max} = \frac{V^2 R_L}{(R_L + R_L)^2}$$

$$= \frac{V^2 R_L}{4R_L^2}$$

$$\boxed{P_{\max} = \frac{V^2}{4R_L}}$$

- b. A black box consisting of generators and impedances where only two output terminals are available gives the following data:
- (i) Open circuit voltage = 120 volts
  - (ii) Short circuit current = 10 Amp
  - (iii) When output terminals are connected to a resistance of  $8\Omega$ , current flowing = 6Amp., determine Thevenin's equivalent generator. (8)

Answer:

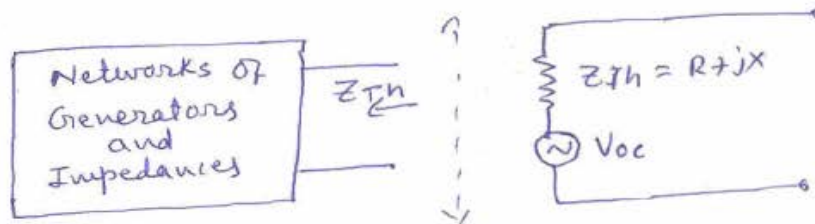
Solution!- From the data given

$$V_{oc} = 120 \text{ V}$$

Short circuit current = 10 Amp.

$$\text{therefore } \frac{120}{\sqrt{R^2 + X^2}} = 10$$

$$\text{or } R^2 + X^2 = 144 \quad \text{--- --- (1)}$$



Similarly from the data given in (iii)

$$\frac{120}{\sqrt{(R+8)^2 + X^2}} = 6$$

$$\text{or } (R+8)^2 + X^2 = 400 \quad \text{--- --- (2)}$$

Subtracting (1) from (2)

$$(R+8)^2 - R^2 = 256$$

$$R^2 + 16R + 64 - R^2 = 256$$

$$\text{or } 16R + 64 = 256$$

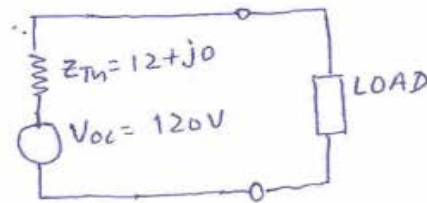
$$\text{or } R = 12 \Omega$$

Put value of R in equation (1), we get

$$12^2 + X^2 = 144$$

$$X = 0 \Omega$$

Thus equivalent Thevenin's generator has an e.m.f. 120V and an internal impedance of  $12 + j0$  as shown in figure.



Q.4 a. What are h- parameters? Draw equivalent circuit using h-parameters and derive equation for calculating h-parameters. (8)

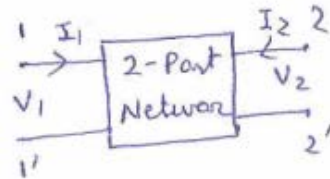
Answer:

### h-Parameters:-

These parameters combine some of properties of the z and y-parameters. In this system of parameters the input voltage and output current are both expressed in terms of input current and output voltage, i.e.

$$V_1 = I_1 h_{11} + V_2 h_{12} \quad \text{--- ①}$$

$$I_2 = I_1 h_{21} + V_2 h_{22} \quad \text{--- ②}$$



In matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where  $h_{11}$ ,  $h_{12}$ ,  $h_{21}$  &  $h_{22}$  are called h-parameters and  $V_1$ , &  $I_1$  Input voltage & current similarly  $V_2$ , &  $I_2$  are output voltage and current.

$h_{11}$  is defined as the input impedance with output short circuited. i.e.  $V_2 = 0$  then from eq. ①

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0}$$

$h_{12}$  is reverse voltage transfer ratio with output open circuited. i.e.  $I_2 = 0$ , from eq. ①

$$h_{12} = \frac{V_2}{V_1} \Big|_{I_2=0}$$

$h_{21}$  is forward current transfer ratio, with output short circuited, i.e.  $V_2 = 0$ , from eq. ②

