#### Q.2 a. Explain (i) Single stub matching and (ii) Double stub matching

(8)

#### Answer:

#### 3-6-1 Single-Stub Matching

Although single-lumped inductors or capacitors can match the transmission line, it is more common to use the susceptive properties of short-circuited sections of transmission lines. Short-circuited sections are preferable to open-circuited ones because a good short circuit is easier to obtain than a good open circuit.

For a lossless line with  $\mathbf{Y}_g = \mathbf{Y}_0$ , maximum power transfer requires  $\mathbf{Y}_{11} = \mathbf{Y}_0$ , where  $\mathbf{Y}_{11}$  is the total admittance of the line and stub looking to the right at point 1-1 (see Fig. 3-6-2). The stub must be located at that point on the line where the real part of the admittance, looking toward the load, is  $\mathbf{Y}_0$ . In a normalized unit  $y_{11}$  must be in the form

$$y_{11} = y_d \pm y_s = 1 \tag{3-6-1}$$



Figure 3-6-2 Single-stub matching for Example 3-6-1.

if the stub has the same characteristic impedance as that of the line. Otherwise

$$\mathbf{Y}_{11} = \mathbf{Y}_d \pm \mathbf{Y}_s = \mathbf{Y}_0 \tag{3-6-2}$$

The stub length is then adjusted so that its susceptance just cancels out the susceptance of the line at the junction.

#### **(ii)**

#### 3-6-2 Double-Stub Matching

Since single-stub matching is sometimes impractical because the stub cannot be placed physically in the ideal location, double-stub matching is needed. Double-stub devices consist of two short-circuited stubs connected in parallel with a fixed length between them. The length of the fixed section is usually one-eighth, three-eighths, or five-eighths of a wavelength. The stub that is nearest the load is used to adjust the susceptance and is located at a fixed wavelength from the constant conductance unity circle (g = 1) on an appropriate constant-standing-wave-ratio circle. Then the admittance of the line at the second stub as shown in Fig. 3-6-4 is

$$y_{22} = y_{d2} \pm y_{s2} = 1 \tag{3-6-3}$$

$$\mathbf{Y}_{22} = \mathbf{Y}_{d2} \pm \mathbf{Y}_{s2} = \mathbf{Y}_0 \tag{3-6-4}$$

In these two equations it is assumed that the stubs and the main line have the same characteristic admittance. If the positions and lengths of the stubs are chosen properly, there will be no standing wave on the line to the left of the second stub measured from the load.



Figure 3-6-4 Double-stub matching for Example 3-6-2.

- b. A lossless line has a characteristic impedance of 50  $\Omega$  and is terminated in a load resistance of 75  $\Omega$ . The line is energised by a generator which has an output impedance of 50  $\Omega$  and an open-circuit output voltage of 30 V (rms). The line is assumed to be 2.25 wavelengths long. Determine: (8)
  - (i) The input impedance
  - (ii) The magnitude of the instantaneous load voltage
  - (iii) The instantaneous power delivered to the load

#### Answer:

a. From Eq. (3-4-26) the line that is 2.25 wavelengths long looks like a quarterwave line. Then

$$\beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

From Eq. (3-4-26) the input impedance is

$$\mathbf{Z}_{\rm in} = \frac{R_0^2}{R_\ell} = \frac{(50)^5}{75} = 33.33 \ \Omega$$

b. The reflection coefficient is

$$\Gamma_{\ell} = \frac{R_{\ell} - R_0}{R_{\ell} + R_0} = \frac{75 - 50}{75 + 50} = 0.20$$

Then the instantaneous voltage at the load is

$$\mathbf{V}_{\ell} = \mathbf{V}_{+} e^{-j\beta\ell} (1 + \Gamma_{\ell}) = 30(1 + 0.20) = 36 \text{ V}$$

c. The instantaneous power delivered to the load is

$$\mathbf{P}_{\ell} = \frac{(36)^2}{75} = 17.28 \text{ W}$$

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- Q.3 a. An air filled rectangular waveguide has dimensions of a = 6 cm and b = 4 cm. The signal frequency is 3 GHz. Compute the following for the  $TE_{10}$  mode; (8)
  - (i) Cut off frequency
  - (ii) Wavelength in the waveguide
  - (iii) Phase velocity
  - (iv) Group velocity

