Q.2 a. Determine the values of power and energy of the following signals. Find whether the signals are power or energy signal.

$$x(n) = \left(\frac{1}{3}\right)^n u(n)$$ **(8)**

Answer:

Energy of the signal $E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} \left|\left(\frac{1}{3}\right)^n\right|^2 = \frac{9}{8}$

Power of the signal $P = \underset{N \to \infty}{lt} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^n = 0$

So, energy of the signal is finite and power is zero. Hence, the signal is an energy signal.

b. Determine if the system described by the following equation is **(8)**
(i) causal or non-causal
(ii) linear or nonlinear.

$$y(n) = x(n) + \frac{1}{x(n-1)}$$

Answer:

(i) $y(n) = x(n) + \frac{1}{x(n-1)}$

For n =-1; $y(-1) = x(-1) + \frac{1}{x(-2)}$

n =0; $y(0) = x(0) + \frac{1}{x(-1)}$

n =1; $y(1) = x(1) + \frac{1}{x(0)}$

For all values of n, the output depends on present and past inputs. Therefore, The **system is causal**

(ii) $y(n) = x(n) + \frac{1}{x(n-1)}$

$y_1(n) = x_1(n) + \frac{1}{x_1(n-1)}$; $y_2(n) = x_2(n) + \frac{1}{x_2(n-1)}$

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)] = a_1x_1(n) + a_2x_2(n) + \frac{1}{a_1x_1(n-1) + a_2x_2(n-1)} \quad (1)$$

On the other hand,

$$a_1y_1(n) + a_2y_2(n) = a_1x_1(n) + \frac{a_1}{x_1(n-1)} + a_2x_2(n) + \frac{a_2}{x_2(n-1)} \quad (2)$$

As (1) $\neq$ (2); So, the system is **non-linear.**

Q.3 a. Find y(n) if $x(n) = n + 2$ for $0 \le n \le 3$ and h(n) =a^nu(n) for all *n*. **(8)**

Answer:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-1)$$

Given $x(n) = n + 2$ for $0 \le n \le 3$

$$y(n) = \sum_{k=0}^{3} x(k)h(n-1)$$

$$= \sum_{k=0}^{3} (k+2)a^{n-k}u(n-1)$$

$$= 2a^n u(n) + 3a^{n-1}u(n-1) + 4a^{n-2}u(n-2) + 5a^{n-3}u(n-3)$$

b. Write the advantages, disadvantages and application of Digital Signal Processing. **(8)**

Answer:

<u>Advantage:</u>

Greater accuracy
Cheaper
Ease of data storage
Flexibility in configuration
Applicability of very low frequency signal
Time sharing

<u>Disadvantage:</u>

System complexity
Bandwidth limited by sampling rate
Power consumption issues

<u>Applications:</u>

Telecommunication
Consumer electronics
Instrumentation and control
Image processing
Medicine
Speech processing
Seismology
Military

Q.4 a. **Determine the solution of the differential solution** $y(n)=\frac{5}{6}y(n-1)-\frac{1}{6}y(n-2)+x(n)$ for $x(n)=2^n u(n)$. **Assume the system is initially relaxed.** **(8)**

Ans:-

4(a)

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n) \quad \text{——} \; (1)$$

$$y(n) = y_h(n) + y_p(n)$$

$$y_p(n) = K\,2^n u(n)$$

Substituting in (1)

$$K\,2^n u(n) = \frac{5}{6}K\,2^{n-1}u(n-1) - \frac{1}{6}K\,2^{n-2}u(n-2) + 2^n u(n)$$

$$n = 2 \;; \quad K.\,2^2 = \frac{5}{6}.\,2K - \frac{K}{6} + 4$$

So, $K = \frac{8}{5}$

So, $$y_p(n) = \frac{8}{5}.2^n u(n) \quad \text{——} \; (2)$$

let $y_h(n) = \lambda^n$

So, $$\lambda^n - \frac{5}{6}\lambda^{n-1} + \frac{1}{6}\lambda^{n-2} = 0$$

or $\lambda_1 = \frac{1}{2}$; $\lambda_2 = \frac{1}{3}$

So, $$y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n$$

So, $$y(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n + \frac{8}{5}2^n u(n)$$

$$y(0) = c_1 + c_2 + \frac{8}{5}$$

$$y(1) = \frac{1}{2}c_1 + \frac{1}{3}c_2 + \frac{16}{5}$$

So, $c_1 = -1$; $c_2 = \frac{2}{5}$

So, $$y(n) = -\left(\frac{1}{2}\right)^n u(n) + \frac{2}{5}\left(\frac{1}{3}\right)^n u(n) + \frac{8}{5}2^n u(n)$$