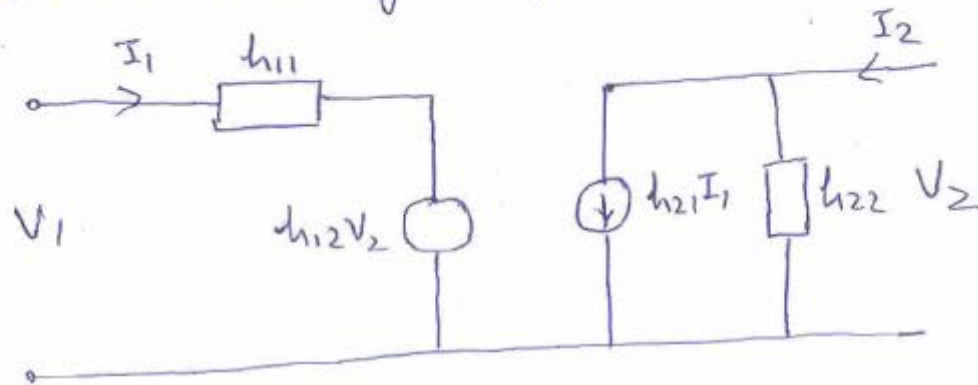
$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0}$$



$h_{22}$  is the output admittance with input open circuited i.e.

$$h_{22} = \frac{I_1}{V_1} \Big|_{I_2 = 0}$$

Equivalent circuit using  $h$ -parameters is



b. Find the equivalent  $\pi$ -network for the T-network shown in Fig.1.

(8)

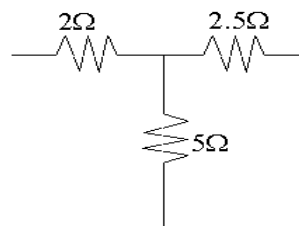
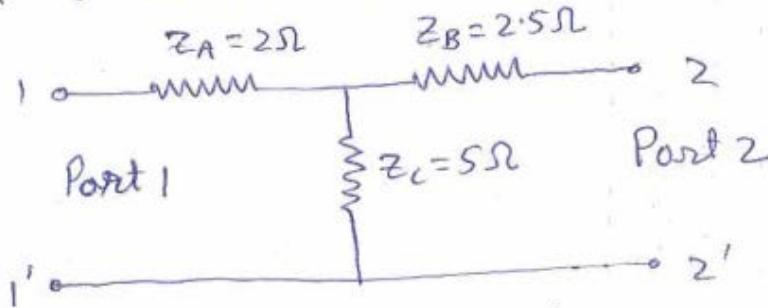


Fig.1

Answer:

Given T-network is



Let the equivalent  $\pi$ -network has  $Y_3$  as admittance in series and  $Y_1$  and  $Y_2$  as shunt admittance at Port 1 and Port 2 respectively.

Then 
$$Y_A = \frac{1}{Z_A} = 0.5 \Omega$$

$$Y_B = \frac{1}{Z_B} = 0.4 \Omega$$

$$Y_C = \frac{1}{Z_C} = 0.2 \Omega$$

Therefore 
$$Y_1 = \frac{Y_A Y_C}{Y_A + Y_B + Y_C} = \frac{0.5 \times 0.2}{0.5 + 0.4 + 0.2} = \frac{1}{1.1} \Omega$$

**Q.5 a. Determine the relationship between the resonant frequency  $f_0$  and the half-power frequencies  $f_1$  and  $f_2$  in a series resonating circuit. (8)**

**Answer:**

→ × ————— × ←

The current in series RLC circuit is

$$I = \frac{V}{\sqrt{(R)^2 + (\omega L - \frac{1}{\omega C})^2}}$$

At half power points

$$I = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \frac{V}{R}$$

Since at resonance  $I_0 = \frac{V}{R}$  (as  $\omega L = \frac{1}{\omega C}$ )

$$\therefore \frac{1}{\sqrt{2}} \frac{V}{R} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\text{or } \sqrt{R^2 + \left[ \omega L - \frac{1}{\omega c} \right]^2} = \sqrt{2} R$$

