Q.1 a. Show that the truth values of the following compound proposition is independent of the truth values of their components
$\{p \wedge (p \rightarrow q)\} \rightarrow q$

Answer:

P	Q	p→q	r ≡{p Λ (p → q)}	r→q
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

b. If A,B,C are finite sets, prove the extended addition principle
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

Answer:

$|A \cup B \cup C| = |A \cup D|$ where $D = |B \cup C|$

$= |A| + |D| - |A \cap D|$

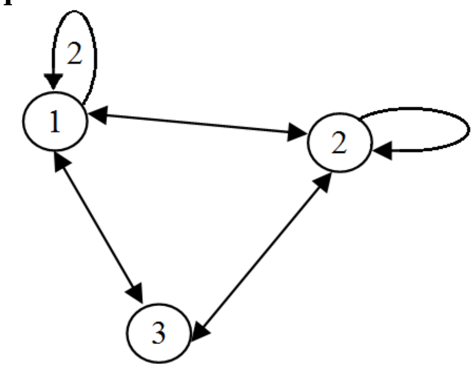
$= |A| + |B \cup C| - |A \cap (B \cup C)|$

$= |A| + |B| + |C| - |B \cap C| - |(A \cap B \cup (A \cap C)|$

$= |A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap C| + |(A \cap B) \cap (A \cap C)$

$= |A| + |B| + |C| - |B \cap C| - |A \cap C| - |A \cap B| + |A \cap B \cap C|$

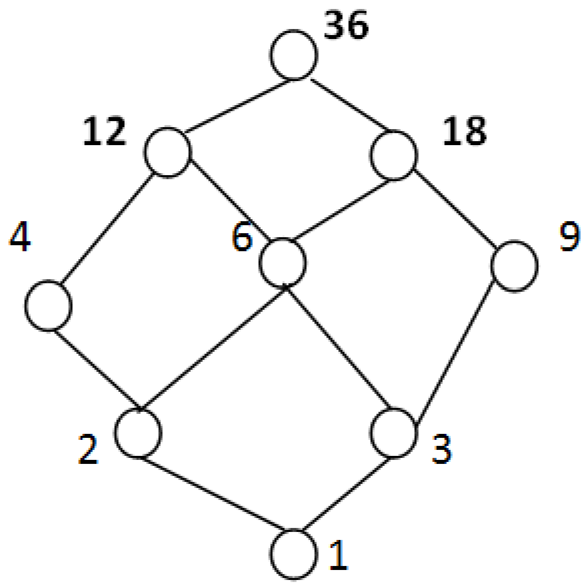
c The digraph of a relation R on the set {1,2,3} is as given below. Determine whether R is an equivalence relation or not?



Answer: (3,3) is not in R

d. Draw the Hasse diagram representing the positive divisors of 36.

Answer:

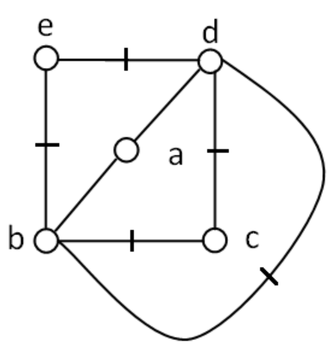


e. State and prove Demorgan's law in Boolean algebra B.

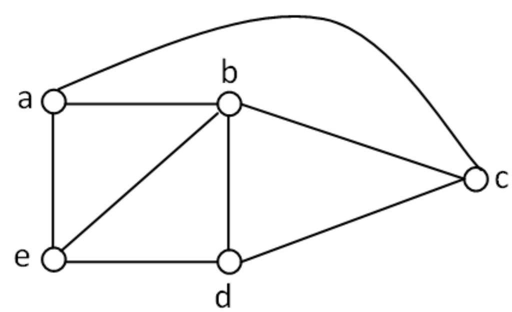
Answer: Let a be any element of a Boolean algebra B then $a + x = 1$ and $a.\ x = 0$ then $x = a$

f. Give an example of a graph that has
(i) An Euler Circuit but no Hamiltonian cycle
(ii)An Hamiltonian cycle but no Euler circuit

Answer:



(i) e d a b c d b e



(ii) a b c d e a

g. Give a regular expression for the language of all strings in $\{0,1,2\}^*$ containing exactly two 2's. **(7×4)**

Answer:

$(0+1)^*2\ (0+1)^*2\ (0+1)^*$

Q.2 a. Obtain PDNF of following: **(5)**
$(P \vee (R \rightarrow \sim (Q \vee P))) \wedge R$ without using truth table.