#### Answer:

- (i) Cutoff frequency  $(f_{\sigma}) = \frac{\sigma}{2\alpha} = \frac{3X10^8}{2X6X10^{-2}} = 2.5 \ GHz$ (ii) Wavelength  $(\lambda_g) = \frac{\lambda_g}{\sqrt{1 - (\frac{f_g}{f})^2}} = 18.09 \ cm$
- (iii) Phase velocity  $(v_p) = c \sqrt{1 (\frac{f_c}{f})^2} = 1.65X10^8$
- (iv) Group velocity (  $v_g$ ) =  $\frac{c}{\sqrt{1 (\frac{fc}{f})^2}} = 5.42X10^8$
- b. A  $TE_{11}$  mode is propagating through a circular waveguide. The radius of the guide is 5 cm and the guide contains an air dielectric. (Given that  $X'_{11} = 1.841 = k_c a$ ). (8)
  - (i) Determine the cutoff frequency.
  - (ii) Determine the wavelength  $\lambda_{_g}$  in the guide for an operating frequency

of 3GHz.

(iii) Determine the wave impedance  $Z_{p}$  in the guide.

#### Answer:

**a.** From Table 4-2-1 for TE<sub>11</sub> mode, n = 1, p = 1, and  $X'_{11} = 1.841 = k_c a$ . The cutoff wave number is

$$k_c = \frac{1.841}{a} = \frac{1.841}{5 \times 10^{-2}} = 36.82$$

The cutoff frequency is

$$f_c = \frac{k_c}{2\pi \sqrt{\mu_0 \epsilon_0}} = \frac{(36.82)(3 \times 10^8)}{2\pi} = 1.758 \times 10^9 \text{ Hz}$$

b. The phase constant in the guide is

$$\beta_{g} = \sqrt{\omega^{2} \mu_{0} \epsilon_{0} - k_{c}^{2}}$$
  
=  $\sqrt{(2\pi \times 3 \times 10^{9})^{2} (4\pi \times 10^{-7} \times 8.85 \times 10^{-12}) - (36.82)^{2}}$   
= 50.9 rads/m

The wavelength in the guide is

$$\lambda_g = \frac{2\pi}{\beta_g} = \frac{6.28}{50.9} = 12.3 \text{ cm}$$

c. The wave impedance in the guide is

$$Z_g = \frac{\omega\mu_0}{\beta_g} = \frac{(2\pi \times 3 \times 10^9)(4\pi \times 10^{-7})}{50.9} = 465 \ \Omega$$

Q.4 a. Explain the working of Microwave Circulator. Explain how a four port Circulator is constructed with two Magic Tees? (10)

#### Answer:

#### 4-6-1 Microwave Circulators

A microwave circulator is a multiport waveguide junction in which the wave can flow only from the *n*th port to the (n + 1)th port in one direction (see Fig. 4-6-2). Although there is no restriction on the number of ports, the four-port microwave circulator is the most common. One type of four-port microwave circulator is a combination of two 3-dB side-hole directional couplers and a rectangular waveguide with two nonreciprocal phase shifters as shown in Fig. 4-6-3.



Figure 4-6-3 Schematic diagram of four-port circulator.

The operating principle of a typical microwave circulator can be analyzed with the aid of Fig. 4-6-3. Each of the two 3-dB couplers in the circulator introduces a phase shift of 90°, and each of the two phase shifters produces a certain amount of phase change in a certain direction as indicated. When a wave is incident to port 1, the wave is split into two components by coupler 1. The wave in the primary guide arrives at port 2 with a relative phase change of 180°. The second wave propagates through the two couplers and the secondary guide and arrives at port 2 with a relative phase shift of 180°. Since the two waves reaching port 2 are in phase, the power transmission is obtained from port 1 to port 2. However, the wave propagates through the primary guide, phase shifter, and coupler 2 and arrives at port 4 with a phase change of  $270^{\circ}$ . The wave travels through coupler 1 and the secondary guide, and it arrives at port 4 with a phase shift of 90°. Since the two waves reaching port 4 are out of phase by 180°, the power transmission from port 1 to port 4 is zero. In general, the differential propagation constants in the two directions of propagation in a waveguide containing ferrite phase shifters should be

$$\omega_1 - \omega_3 = (2m + 1)\pi$$
 rad/s (4-6-10)

$$\omega_2 - \omega_4 = 2n\pi \quad \text{rad/s} \quad (4-6-11)$$

where m and n are any integers, including zeros. A similar analysis shows that a wave incident to port 2 emerges at port 3 and so on. As a result, the sequence of power flow is designated as  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$ .

Many types of microwave circulators are in use today. However, their principles of operation remain the same. Figure 4-6-4 shows a four-port circulator constructed of two magic tees and a phase shifter. The phase shifter produces a phase shift of 180°. The explanation of how this circulator works is left as an exercise for the reader.