Squaring both side

$$R^2 + \left[ \omega L - \frac{1}{\omega c} \right]^2 = 2 R^2$$

$$\omega L - \frac{1}{\omega c} = \pm R \quad \text{--- (1)}$$

This quadratic equation has two value of  $\omega$ . If  $\omega_1$  and  $\omega_2$  are the power frequencies, then

$$\omega_2 L - \frac{1}{\omega_2 c} = R \quad \text{--- (2)}$$

$$\omega_1 L - \frac{1}{\omega_1 c} = -R \quad \text{--- (3)}$$

on Adding eq. (2) & (3)

$$(\omega_2 + \omega_1) L - \frac{1}{c} \left[ \frac{1}{\omega_2} + \frac{1}{\omega_1} \right] = 0$$

$$(\omega_2 + \omega_1) L - \frac{1}{c} \left[ \frac{\omega_2 + \omega_1}{\omega_2 \omega_1} \right] = 0$$

$$\text{or } (\omega_2 + \omega_1) \left[ L - \frac{1}{c \omega_1 \omega_2} \right] = 0$$

$$\text{or } \omega_2 + \omega_1 \left[ \frac{L \omega_1 \omega_2 - 1}{c \omega_1 \omega_2} \right] = 0$$

$$\text{or } \omega_2 \omega_1 = \frac{1}{L c}$$

$$\text{But at resonance } \omega_0 = \frac{1}{\sqrt{L c}}$$

$$\therefore \omega_2 \omega_1 = \omega_0^2$$

$$\text{or } \boxed{f_1 \cdot f_2 = f_0^2}$$

- b. A coil with resistance of 20 ohms and induction of 0.2 H is connected in parallel with a 100  $\mu$ F capacitor. Calculate the frequency at resonance ( $f_0$ ) and Q factor. (8)

Answer:

Circuit will act as non-inductive & non-capacitive at resonance frequency.

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$\omega_0^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$$

$$= \frac{1}{0.2 \times 100 \times 10^{-6}} - \left(\frac{20}{0.2}\right)^2$$

$$= 50,000 - 10,000 = 40,000$$

$$\text{or } \omega_0 = \sqrt{40,000} = 200 \text{ rad/sec}$$

$$\text{or } f_0 = \frac{200}{2\pi} = 31.8 \text{ Hz.}$$

$$\text{and } Q = \frac{\omega_0 L}{R} = \frac{200 \times 0.2}{20} = 2$$

$$\therefore \boxed{f_0 = 31.8 \text{ Hz} \quad \text{and} \quad Q = 2}$$

- Q.6 a. Define and explain the term characteristic impedance and propagation constant of a transmission line. (8)

Answer:

### Characteristic Impedance

Characteristic impedance of a uniform transmission line may be defined as the steady-state vector ratio of the voltage to the current at the input of an infinite line. It can be defined as the impedance looking into an infinite length of the line. For a lossless line, the characteristic impedance  $Z_0$  is called "surge impedance".  $Z_0$  does not depend on the length of the line or on the character of the terminating load.

The value of  $Z_0$  for two types of basic transmission lines are given below (1) open wire line.

OPEN WIRE LINE!

The characteristic impedance  $Z_0$  for open wire or parallel wire line is given by

$$Z_0 = 276 \log_{10} \frac{S}{r} \text{ ohms}$$

where 'S' is the spacing between two wire - centre to centre and  $r$  is the radius of either of the wire.

Coaxial Cable!

$$Z_0 = 138 \log_{10} \frac{D}{d} \text{ ohms}$$

where 'D' is the inner diameter of the conductor and 'd' is the diameter of the inner conductor.

By definition, the input impedance

$$\frac{V_{si}}{I_{si}} = Z_0 = \text{characteristic Impedance.}$$

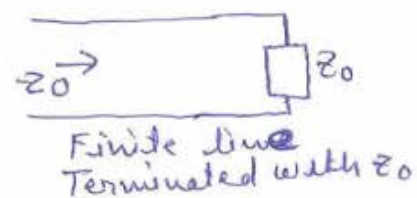
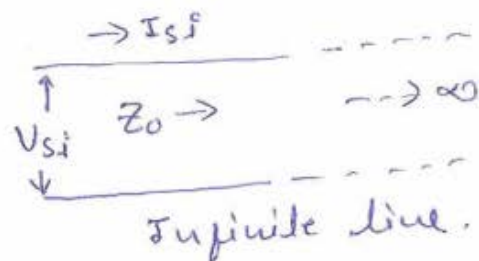
$\therefore$  terms of Primary  $\cos A d$  -

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

where all symbol  $R, L, G, C$  have their usual meaning.