Answer: $(P \wedge R \wedge Q) \vee (P \wedge R \wedge \sim Q) \vee (\sim Q \wedge \sim P \wedge R)$

b. Show that Q is a valid conclusion for the premises: **(5)**
$\sim S$, $P \vee (Q \wedge R)$, $P \leftrightarrow S$

Answer:

pf ~P
P↔S
P → S ∧S → P
P → S
~S →~ P
~ P
PV (Q∧R)
(Q∧R)
Q

c. Let A be any set and P(A) be the power set of A. Show that it is a lattice under the partial order defined as $X \leq Y \leftrightarrow X \subseteq Y$. **(8)**

Answer:

$\subseteq$ is known to be partial order set on P(A).Consider X,Y $\in$ P(A).now we need to show that sup{X,Y} = X V Y exists.
By def, X VY must contain X and Y. Further if any other set contains X and Y, it must contain X V Y .Thus it is clear that X V Y has to be $X \cup Y$. Similarly $X \wedge Y$ = inf {X,Y} = $X \cap Y$. Thus P(A) is a lattice.

Q.3 a. Show that if any 5 numbers are chosen from {1 to 8}, then two of them will add upto 9. **(6)**

Answer:

Consider the pairs (1,8),(2,7),?(3,6),(4,5) which are four in number.if we chose five numbers from 1 to eight by pigeon hole principle two of them must belong to one of the above pairs.Hence their sum must be nine.

b. Show that: **(6)**
$\forall P(x) \wedge \exists Q(x) \Rightarrow \exists(P(X) \wedge Q(X))$

Answer:

$\forall P(x)$ simplification
P(a) US
$\exists Q(x)$ simplification
Q(a) ES
$P(a) \wedge Q(a)$
$\exists(P(X) \wedge Q(X))$ EG

c. Let f : R → R be defined as **(6)**
$f(x) = 2x - 3$ if $x \leq 9$
$= x^2 - 4x + 7$, if $9 < x < 100$
$= \cos \Pi x$ if $100 \leq x$
Find f(7), f(8), f(9), f(10) and f (100)

Answer:

f(7) = 11,f(8) = 13
f(9) = 15, f(10)= 67 f(100) = cos 100Π = $(-1)^{100}$ =1

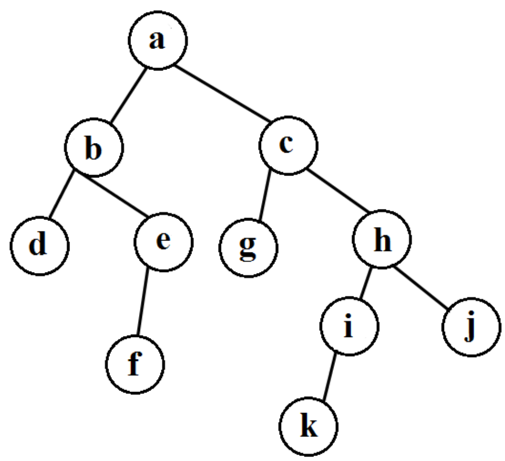
Q.4 a. If L1 and L2 are regular languages ever Σ show that L1∪L2 is regular. (6)

Answer:
Construct new automata with Q = Q1 X Q2 q0 = (q1,q2) and F = F1 X F2
Transition function is defined as
$\delta((u,v),x) = (\delta 1(u,x),\delta 2(v,x))$
Where Q1,Q2 are starting states of first and second automata,q1 and q2 are respective starting states,F1 and F2 are final states of two automata.(δ1,δ2) are respective transition functions.