Ans:-

b. State and Prove the Parseval's Theorem. (8)

Ans:-
4.(b)

If $F[x(n)] = X(e^{jw})$

Then $$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw$$

Proof

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} x(n)\, x^*(n)$$

$$= \sum_{n=-\infty}^{\infty} x(n) \left[\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})\, e^{jwn}\right]^* dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{jw}) \left[\sum_{n=-\infty}^{\infty} x(n)\, e^{-jwn}\right] dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{jw})\, X(e^{jw})\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw$$ (Proved)

Q.5 a. Determine the magnitude response of $y(n) = \frac{1}{2}[x(n) + x(n-2)]$**.** **(8)**

Ans:- 5. (a).

$$y(n) = \frac{1}{2}\left[x(n) + x(n-2)\right]$$

$$Y(e^{jw}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-jwn}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty}\left[x(n) + x(n-2)\right] e^{-jwn}$$

$$= \frac{1}{2}\left[\sum_{n=-\infty}^{\infty} x(n)\, e^{-jwn} + \sum_{n=-\infty}^{\infty} x(n-2)\, e^{-jwn}\right]$$

$$= \frac{1}{2}\left[X(e^{jw}) + e^{-2jw} X(e^{jw})\right]$$

$$= \frac{X(e^{jw})}{2}\left[1 + e^{-2jw}\right]$$

So,
$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{1 + e^{-2jw}}{2}$$

$$= \frac{1 + \cos 2w - j\sin 2w}{2}$$

$$\left|H(e^{jw})\right| = \cos w$$ Ans:-

b. For the following system, determine whether or not the system is time invariant. $y(n)=\sum_{k=0}^{m}a(k)x(n-k)-\sum_{k=1}^{n}b(k)y(n-k)$. **(8)**

Ans:- 5. (b)

$$y(n) = \sum_{k=0}^{m} a_k x(n-k) - \sum_{k=1}^{n} b(k)\, y(n-k)$$

If we delay the input by m units in time

$$y(n,m) = T\left[x(n-m)\right]$$

$$= \sum_{k=0}^{m} a(k)\, x(n-k-m) - \sum_{k=1}^{n} y(n-k-m)$$

If we delay the output by m units

$$y(n-m) = \sum_{k=0}^{m} a(k)\, x(n-k-m) - \sum_{k=1}^{n} y(n-k-m)$$

as $y(n,m) = y(n-m)$, the system is time invariant.

Q.6 a. Find the z- transform and the ROC of the signal $x(n) = -b^n u(-n-1)$. **(8)**

Ans:- 6. (a)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} b^n z^{-n} = -\sum_{n=1}^{\infty} (b^{-1} z)^{n}$$

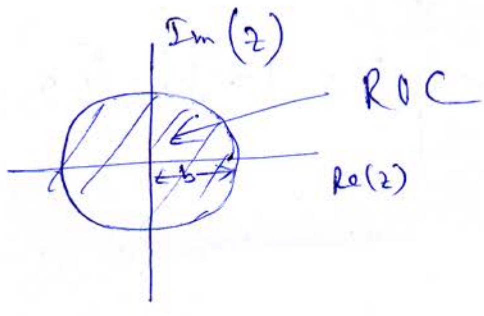
$$= -\left[\sum_{n=0}^{\infty} (b^{-1} z)^{n} - 1\right]$$