Since no part of the power sent on an infinite line returns, no reflection occurs. when there's no reflection, no loss of power.

Also since a line terminated by its  $Z_0$  behaves as an infinite line, will also have no reflection, and is said to be correctly terminated.



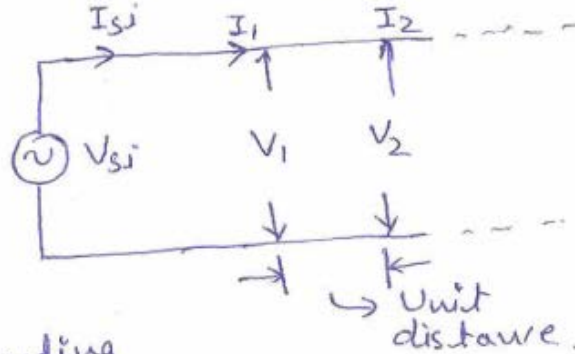
### Propagation Constant

The propagation constant per unit length of a uniform line may be defined as the natural logarithm of the steady state vector ratio of the current or voltage at any point, to that at a point unit distance further from the source, when the line is infinitely long.

Therefore

$$P = \log_e \frac{I_1}{I_2}$$

$$\text{or } P = \log_e \frac{V_1}{V_2}$$



Current  $I_x$  at any point, the distance  $x$  from the sending end is given by

$$I_x = I_{si} e^{-Px} \quad \text{or} \quad e^{Px} = \frac{I_{si}}{I_x}$$

Taking log of both side  $Px = \log_e \frac{I_{si}}{I_x}$

If  $I_{si}$  and  $I_x$  are unit distance apart, as shown in fig,  $x=1$  and  $\log_e \frac{I_{si}}{I_x}$  will become propagation constant, i.e.

$$P = \log_e \frac{I_{si}}{I_x}$$

Propagation constant in terms of primary constant  $R, L, G$  &  $C$  is given by

$$P = \sqrt{(R + j\omega L)(G + j\omega C)}$$

In lossless lines,  $P$  is purely imaginary and is directly proportional the frequency.

- b. An open wire transmission line terminated in its characteristic impedance has the following primary constant at 1 KHz.  $R = 6 \Omega / \text{km}$ ;  $L = 2 \text{ mH} / \text{km}$ ;  $G = 0.5 \mu\Omega / \text{loop km}$  and  $C = 0.005 \mu\text{F} / \text{loop km}$ . Calculate (i) characteristic impedance (ii)



phase velocity and (iii) the attenuation suffered by a signal in a length of 100 km.  
(8)

Answer:

Given that

$$R = 6 \text{ ohms/Km}$$

$$L = 2 \text{ mH/Km}$$

$$G = 0.5 \text{ } \mu\text{S/Km}$$

$$C = 0.005 \text{ } \mu\text{F/loop Km}$$

$$f = 1 \text{ KHz.}$$

Since  $f = 1 \text{ KHz}$

$$\therefore \omega = 2\pi f = 2 \times 3.14 \times 1000 = 6280 \text{ rad/sec.}$$

Series impedance,  $Z = R + j\omega L$

$$= 6 + j \times 6280 \times 2 \times 10^{-3}$$

$$= 6 + j12.56 = \sqrt{6^2 + (12.56)^2} \angle \tan^{-1} \frac{12.56}{6}$$

$$= 13.92 \angle 64.5^\circ$$

Similar Shunt Admittance

$$Y = G + j\omega C = 0.5 \times 10^{-6} + j6280 \times 0.005 \times 10^{-6}$$

$$= 10^{-6} (0.5 + j31.4) = 10^{-6} \sqrt{(0.5)^2 + (31.4)^2} \angle \tan^{-1} \frac{31.4}{0.5}$$

$$= 31.4 \times 10^{-6} \angle \tan^{-1} 62.8 = 31.4 \times 10^{-6} \angle 89.1^\circ$$