Phase shifter

Figure 4-6-4 A four-port circulator.

A perfectly matched, lossless, and nonreciprocal four-port circulator has an S matrix of the form

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$
(4-6-12)

Using the properties of S parameters as described previously, the S matrix in Eq.

(4-6-12) can be simplified to

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$
(4-6-13)

b. What are Directional Couplers? Explain with a neat diagram. Derive Smatrix of Directional Coupler discussed. (6)

Answer:

### 4-5 DIRECTIONAL COUPLERS

A directional coupler is a four-port waveguide junction as shown in Fig. 4-5-1. It consists of a primary waveguide 1-2 and a secondary waveguide 3-4. When all ports are terminated in their characteristic impedances, there is free transmission of power, without reflection, between port 1 and port 2, and there is no transmission of power between port 1 and port 3 or between port 2 and port 4 because no coupling exists between these two pairs of ports. The degree of coupling between port 1 and port 4 and between port 2 and port 3 depends on the structure of the coupler.

The characteristics of a directional coupler can be expressed in terms of its coupling factor and its directivity. Assuming that the wave is propagating from port 1 to port 2 in the primary line, the coupling factor and the directivity are defined,



respectively, by

Coupling factor (dB) =  $10 \log_{10} \frac{P_1}{P_4}$  (4-5-1)

Directivity (dB) = 
$$10 \log_{10} \frac{P_4}{P_3}$$
 (4-5-2)

where  $P_1$  = power input to port 1

 $P_3$  = power output from port 3  $P_4$  = power output from port 4

### 4-5-2 \$ Matrix of a Directional Coupler

In a directional coupler all four ports are completely matched. Thus the diagonal elements of the S matrix are zeros and

$$S_{11} = S_{22} = S_{33} = S_{44} = 0 \tag{4-5-4}$$

As noted, there is no coupling between port 1 and port 3 and between port 2 and port 4. Thus

$$S_{13} = S_{31} = S_{24} = S_{42} = 0 \tag{4-5-5}$$

Consequently, the S matrix of a directional coupler becomes

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$
(4-5-6)

Equation (4-5-6) can be further reduced by means of the zero property of the S matrix, so we have

$$S_{12}S_{14}^* + S_{32}S_{34}^* = 0 \tag{4-5-7}$$

$S_{21}S_{23}^* + S_{41}S_{43}^* = 0$	(4-5-8)
Also from the unity property of the S matrix, we can write	
$S_{12}S_{12}^* + S_{14}S_{14}^* = 1$	(4-5-9)
Equations (4-5-7) and (4-5-8) can also be written	
$ S_{12}  S_{14}  =  S_{32}  S_{34} $	(4-5-10)
$ S_{21}  S_{23}  =  S_{41}  S_{43} $	(4-5-11)
Since $S_{12} = S_{21}$ , $S_{14} = S_{41}$ , $S_{23} = S_{32}$ , and $S_{34} = S_{43}$ , then	
$ S_{12}  =  S_{34} $	(4-5-12)
$ S_{14}  =  S_{23} $	(4-5-13)
Let	
$S_{12} = S_{34} = p$	(4-5-14)
where $p$ is positive and real. Then from Eq. (4-5-8)	
$p(S_{23}^* + S_{41}) = 0$	(4-5-15)
Let	
$S_{23} = S_{41} = jq$	(4-5-16)
where $q$ is positive and real. Then from Eq. (4-5-9)	
$p^2 + q^2 = 1$	(4-5-17)
The S matrix of a directional coupler is reduced to	
$\mathbf{S} = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$	(4-5-18)
Q.5 a. Explain the physical description of Read Diode.	(6)

Answer:

### 8-1 READ DIODE

#### 8-1-1 Physical Description

The basic operating principle of IMPATT diodes can be most easily understood by reference to the first proposed avalanche diode, the Read diode [1]. The theory of this device was presented by Read in 1958, but the first experimental Read diode was reported by Lee et al. in 1965 [3]. A mode of the original Read diode with a doping profile and a dc electric field distribution that exists when a large reverse bias is applied across the diode is shown in Fig. 8-1-1.



The Read diode is an  $n^+$ -p-i- $p^+$  structure, where the superscript plus sign denotes very high doping and the *i* or *v* refers to intrinsic material. The device consists essentially of two regions. One is the thin *p* region at which avalanche multiplication occurs. This region is also called the high-field region or the avalanche region. The other is the *i* or *v* region through which the generated holes must drift in moving to the  $p^+$  contact. This region is also called the intrinsic region or the drift region. The p region is very thin. The space between the  $n^+$ -p junction and the i- $p^+$  junction is called the space-charge region. Similar devices can be built in the  $p^+$ -n-i- $n^+$  structure, in which electrons generated from avalanche multiplication drift through the *i* region.