The above series converges for $|b^{-1} z| < 1$,

i.e., $|z| < b$

$$X(z) = -\left[\frac{1}{1 - b^{-1} z} - 1\right]$$

$$= \frac{z}{z-b} \qquad \text{ROC: } |z| < b$$



b. Find the DFT of a sequence $x(n)=\{1,\ 1,\ 0,\ 0\ \}$**.** **(8)**

Ans:- 6.(b)

let us assume $N = L = 4$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}, \quad k=0,1 \cdots N-1$$

$$X(0) = \sum_{n=0}^{3} x(n) = x(0)+x(1)+x(2)+x(3) = 1+1+0+0 = 2$$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n/2} = x(0)+x(1)\,e^{-j\pi/2}+x(2)\,e^{-j\pi}+x(3)\,e^{-j\frac{3\pi}{2}}$$

$$= 1+\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$$

$$= 1-j$$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n} = 1+\cos\pi - j\sin\pi = 1-1 = 0$$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{3n\pi}{2}} = 1+\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}$$

$$= 1+j$$

So, $X(k) = \{2,\ 1-j,\ 0,\ 1+j\}$ Ans:-

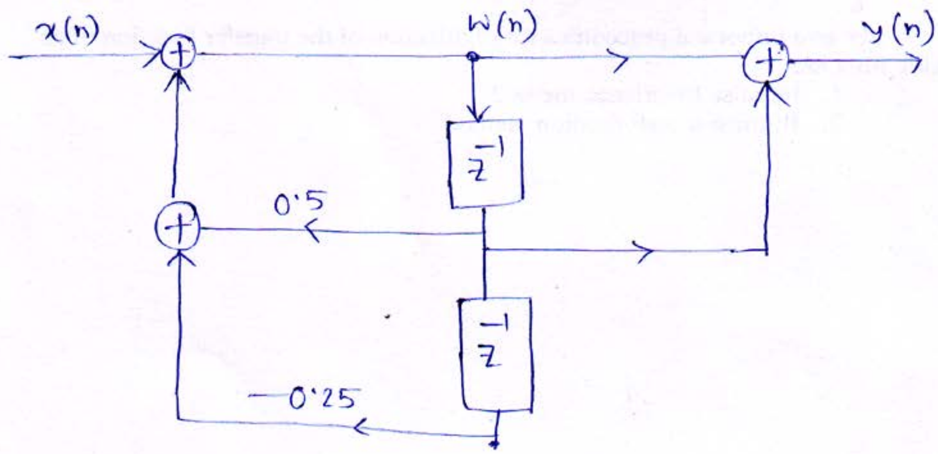
Q.7 a. Determine the direct form II and Transposed direct form II for the given system

$$y(n)=\frac{1}{2}y(n-1)-\frac{1}{4}y(n-2)+x(n)+x(n-1).$$ **(8)**

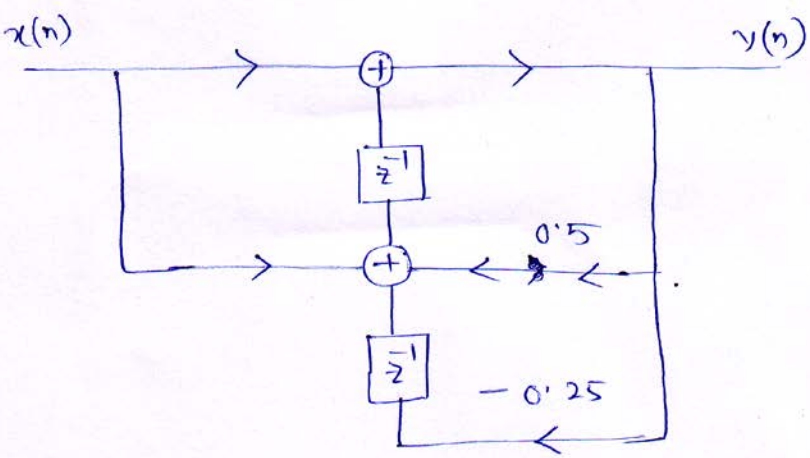
Ans:- 7.(a)

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+z^{-1}}{1-0.5z^{-1}+0.25z^{-2}}$$

Direct form II



Transposed Structure



b. (i) Explain the procedure of designing digital filters from analog filters. (4)
(ii) Mention any two procedures for digitizing the transfer function of an analog filter. (4)

Answer:

(i)

1. Map the desired digital filter specifications into those for the equivalent analog filter.
2. Derive the analog transfer function for the analog prototype.
3. Transform the transfer function of the analog prototype into an equivalent digital filter transfer function.