$\therefore$  Characteristic Impedance  $Z_0 = \sqrt{\frac{Z}{Y}}$

$$= \sqrt{\frac{13.92 \angle 64.5^\circ}{31.4 \times 10^{-6} \angle 89.1^\circ}}$$

$$= \sqrt{443.312 \times 10^3 \angle \frac{64.5 - 89.1}{2}}$$

$$Z_0 = 665.8 \angle -12.3^\circ$$

Propagation Constant  $P = \sqrt{Z \times Y} = \sqrt{13.92 \angle 64.5^\circ \times 31.4 \times 10^{-6} \angle 89.1^\circ}$

$$= \sqrt{13.92 \times 31.4 \times 10^{-6} \angle \frac{64.5 + 89.1}{2}}$$

$$= 20.9 \times 10^{-3} \angle 76.8^\circ = 0.0209 (\cos 76.8^\circ + j \sin 76.8^\circ)$$

$$= 0.0209 (0.2284 + j0.9736)$$

$$P = 0.004768 + j0.2034$$

real part of this will give the attenuation constant  $\alpha$ , while the imaginary part will give phase constant  $\beta$ . Therefore

$$\alpha = 0.004768 \text{ neper/Km}$$

$$\beta = 0.02034 \text{ radians/Km}$$

Phase velocity is given by

$$v_p = \frac{\omega}{\beta} = \frac{6280}{0.02034} = 308.8 \text{ Km/sec}$$

$$\text{Attenuation for 100 Km} = 0.004768 \times 100$$

$$\alpha = 0.4768 \text{ nepers}$$

Q.7 a. Define VSWR for transmission line.

(3)

Answer:

Voltage Standing wave ratio (VSWR)

This is defined as the ratio of magnitudes maximum and minimum voltage of the standing wave

$$\text{i.e. } VSWR = S = \frac{|V_{\max}|}{|V_{\min}|}$$

In terms of reflection coefficient  $K$  it is given by relation

$$S = \frac{1+|K|}{1-|K|}$$

- b. Open and short circuit of a transmission line at 1.6 kHz are  $900\angle -30^\circ$  ohms and  $400\angle -10^\circ$  ohms respectively. Calculate its characteristic impedance. (6)

Answer:

It is given that

$$Z_{oc} = 900\angle -30^\circ$$

$$Z_{sc} = 400\angle -10^\circ$$

Characteristic impedance in terms of  $Z_{oc}$  &  $Z_{sc}$  is given by

$$Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}} = \sqrt{900\angle -30^\circ \times 400\angle -10^\circ}$$

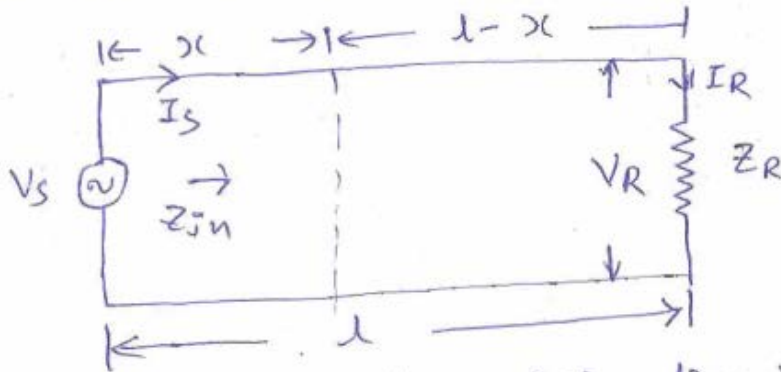
$$= \sqrt{900 \times 400} \angle \frac{-30^\circ - 10^\circ}{2} = 600\angle -20^\circ \Omega$$

$$\boxed{Z_0 = 600\angle -20^\circ \Omega}$$

- c. Derive an expression for the input impedance of a lossless transmission line when line is terminated with any impedance  $Z_R$ . (7)

Answer:

Considering a transmission line of length  $l$ , terminating in an impedance  $Z_R$ . Let  $V_R$  is the Voltage across  $Z_R$  and  $I_R$  be the current flowing through it.