The Read diode oscillator consists of an  $n^+$ -p-i- $p^+$  diode biased in reverse and mounted in a microwave cavity. The impedance of the cavity is mainly inductive and is matched to the mainly capacitive impedance of the diode to form a resonant circuit. The device can produce a negative ac resistance that, in turn, delivers power from the dc bias to the oscillation.

b. Explain the principle of operation TRAPATT Diodes.

(10)

#### Answer:

#### 8-3-2 Principles of Operation

Approximate analytic solutions for the TRAPATT mode in  $p^+$ -n-n<sup>+</sup> diodes have been developed by Clorfeine et al. [8] and DeLoach [9] among others. These analyses have shown that a high-field avalanche zone propagates through the diode and fills the depletion layer with a dense plasma of electrons and holes that become trapped in the low-field region behind the zone. A typical voltage waveform for the TRAPATT mode of an avalanche  $p^+$ -n-n<sup>+</sup> diode operating with an assumed squarewave current drive is shown in Fig. 8-3-1. At point A the electric field is uniform throughout the sample and its magnitude is large but less than the value required for avalanche breakdown. The current density is expressed by

$$J = \epsilon_s \frac{dE}{dt} \tag{8-3-1}$$

where  $\epsilon_s$  is the semiconductor dielectric permittivity of the diode.

At the instant of time at point A, the diode current is turned on. Since the only charge carriers present are those caused by the thermal generation, the diode initially charges up like a linear capacitor, driving the magnitude of the electric field above the breakdown voltage. When a sufficient number of carriers is generated, the particle current exceeds the external current and the electric field is depressed throughout the depletion region, causing the voltage to decrease. This portion of the cycle is shown by the curve from point B to point C. During this time interval the electric field is sufficiently large for the avalanche to continue, and a dense plasma of electrons and holes is created. As some of the electrons and holes drift out of the ends of the depletion layer, the field is further depressed and "traps" the remaining plasma. The voltage decreases to point D. A long time is required to remove the plasma because the total plasma charge is large compared to the charge per unit time



in the external current. At point E the plasma is removed, but a residual charge of electrons remains in one end of the depletion layer and a residual charge of holes in the other end. As the residual charge is removed, the voltage increases from point E to point F. At point F all the charge that was generated internally has been removed. This charge must be greater than or equal to that supplied by the external current; otherwise the voltage will exceed that at point A. From point F to point G the diode charges up again like a fixed capacitor. At point G the diode current goes to zero for half a period and the voltage remains constant at  $V_A$  until the current comes back on and the cycle repeats. The electric field can be expressed as

$$E(x, t) = E_m - \frac{qN_A}{\epsilon_s}x + \frac{Jt}{\epsilon_s}$$
(8-3-2)

where  $N_A$  is the doping concentration of the *n* region and *x* is the distance.

Thus the value of t at which the electric field reaches  $E_m$  at a given distance x into the depletion region is obtained by setting  $E(x, t) = E_m$ , yielding

$$t = \frac{qN_A}{J}x \tag{8-3-3}$$

Differentiation of Eq. (8-3-3) with respect to time t results in

$$v_z \equiv \frac{dx}{dt} = \frac{J}{qN_A} \tag{8-3-4}$$

where  $v_z$  is the avalanche-zone velocity.

Thus the avalanche zone (or avalanche shock front) will quickly sweep across most of the diode, leaving the diode filled by a highly conducting plasma of holes and electrons whose space charge depresses the voltage to low values. Because of the dependence of the drift velocity on the field, the electrons and holes will drift at velocities determined by the low-field mobilities, and the transit time of the carriers can become much longer than

$$\tau_s = \frac{L}{v_s} \tag{8-3-5}$$

where  $v_s$  is the saturated carrier drift velocity.

Thus the TRAPATT mode can operate at comparatively low frequencies, since the discharge time of the plasma—that is, the rate Q/I of its charge to its current can be considerably greater than the nominal transit time  $\tau_s$  of the diode at high field. Therefore the TRAPATT mode is still a transit-time mode in the real sense that the time delay of carriers in transit (that is, the time between injection and collection) is utilized to obtain a current phase shift favorable for oscillation.