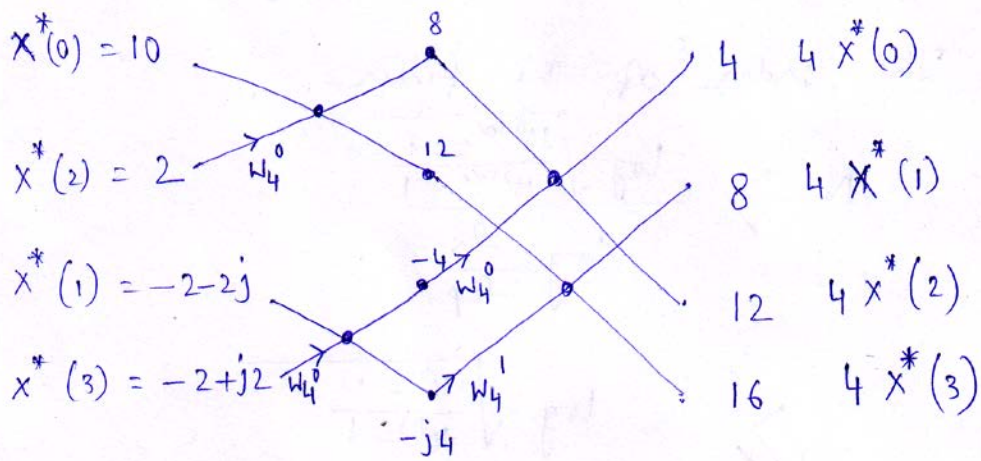
(ii) The two important procedures for digitization of the transfer function of an analog filter are:

1. Impulse Invariance method
2. Bilinear transformation method

Q.8 a. Find the IDFT of the sequence $X(k) = \{10, -2+j2, -2, -2-j2\}$. (8)

Ans:- 8.(a).

The twiddle factors are $W_4^0 = 1$; $W_4^1 = -j$



The output $N x^*(n)$ is normal order

Therefore, $x(n) = \{1, 2, 3, 4\}$

b. Determine the order of lowpass. Butterworth filter that has a 3dB attenuation at 500Hz and an attenuation of 40dB at 1000Hz. (8)

Ans:- 8.(b).

Given data $\alpha_p = 3\,dB$; $\alpha_s = 40\,dB$; $\Omega_p = 2\times\pi\times 500 = 1000\pi\ rad/s$

$\Omega_s = 2\times\pi\times 1000 = 2000\pi\ rad/sec.$

The order of the filter

$$N \geq \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\frac{\Omega_s}{\Omega_p}}$$

$$\geq \frac{\log.\sqrt{\frac{10^{4}-1}{10^{0.3}-1}}}{\log\frac{2000\pi}{1000\pi}} \simeq 6.6$$

Rounding 'N' to nearest higher value we get $\underline{N = 7}$. (Ans:-)

Q.9 a. Explain Hibbert transform relations for complex sequences with suitable illustrations. (8)

Answer:

Thus far, we have considered Hilbert transform relations for the Fourier transform of causal sequences and the discrete Fourier transform of periodic sequences that are "periodically causal" in the sense that they are zero in the second half of each period. In this section, we consider *complex sequences* for which the real and imaginary components can be related through a discrete convolution similar to the Hilbert transform relations derived in the previous sections. These relations are particularly useful in representing bandpass signals as complex signals in a manner completely analogous to the analytic signals of continuous-time signal theory.

As mentioned previously, it is possible to base the derivation of the Hilbert transform relations on a notion of causality or one-sidedness. Since we are interested in relating the real and imaginary parts of a complex sequence, one-sidedness will be applied to the Fourier transform of the sequence. We cannot, of course, require that the Fourier transform be zero for $\omega < 0$, since it must be periodic. Instead, we consider sequences for which the Fourier transform is zero in the second half of each period; i.e., the z-transform is zero on the bottom half $(-\pi \le \omega < 0)$ of the unit circle. Thus, with $x[n]$ denoting the sequence and $X(e^{j\omega})$ its Fourier transform, we require that

$$X(e^{j\omega}) = 0, \qquad -\pi \le \omega < 0. \tag{11.57}$$

(We could just as well assume that $X(e^{j\omega})$ is zero for $0 < \omega \le \pi$.) The sequence $x[n]$ corresponding to $X(e^{j\omega})$ must be complex, since, if $x[n]$ were real, $X(e^{j\omega})$ would be conjugate symmetric, i.e., $X(e^{j\omega}) = X^*(e^{-j\omega})$. Therefore, we express $x[n]$ as

$$x[n] = x_r[n] + jx_i[n], \tag{11.58}$$

where $x_r[n]$ and $x_i[n]$ are real sequences. In continuous-time signal theory, the comparable signal is an analytic function and thus is called an *analytic signal*. Although analyticity has no formal meaning for sequences, we will nevertheless apply the same terminology to complex sequences whose Fourier transforms are one sided.