Input impedance of transmission line is defined as the impedance measured across the input terminals of the transmission line.

$$\text{Thus } Z_{in} = \frac{V_s}{I_s}$$

But for general line equation

$$V_s = V_R \cosh Pl + I_R Z_0 \sinh Pl$$

where  $P = \text{Propagation Constant}$ .

$$\text{and } I_s = \frac{V_R}{Z_0} \sinh \beta l + I_R \cosh \beta l$$

$$\text{Therefore } Z_{in} = \frac{V_s}{I_s} = \frac{V_R \cosh \beta l + I_R Z_0 \sinh \beta l}{\frac{V_R}{Z_0} \sinh \beta l + I_R \cosh \beta l}$$

Multiplying numerator and denominator by  $\frac{Z_0}{I_R}$  we get.

$$Z_{in} = Z_0 \frac{\frac{V_R}{I_R} \cosh \beta l + Z_0 \sinh \beta l}{\frac{V_R}{I_R} \sinh \beta l + Z_0 \cosh \beta l}$$

$$\text{But } Z_R = \frac{V_R}{I_R} \quad Z_R \cosh \beta l + Z_0 \sinh \beta l$$

$$\therefore Z_{in} = Z_0 \frac{Z_R \cosh \beta l + Z_0 \sinh \beta l}{Z_0 \cosh \beta l + Z_R \sinh \beta l}$$

Dividing numerator and denominator by  $\cosh \beta l$  we get.

$$Z_{in} = Z_0 \frac{Z_R + Z_0 \tanh \beta l}{Z_0 + Z_R \tanh \beta l}$$

For lossless,  $\alpha = 0$ . therefore  $\beta = \alpha + j\beta = j\beta$

$$\text{hence } Z_{in} = Z_0 \frac{Z_R + Z_0 \tanh j\beta l}{Z_0 + Z_R \tanh j\beta l}$$

$$\text{But } \tanh j\beta l = j \tan \beta l$$

$$\therefore Z_{in} = Z_0 \frac{Z_R + j Z_0 \tan \beta l}{Z_0 + j Z_R \tan \beta l}$$

$$\text{Since } \beta = \frac{2\pi}{\lambda}$$

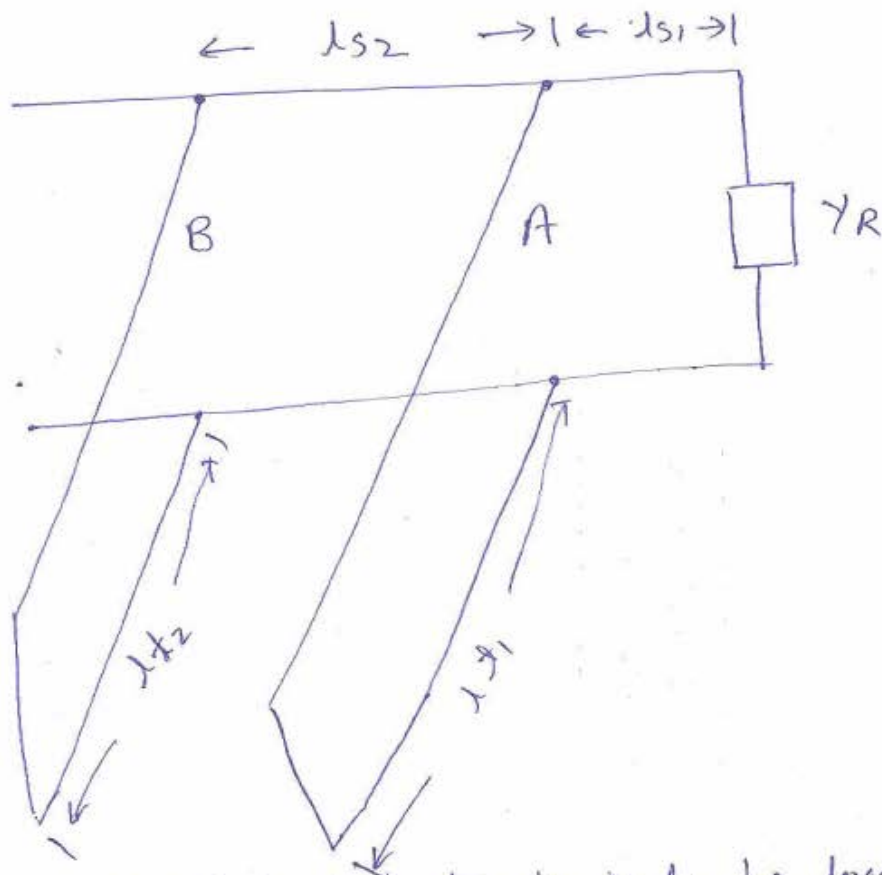
$$\therefore Z_{in} = Z_0 \frac{Z_R + j Z_0 \tan \frac{2\pi l}{\lambda}}{Z_0 + j Z_R \tan \frac{2\pi l}{\lambda}} \quad \underline{\underline{\text{Ans}}}$$