# Q.6 a. Explain the effect of Lead-inductance and interelectrode capacitance in vacuum tubes at microwave frequencies. (8)

#### **Answer:**

#### 9-1-1 Lead-Inductance and Interelectrode-Capacitance Effects

At frequencies above 1 GHz conventional vacuum tubes are impaired by parasiticcircuit reactances because the circuit capacitances between tube electrodes and the circuit inductance of the lead wire are too large for a microwave resonant circuit. Furthermore, as the frequency is increased up to the microwave range, the real part of the input admittance may be large enough to cause a serious overload of the input circuit and thereby reduce the operating efficiency of the tube. In order to gain a better understanding of these effects, the triode ciruit shown in Fig. 9-1-1 should be studied carefully.



Figure 9-1-1 Triode circuit (a) and its equivalent (b).

Figure 9-1-1(b) shows the equivalent circuit of a triode circuit under the assumption that the interelectrode capacitances and cathode inductance are the only parasitic elements. Since  $C_{gp} \ll C_{gk}$  and  $\omega L_k \ll 1/(\omega C_{gk})$ , the input voltage  $V_{in}$  can be written as

$$\mathbf{V}_{in} = V_{g} + V_{k} = V_{g} + j\omega L_{k} g_{m} V_{g}$$
(9-1-1)

and the input current as

$$\mathbf{I}_{\rm in} = j\omega C_{gk} V_g \tag{9-1-2}$$

Substitution of Eq. (9-1-2) in Eq. (9-1-1) yields

$$\mathbf{V}_{\rm in} = \frac{\mathbf{I}_{\rm in}(1 + j\omega L_k g_m)}{j\omega C_{gk}} \tag{9-1-3}$$

The input admittance of the tube is approximately

$$\mathbf{Y}_{in} = \frac{\mathbf{I}_{in}}{\mathbf{V}_{in}} = \frac{j\omega C_{gk}}{1 + j\omega L_k g_m} = \omega^2 L_k C_{gk} g_m + j\omega C_{gk}$$
(9-1-4)

in which  $\omega L_k g_m \ll 1$  has been replaced. The inequality is almost always true, since the cathode lead is usually short and is quite large in diameter, and the transconductance  $g_m$  is generally much less than one millimho.

The input impedance at very high frequencies is given by

$$\mathbf{Z}_{in} = \frac{1}{\omega^2 L_k C_{gk} g_m} - j \frac{1}{\omega^3 L_k^2 C_{gk} g_m^2}$$
(9-1-5)

The real part of the impedance is inversely proportional to the square of the frequency, and the imaginary part is inversely proportional to the third order of the frequency. When the frequencies are above 1 GHz, the real part of the impedance becomes small enough to nearly short the signal source. Consequently, the output power is decreased rapidly. Similarly, the input admittance of a pentode circuit is expressed by

$$Y_{in} = \omega^2 L_k C_{gk} g_m + j \omega (C_{gk} + C_{gs})$$
(9-1-6)

where  $C_{gs}$  is the capacitance between the grid and screen, and its input impedance is given by

$$\mathbf{Z}_{in} = \frac{1}{\omega^2 L_k C_{gk} g_m} - j \frac{C_{gk} + C_{gs}}{\omega^3 L_k^2 C_{gk}^2 g_m^2}$$
(9-1-7)

There are several ways to minimize the inductance and capacitance effects, such as a reduction in lead length and electrode area. This minimization, however, also limits the power-handling capacity.

b. Explain the velocity modulation process in Two cavity Klystron. (8) Answer:

#### 9-2-2 Velocity-Modulation Process

When electrons are first accelerated by the high dc voltage  $V_0$  before entering the buncher grids, their velocity is uniform:

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0}$$
 m/s (9-2-10)

In Eq. (9-2-10) it is assumed that electrons leave the cathode with zero velocity. When a microwave signal is applied to the input terminal, the gap voltage between the buncher grids appears as

$$V_s = V_1 \sin(\omega t) \tag{9-2-11}$$

where  $V_1$  is the amplitude of the signal and  $V_1 \ll V_0$  is assumed.

In order to find the modulated velocity in the buncher cavity in terms of either the entering time  $t_0$  or the exiting time  $t_1$  and the gap transit angle  $\theta_g$  as shown in Fig. 9-2-2 it is necessary to determine the average microwave voltage in the buncher gap as indicated in Fig. 9-2-6.

Since  $V_1 \ll V_0$ , the average transit time through the buncher gap distance d is

$$\tau \approx \frac{d}{v_0} = t_1 - t_0$$
 (9-2-12)



Figure 9-2-6 Signal voltage in the buncher gap.