If $X_r(e^{j\omega})$ and $X_i(e^{j\omega})$ denote the Fourier transforms of the real sequences $x_r[n]$ and $x_i[n]$, respectively, then

$$X(e^{j\omega}) = X_r(e^{j\omega}) + jX_i(e^{j\omega}), \tag{11.59a}$$

and it follows that

$$X_r(e^{j\omega}) = \frac{1}{2}[X(e^{j\omega}) + X^*(e^{-j\omega})] \tag{11.59b}$$

and

$$jX_i(e^{j\omega}) = \frac{1}{2}[X(e^{j\omega}) - X^*(e^{-j\omega})]. \tag{11.59c}$$

Note that Eq. (11.59c) gives an expression for $jX_i(e^{j\omega})$, which is the Fourier transform of the imaginary signal $jx_i[n]$. Note also that $X_r(e^{j\omega})$ and $X_i(e^{j\omega})$ are both complex-valued functions in general, and the complex transforms $X_r(e^{j\omega})$ and $jX_i(e^{j\omega})$ play a role similar to that played in the previous sections by the even and odd parts, respectively, of causal sequences. However, $X_r(e^{j\omega})$ is conjugate symmetric, i.e., $X_r(e^{j\omega}) = X_r^*(e^{-j\omega})$. Similarly, $jX_i(e^{j\omega})$ is conjugate antisymmetric, i.e., $jX_i(e^{j\omega}) = -jX_i^*(e^{-j\omega})$.

Figure 11.4 depicts an example of a complex one-sided Fourier transform of a complex sequence $x[n] = x_r[n] + jx_i[n]$ and the corresponding two-sided transforms of the real sequences $x_r[n]$ and $x_i[n]$. This figure shows pictorially the cancellation implied by Eqs. (11.59).

If $X(e^{j\omega})$ is zero for $-\pi \le \omega < 0$, then there is no overlap between the nonzero portions of $X(e^{j\omega})$ and $X^*(e^{-j\omega})$. Thus, $X(e^{j\omega})$ can be recovered from either $X_r(e^{j\omega})$ or $X_i(e^{j\omega})$. Since $X(e^{j\omega})$ is assumed to be zero at $\omega = \pm\pi$, $X(e^{j\omega})$ is totally recoverable from $jX_i(e^{j\omega})$. This is in contrast to the situation in Section 11.2, in which the causal sequence could be recovered from its odd part except at the endpoints.

In particular,

$$X(e^{j\omega}) = \begin{cases} 2X_r(e^{j\omega}), & 0 \le \omega < \pi, \\ 0, & -\pi \le \omega < 0, \end{cases} \tag{11.60}$$

and

$$X(e^{j\omega}) = \begin{cases} 2jX_i(e^{j\omega}), & 0 \le \omega < \pi, \\ 0, & -\pi \le \omega < 0. \end{cases} \tag{11.61}$$

Alternatively, we can relate $X_r(e^{j\omega})$ and $X_i(e^{j\omega})$ directly by

$$X_i(e^{j\omega}) = \begin{cases} -jX_r(e^{j\omega}), & 0 < \omega < \pi, \\ jX_r(e^{j\omega}), & -\pi \le \omega < 0, \end{cases} \tag{11.62}$$

or

$$X_i(e^{j\omega}) = H(e^{j\omega})X_r(e^{j\omega}), \tag{11.63a}$$

where

$$H(e^{j\omega}) = \begin{cases} -j, & 0 < \omega < \pi, \\ j, & -\pi < \omega < 0. \end{cases} \tag{11.63b}$$