b. Construct a finite automata to accept all the strings of odd lengths on the alphabet {a,b,c}. (6)

Answer: Construct a finite automata to accept all the strings of odd lengths on the alphabet {a,b,c}.

c. Traverse the tree using
(i) Preorder traversal
(ii) Inorder traversal
(iii) Post order traversal (6)

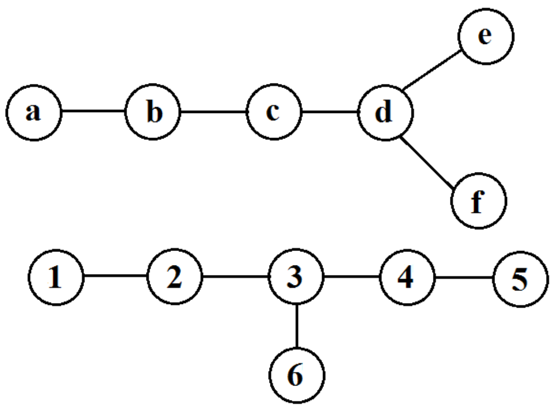


Answer:
(i) Preorder traversal a b d e f c g h i k j
(ii) Inorder traversal d b f e a g c k i g h
(iii) Preorder traversal d f e b g k i j h c a

Q.5 a. In a tree (with two or more vertices), prove that there are at least two pendant vertices. (5)

Answer:
Sum of the degrees of n vertices of the tree of size n,is 2(n-1).Now 2(n-1) degrees are to be divided amongst n vertices so that no vertex is of degrezero.If all the vertices are of degree 2 then sum of degrees of all vertices will be 2n,which is not true..If there is only one vertex of degree one, then the sum of degrees of all vertices will be 2n-1 which is also not true. If there are two vertices of degree 1 then this sum will be 2n-2.
Hence at least two nodes of degree one is possible.

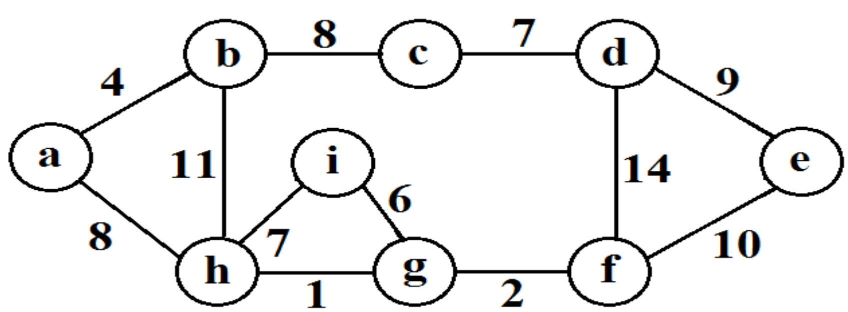
b. Define isomorphism of graphs. Verify following graphs for isomorphism. (5)



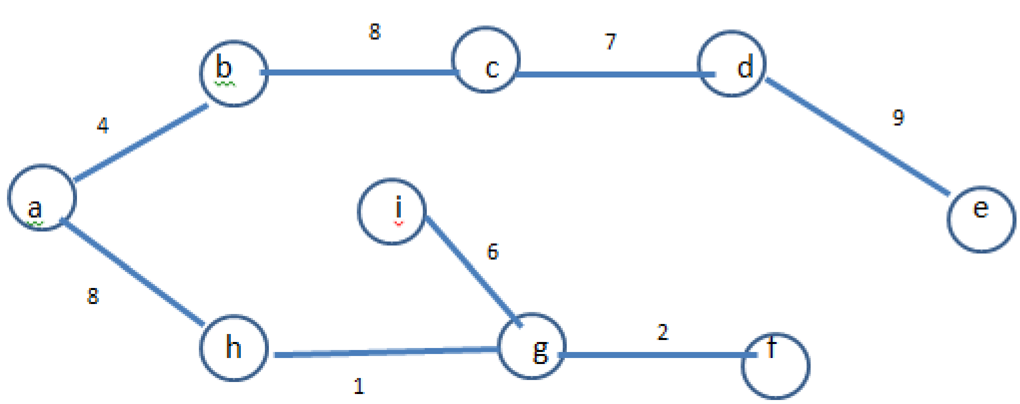
Answer:

The above two graphs are not isomorphic :only node of degree 3 in first graph has one pendant vertex as adjacent, but in the other graph vertex of degree three node 3 has only one adjacent pendant vertex.

c. Define a spanning tree of a graph. Does every graph have a spanning tree? Find the minimum spanning tree of the following graph using Kruskal's algorithm. (8)



Answer:



Q.6 a. Explain how Binary Search method fails to find 43 in the following given sorted array: (6)
8, 12, 25, 26, 35, 48, 57, 78, 86, 93, 97, 108, 135, 168, 201

Answer:

The value to be searched is 43. As explained earlier, in the first iteration
low = 1, high = 15 and mid = 8
As 43 <A[8] = 78, therefore, as in part (i)
low = 1, high = 8 − 1 = 7 and mid = 4
As 43 >A[4] = 26, the algorithm makes another iteration in which
low = mid + 1 = 5 high = 7 and (new) mid = (5 + 7)/2 = 6
Next, as 43 <A[6] = 48, the algorithm makes another iteration, in which
low = 5 high = 6 − 1 = 5 hence mid = 5, and A[5] = 35 As 43 > A[5], hence value ≠ A[5].
But, at this stage, low is not less than high and hence the algorithm returns − 1, indicating failure to find the given value in the array.

b. How can the output of the Floyd-Warshall algorithm be used to detect the presence of a negative-weight cycle? (6)

Answer:

One could just run the normal FLOYD-WARSHALL algorithm one extra iteration to see if any of the d values change. If there are negative cycles, then some shortest-path cost will be cheaper. If there are no such cycles, then no d values will change because the algorithm gives the correct shortest paths.

c. Define Cartesian product on sets. For the given sets X = {1, 2}, Y = {a, b, c} and Z = {c, d}, find (X×Y)∩(X×Z). (6)

Answer:

Given X = {1, 2}, Y = {a, b, c} and Z = {c, d}, then

$$X\times Y = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

$$X\times Z = \{(1, c), (1, d), (2, c), (2, d)\}$$

Q.7 a. Draw the ordered rooted tree that represent the expression
$((x + y)\uparrow 2) + ((x - 4)/3)$.
How do you find the equivalent prefix and postfix expressions from the tree?(6)

Answer:

Given f : R → R as f(x) = |x|.

f is not one-one: clearly f is not a one-one function as f(x) = |x| = x =|-x| = f(-x) but x ≠ -x.

For eg. |2| = 2 and |-2| = 2 but 2 ≠ -2

4

f is not onto: A function is said to be onto if Range(f) = co domain of (f). Here the co-domain is R (real line) and range of function f(x) = |x| for all x in R.

Range(f) = {|x|; for all x in R} = {+ve x; for all x in R as |x| gives +ve values} = R^+ 4

Clearly Range(f) = R^+ ≠ co-domain(f) = R.

Hence f is neither one-one nor onto function.

b. Prove that deterministic and nondeterministic finite automata are equivalent.(6)

Answer:

c. Define regular expression. (i) Let *L* = {*w*∈ {a, b}* : |*w*| $\equiv_3$ 0}. List the first six elements in a lexicographic enumeration of *L*. (ii) *L* = {*w*∈ {a, b}* : all prefixes of *w* end in a}. List the elements of *L*. (6)

Answer:

ε, aaa, aab, aba, abb, baa

(ii) L = {*w*∈ {a, b}* : all prefixes of *w* end in a}.List the elements of L.

L= ∅, since ε is a prefix of every string and it doesn't end in a. So all strings are not in *L*, including a and aa.