The average gap transit angle can be expressed as

$$\theta_g = \omega \tau = \omega (t_1 - t_0) = \frac{\omega d}{v_0} \qquad (9-2-13)$$

The average microwave voltage in the buncher gap can be found in the following way:

$$\langle V_s \rangle = \frac{1}{\tau} \int_{t_0}^{t_1} V_1 \sin(\omega t) dt = -\frac{V_1}{\omega \tau} [\cos(\omega t_1) - \cos(\omega t_0)]$$
  
$$= \frac{V_1}{\omega \tau} \left[ \cos(\omega t_0) - \cos\left(\omega_0 + \frac{\omega d}{v_0}\right) \right] \qquad (9-2-14)$$

Let

$$\omega t_0 + \frac{\omega d}{2v_0} = \omega t_0 + \frac{\theta_g}{2} = A$$

and

$$\frac{\omega d}{2v_0}=\frac{\theta_g}{2}=B$$

Then using the trigonometric identity that  $\cos (A - B) - \cos (A + B) = 2 \sin A \sin B$ , Eq. (9-2-14) becomes

$$\langle V_s \rangle = V_1 \frac{\sin\left[\omega d/2v_0\right]}{\omega d/(2v_0)} \sin\left(\omega t_0 + \frac{\omega d}{2v_0}\right) = V_1 \frac{\sin\left(\theta_g/2\right)}{\theta_g/2} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \quad (9-2-15)$$

It is defined as

$$\boldsymbol{\beta}_{i} = \frac{\sin\left[\omega d/(2v_{0})\right]}{\omega d/(2v_{0})} = \frac{\sin\left(\theta_{g}/2\right)}{\theta_{g}/2}$$
(9-2-16)

Note that  $\beta_i$  is known as the *beam-coupling coefficient* of the input cavity gap (see Fig. 9-2-7).

It can be seen that increasing the gap transit angle  $\theta_s$  decreases the coupling between the electron beam and the buncher cavity; that is, the velocity modulation





Figure 9-2-7 Beam-coupling coefficient versus gap transit angle.

of the beam for a given microwave signal is decreased. Immediately after velocity modulation, the exit velocity from the buncher gap is given by

$$v(t_1) = \sqrt{\frac{2e}{m} \left[ V_0 + \beta_i V_1 \sin \left( \omega t_0 + \frac{\theta_s}{2} \right) \right]}$$
$$= \sqrt{\frac{2e}{m} V_0 \left[ 1 + \frac{\beta_i V_1}{V_0} \sin \left( \omega t_0 + \frac{\theta_s}{2} \right) \right]}$$
(9-2-17)

where the factor  $\beta_i V_1/V_0$  is called the *depth of velocity modulation*.

Using binomial expansion under the assumption of

$$\boldsymbol{\beta}_i V_1 \ll V_0 \tag{9-2-18}$$

Eq. (9-2-17) becomes

8

$$v(t_1) = v_0 \left[ 1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_0 + \frac{\theta_s}{2}\right) \right] \qquad (9-2-19)$$

Equation (9-2-19) is the equation of velocity modulation. Alternatively, the equation of velocity modulation can be given by

$$v(t_1) = v_0 \left[ 1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_s}{2}\right) \right]$$
(9-2-20)

Answer:

Q.7 a. Explain the power output and efficiency of a Magnetron.

(8)

Power output and efficiency. The efficiency and power output of a mag-

netron depend on the resonant structure and the dc power supply. Figure 10-1-4 shows an equivalent circuit for a resonator of a magnetron.

where  $Y_e$  = electronic admittance

 $V = \mathbf{RF}$  voltage across the vane tips

C = capacitance at the vane tips

L = inductance of the resonator

 $G_r$  = conductance of the resonator

G =load conductance per resonator



Figure 10-1-4 Equivalent circuit for one resonator of a magnetron.

Each resonator of the slow-wave structure is taken to comprise a separate resonant circuit as shown in Fig. 10-1-4. The unloaded quality factor of the resonator is given by

$$Q_{\rm un} = \frac{\omega_0 C}{G_r} \tag{10-1-22}$$

where  $\omega_0 = 2\pi f_0$  is the angular resonant frequency. The external quality factor of the load ciruit is

$$Q_{\rm ex} = \frac{\omega_0 C}{G_\ell} \tag{10-1-23}$$

Then the loaded  $Q_\ell$  of the resonant circuit is expressed by

$$Q_{\ell} = \frac{\omega_0 C}{G_r + G_{\ell}} \tag{10-1-24}$$

The circuit efficiency is defined as

$$\eta_c = \frac{G_\ell}{G_\ell + G_r}$$

$$= \frac{G_\ell}{G_{\text{ex}}} = \frac{1}{1 + Q_{\text{ex}}/Q_{\text{un}}}$$
(10-1-25)

The maximum circuit efficiency is obtained when the magnetron is heavily loaded, that is, for  $G_{\ell} \ge G_r$ . Heavy loading, however, makes the tube quite sensitive to the load, which is undesirable in some cases. Therefore, the ratio of  $Q_{\ell}/Q_{ex}$  is often chosen as a compromise between the conflicting requirements for high circuit efficiency and frequency stability.