Equations (11.63) are illustrated by comparing Figures 11.4(c) and 11.4(d). Now $X_i(e^{j\omega})$ is the Fourier transform of $x_i[n]$, the imaginary part of $x[n]$, and $X_r(e^{j\omega})$ is the Fourier transform of $x_r[n]$, the real part of $x[n]$. Thus, according to Eqs. (11.63), $x_i[n]$ can be obtained by processing $x_r[n]$ with a linear time-invariant discrete-time system with frequency response $H(e^{j\omega})$, as given by Eq. (11.63b). This frequency response has unity magnitude, a phase angle of $-\pi/2$ for $0 < \omega < \pi$, and a phase angle of $+\pi/2$ for $-\pi < \omega < 0$. Such a system is called an ideal 90-degree phase shifter. Alternatively,

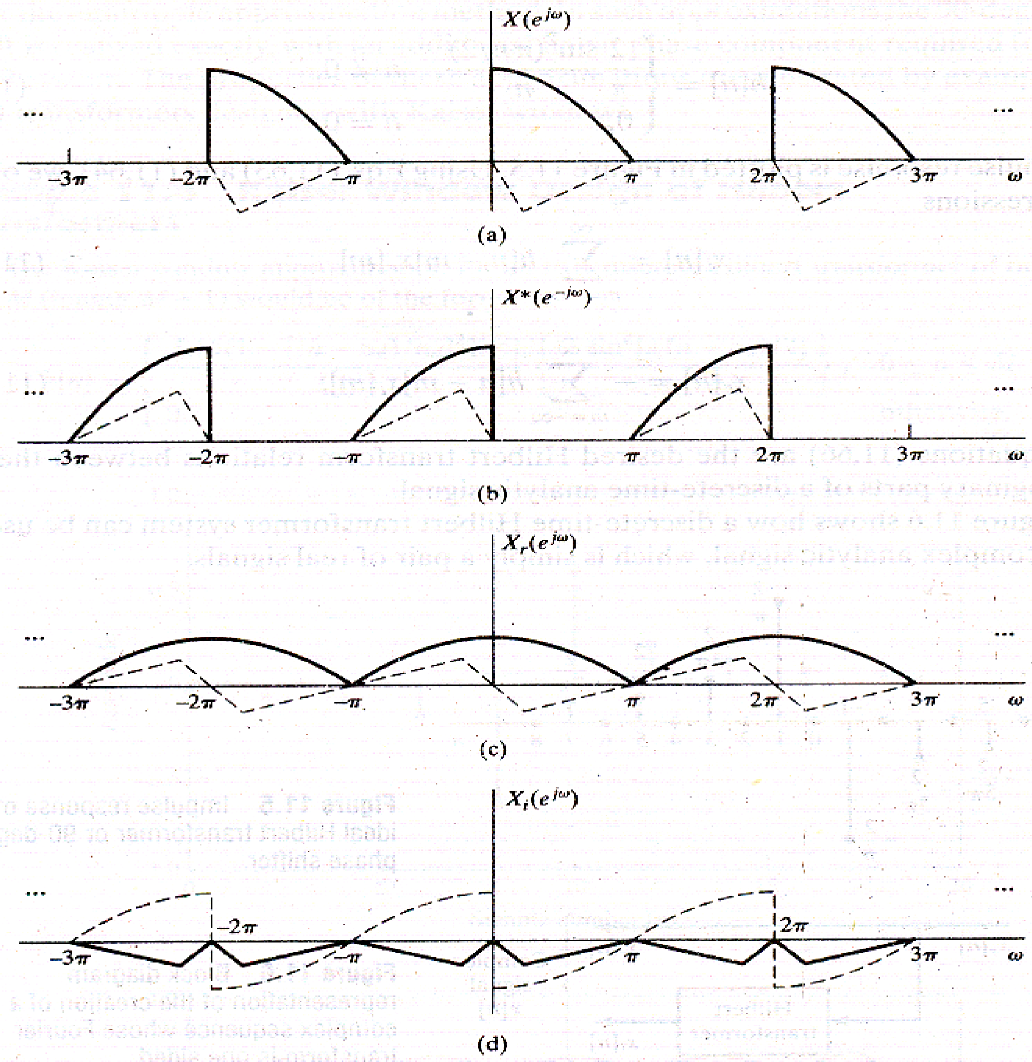


Figure 11.4 Illustration of decomposition of a one-sided Fourier transform. (Solid curves are real parts and dashed curves are imaginary parts.)

when it is clear that we are considering an operation on a sequence, the 90-degree phase shifter is also called a *Hilbert transformer*. From Eqs. (11.63), it follows that

$$X_r(e^{j\omega}) = \frac{1}{H(e^{j\omega})}X_i(e^{j\omega}) = -H(e^{j\omega})X_i(e^{j\omega}). \tag{11.64}$$

Thus, $-x_r[n]$ can also be obtained from $x_i[n]$ with a 90-degree phase shifter.