- Q.8 a. Describe double stub matching of a transmission line. What are the advantages of this method over single stub matching? (8)

Answer:

Double stub matching :-

It is matching with two short circuited stubs whose lengths are adjustable independently but whose position are fixed, may be use as shown in figure below



Let the first stub whose length is  $l_{s1}$  be located at a point A and a distance  $l_{s1}$  from the load end. The normalised input admittance at that point will be

$$Y_A = \frac{Y_A}{Y_0} = \frac{Y_R + j \tan \beta l_{s1}}{1 + j Y_R \tan \beta l_{s1}} \quad \text{--- (1)}$$

$Y_R$  can be assumed to be pure conductance. Rationalising equation (1) we get.

$$\begin{aligned}
 Y_A &= \frac{Y_r + j \tan \beta l_{s1} (1 - j Y_r \tan \beta l_{s1})}{1 + j Y_r \tan \beta l_{s1} (1 - j Y_r \tan \beta l_{s1})} \\
 &= \frac{Y_r (1 + \tan^2 \beta l_{s1}) + j (1 - Y_r^2) \tan \beta l_{s2}}{1 + Y_r^2 \tan^2 \beta l_{s1}} \\
 &= \frac{Y_r \sec^2 \beta l_{s1} + j (1 - Y_r^2) \tan \beta l_{s2}}{1 + Y_r^2 \tan^2 \beta l_{s1}} \\
 &= g_A + j b_A
 \end{aligned}$$

where  $g_A = \frac{Y_r \sec^2 \beta l_{s1}}{1 + Y_r^2 \tan^2 \beta l_{s1}}$

&  $b_A = \frac{(1 - Y_r^2) \tan \beta l_{s1}}{1 + Y_r^2 \tan^2 \beta l_{s1}}$

When a stub having a susceptance  $b_1$  is added at this point, the new admittance value will be

$$Y_A' = g_A + j b_A'$$

Since only the susceptance value is altered by the addition of the stub, the conductance part remains unchanged.  $Y_A'$  should be such value that the admittance  $Y_B$  equals  $1 + j b_2$ . The stub length at B is adjusted such that new value of  $Y_A'$  is 1 to effect matching.

There are some restrictions which are often encountered in practice. The distance  $l_{s1}$  can never be more than or equal to  $\lambda/2$ . Advantage of double stub is that due to change in frequency lengths are adjustable independently.

- b. A low loss transmission line has characteristic impedance of  $70 \Omega$  and is terminated by another impedance of  $(115 - j80) \Omega$ . Find (i) reflection co-efficient and (ii) standing wave ratio. (8)

Answer:

Given that

$$Z_0 = 70 \Omega$$

$$Z_R = (115 - j80) = \sqrt{115^2 + 80^2} \angle \tan^{-1} \frac{-80}{115}$$

$$= 140 \angle -34.8^\circ$$

Reflection coefficient is given by

$$K = \frac{Z_0 - Z_R}{Z_0 + Z_R} = \frac{70 - (115 - j80)}{70 + (115 - j80)}$$

$$K = \frac{-45 + j80}{185 - j80} = \frac{91.79 \angle -60.6^\circ}{201.5 \angle -23.38^\circ}$$