The electronic efficiency is defined as

$$\eta_{e} = \frac{P_{gen}}{P_{dc}} = \frac{V_0 I_0 - P_{lost}}{V_0 I_0}$$
(10-1-26)

where  $P_{gen} = RF$  power induced into the anode circuit

 $P_{\rm dc} = V_0 I_0$  power from the dc power supply

 $V_0$  = anode voltage

 $I_0$  = anode current

 $P_{\text{lost}}$  = power lost in the anode circuit

The RF power generated by the electrons can be written as

$$P_{gen} = V_0 I_0 - P_{lost}$$
  
=  $V_0 I_0 - I_0 \frac{m}{2e} \frac{\omega_0^2}{\beta^2} + \frac{E_{max}^2}{B_z^2}$  (10-1-27)  
=  $\frac{1}{2} N |V|^2 \frac{\omega_0 C}{O_\ell}$ 

where N = total number of resonators

V = RF voltage across the resonator gap

 $E_{\text{max}} = M_1 |V| / L$  is the maximum electric field

$$M_1 = \sin\left(\frac{\beta_n \frac{\sigma}{2}}{2}\right) / \left(\frac{\beta_n \frac{\sigma}{2}}{2}\right) = 1$$
 for small  $\delta$  is the gap factor for the  $\pi$ -mode operation

 $\beta$  = phase constant

 $B_z$  = magnetic flux density

L = center-to-center spacing of the vane tips

The power generated may be simplified to

 $P_{\rm gen} = \frac{NL^2\omega_0 C}{2M_1^2 Q\ell} E_{\rm max}^2$ (10 - 1 - 28)The electronic efficiency may be rewritten as  $\eta_{\epsilon} = \frac{P_{\text{gen}}}{V_0 I_0} = \frac{1 - \frac{m\omega_0^2}{2eV_0\beta^2}}{1 + \frac{I_0 mM_1^2 Q_{\ell}}{B_1 e NL^2 \omega_0 C}}$ 

b. Explain the principle of operation Forward Wave Cross-Field Amplifier.

(8)

### **Answer:** 10-2-1 Principles of Operation

In the emitting-sole tube, the current emanated from the cathode is in response to the electric field forces in the space between the cathode and anode. The amount of

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Figure 10-2-1 Schematic diagrams of CFAs. (After J. F. Skowron [2], reprinted by permission of IEEE, Inc.)

![](_page_18_Figure_2.jpeg)

Figure 10-2-2 Diagram of a strapped CFA. (After J. F. Skowron [2], reprinted by permission of IEEE, Inc.)

current is a function of the dimension, the applied voltage, and the emission properties of the cathode. The perveance of the interaction geometry tends to be quite high, about 5 to  $10 \times 10^{-10}$ , which results in a high-current and high-power capability at relative low voltage. In the injected-beam tube the electron beam is produced in a separate gun assembly and is injected into the interaction region.

The beam-circuit interaction features are similar in both the emitting-sole and the injected-beam tubes. Favorably phased electrons continue toward the positively polarized anode and are ultimately collected, whereas unfavorably phased electrons are directed toward the negative polarized electrode.

In linear-beam interaction, as discussed for traveling-wave tubes in Sec. 9-5, the electron stream is first accelerated by an electric gun to the full dc velocity; the dc velocity is approximately equal to the axial phase velocity of the RF field on the slow-wave structure. After interaction occurs, the spent electron beam leaves the interaction region with a low-average velocity. The difference in velocity is accounted for by the RF energy created on the microwave circuit. In the CFA, the electron is exposed to the dc electric field force, magnetic field force, and the electric field force of the RF field, and even to the space-charge force from other electrons. The last force is normally not considered in analytic approaches because of its complexity. Under the influence of the three forces, the electrons travel in spiral trajectories in a direction tending along equipotentials. The exact motion has been subject to much analysis by means of a computer. Figure 10-2-3 shows the pattern of the electron flow in the CFA by computerized techniques [2]. It can be seen that when the spoke is positively polarized or the RF field is in the positive half cycle, the electron speeds up toward the anode; while the spoke is negatively polarized or the RF field is

![](_page_19_Figure_2.jpeg)

Figure 10-2-3 Motion of electrons in CFA. (After J. F. Skowron [2], reprinted by permission of IEEE, Inc.)

in the negative half cycle, the electrons are returned toward the cathode. Consequently, the electron beam moves in a spiral path in the interaction region.