The impulse response $h[n]$ of a 90-degree phase shifter, corresponding to the frequency response $H(e^{j\omega})$ given in Eq. (11.63b), is

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{0} je^{j\omega n}d\omega - \frac{1}{2\pi}\int_{0}^{\pi} je^{j\omega n}d\omega,$$

or

$$h[n] = \begin{cases} \dfrac{2}{\pi}\dfrac{\sin^2(\pi n/2)}{n}, & n \neq 0, \\ 0, & n = 0. \end{cases} \tag{11.65}$$

The impulse response is plotted in Figure 11.5. Using Eqs. (11.63) and (11.64), we obtain the expressions

$$x_i[n] = \sum_{m=-\infty}^{\infty} h[n-m]x_r[m] \tag{11.66a}$$

and

$$x_r[n] = -\sum_{m=-\infty}^{\infty} h[n-m]x_i[m]. \tag{11.66b}$$

Equations (11.66) are the desired Hilbert transform relations between the real and imaginary parts of a discrete-time analytic signal.

Figure 11.6 shows how a discrete-time Hilbert transformer system can be used to form a complex analytic signal, which is simply a pair of real signals.

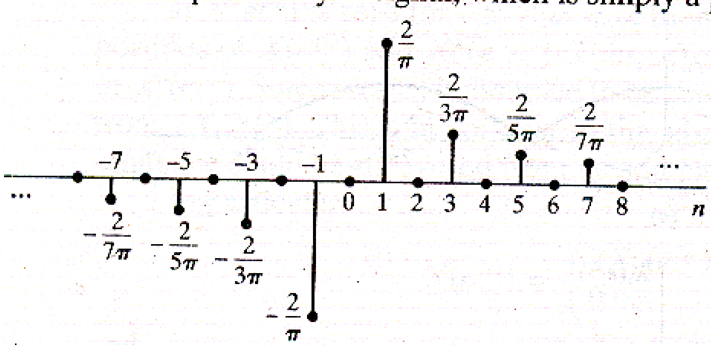


Figure 11.5 Impulse response of an ideal Hilbert transformer or 90-degree phase shifter.

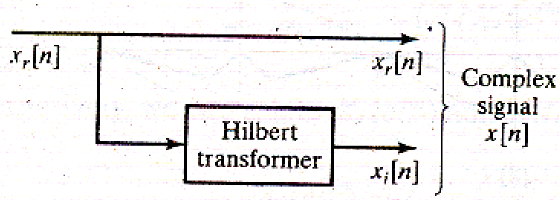


Figure 11.6 Block diagram representation of the creation of a complex sequence whose Fourier transform is one sided.

b. Find the inverse Z-transform of

$$X(z)=\frac{z^{-1}}{3-4z^{-1}+z^{-2}}\text{; ROC } |z|>1$$ **(8)**

Ans: 9. (b)

Given $x(z) = \dfrac{z^{-1}}{3-4z^{-1}+z^{-2}}$; ROC $|z|>1$

$x(z)$ can be simplified to

$$x(z) = \frac{z}{3z^2-4z+1}$$

using partial fraction

$$x(z) = \frac{1}{2}\left[\frac{z}{z-1} - \frac{z}{z-\left(\frac{1}{3}\right)}\right]; \quad \text{ROC } |z|>1$$

Since ROC is $|z|>1$; both the sequences must be causal. Therefore, taking inverse z - transform,

$$x(n) = \frac{1}{2}\left[u(n) - \left(\frac{1}{3}\right)^n u(n)\right]; \quad \text{ROC } |z|>1$$

Ans:

TEXT-BOOK

I. **Discrete-Time Signal Processing (1999), Oppenheim, A. V., and Schafer, R. W., with J. R. Buck, Second Edition, Pearson Education, Low Price Edition.**