$$= 0.46 \angle -37.22^\circ$$

$$\therefore |K| = 0.46$$

$$\text{Therefore } S = \frac{1+K}{1-K} = \frac{1.46}{0.54} = 2.7$$

$\therefore$  reflection co-efficient  $K = 0.46 \angle -37.22^\circ$

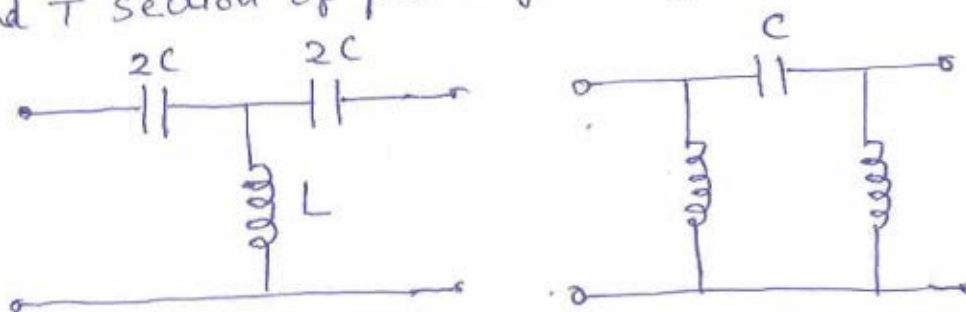
& standing wave ratio =  $S = 2.7$

- Q.9 a. Draw T and  $\pi$  sections of a constant - K high pass filter. Derive an expression for cut-off frequency. (4+4)

Answer:



High pass constant K filter,  
 $\pi$  and T Section of prototype High pass filter is



Now

$$Z_1 = -\frac{j}{\omega C}$$

$$Z_2 = j\omega L$$

$$\text{Hence } Z_1 Z_2 = -\frac{j}{\omega C} \times j\omega L = \frac{L}{C}$$

Since the product is  $Z_1 \times Z_2$  is independent of frequency, the filter is called constant K filter.

$$Z_1 Z_2 = R_K^2 = \frac{L}{C}$$

$$\text{or } R_K = \sqrt{\frac{L}{C}}$$

and characteristic impedance of T section is given by

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_2 Z_1}$$

$$Z_{OT} = \sqrt{\frac{-1}{4\omega^2 C^2} + \frac{L}{C}} = \sqrt{\frac{L}{C}} \sqrt{1 - \frac{1}{4\omega^2 LC}}$$

i.e.  $Z_{OT}$  is real when  $\frac{1}{4\omega^2 LC} \Rightarrow$  is greater than 1 and the filter works in the pass band. However if  $4\omega^2 LC < 1$ ,  $Z_{OT}$  is imaginary, and the filter lies in attenuation band. So, the cut off frequency is given by

$$4\omega_c^2 LC = 1$$

$$\text{or } \omega_c = \frac{1}{2\sqrt{LC}}$$

$$\text{or } \boxed{f_c = \frac{1}{4\pi\sqrt{LC}}} \text{ Ans.}$$

- b. Design a symmetrical Bridged-T network with an attenuation of 40 dB and an impedance of 600 ohms. (8)

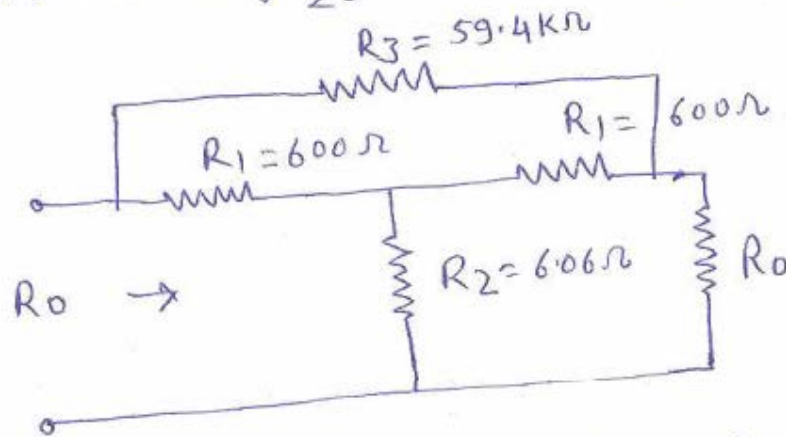
Answer:

It is given that

$$D = 40 \text{ dB}$$

$$R_0 = 600 \Omega$$

$$\therefore N = \text{Antilog} \frac{D}{20} = \text{Antilog} \frac{40}{20} = 100$$



$$R_3 = R_0 (N-1) = 600 (100-1) \\ = 600 \times 99$$

or  $R_3 = 59.4 \text{ k}\Omega$

$$R_2 = \frac{R_0}{N-1} = \frac{600}{100-1} = \frac{600}{99} = 6.06 \Omega$$

$$R_1 = R_0 = 600 \Omega$$

### TEXT BOOK

I. Transmission Lines and Networks; Umesh Sinha, 8<sup>th</sup> Edition; Reprint 2004, Satya Prakashan, Incorporating Tech India Publications, New Delhi