Q.8 a. Explain (i) Dielectric Losses (ii) Ohmic Losses in a microstrip line. (8)

Answer:

**Dielectric losses.** As stated in Section 2-5-3, when the conductivity of a dielectric cannot be neglected, the electric and magnetic fields in the dielectric are no longer in time phase. In that case the dielectric attenuation constant, as expressed in Eq. (2-5-20), is given by

$$\alpha_d = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}}$$
 Np/cm (11-1-15)

where  $\sigma$  is the conductivity of the dielectric substrate board in U/cm. This dielectric constant can be expressed in terms of dielectric loss tangent as shown in Eq. (2-5-17):

$$\tan\theta = \frac{\sigma}{\omega\epsilon} \tag{11-1-16}$$

Then the dielectric attenuation constant is expressed by

$$\alpha_d = \frac{\omega}{2} \sqrt{\mu\epsilon} \tan \theta$$
 Np/cm (11-1-17)

Since the microstrip line is a nonmagnetic mixed dielectric system, the upper dielectric above the microstrip ribbon is air, in which no loss occurs. Welch and Pratt [9] derived an expression for the attenuation constant of a dielectric substrate. Later on, Pucel and his coworkers [10] modified Welch's equation [9]. The result is

$$\alpha_{d} = 4.34 \frac{q\sigma}{\sqrt{\epsilon_{re}}} \sqrt{\frac{\mu_{0}}{\epsilon_{0}}}$$
$$= 1.634 \times 10^{3} \frac{q\sigma}{\sqrt{\epsilon_{re}}} \qquad \text{dB/cm} \qquad (11-1-18)$$

In Eq. (11-1-18) the conversion factor of 1 Np = 8.686 dB is used,  $\epsilon_{re}$  is the effective dielectric constant of the substrate, as expressed in Eq. (11-1-5), and q denotes the dielectric filling factor, defined by Wheeler [3] as

$$q = \frac{\epsilon_{re} - 1}{\epsilon_r - 1} \tag{11-1-19}$$

We usually express the attenuation constant per wavelength as

$$\alpha_d = 27.3 \left(\frac{q\epsilon_r}{\epsilon_{re}}\right) \frac{\tan\theta}{\lambda_g} \qquad dB/\lambda_g \qquad (11-1-20)$$

where  $\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{re}}}$  and  $\lambda_0$  is the wavelength in free space, or  $\lambda_g = \frac{c}{f\sqrt{\epsilon_{re}}}$  and c is the velocity of light in vacuum.

If the loss tangent, tan  $\theta$ , is independent of frequency, the dielectric attenuation per wavelength is also independent of frequency. Moreover, if the substrate conductivity is independent of frequency, as for a semiconductor, the dielectric attenuation per unit is also independent of frequency. Since q is a function of  $\epsilon_r$  and w/h, the filling factors for the loss tangent  $q\epsilon_n/\epsilon_{re}$  and for the conductivity  $q/\sqrt{\epsilon_{re}}$ are also functions of these quantities. Figure 11-1-5 shows the loss-tangent filling factor against w/h for a range of dielectric constants suitable for microwave inte-

![](_page_21_Figure_2.jpeg)

![](_page_21_Figure_3.jpeg)

grated circuits. For most practical purposes, this factor is considered to be 1. Figure 11-1-6 illustrates the product  $\alpha_d \rho$  against w/h for two semiconducting substrates, silicon and gallium arsenide, that are used for integrated microwave circuits. For design purposes, the conductivity filling factor, which exhibits only a mild dependence on w/h, can be ignored.

**Ohmic losses.** In a microstrip line over a low-loss dielectric substrate, the predominant sources of losses at microwave frequencies are the nonperfect conductors. The current density in the conductors of a microstrip line is concentrated in a sheet that is approximately a skin depth thick inside the conductor surface and exposed to the electric field. Both the strip conductor thickness and the ground plane thickness are assumed to be at least three or four skin depths thick. The current density in the strip conductor and the ground conductor is not uniform in the transverse plane. The microstrip conductor contributes the major part of the ohmic loss. A diagram of the current density J for a microstrip line is shown in Fig. 11-1-7.

Because of mathematical complexity, exact expressions for the current density of a microstrip line with nonzero thickness have never been derived [10]. Several researchers [8] have assumed, for simplicity, that the current distribution is uniform and equal to I/w in both conductors and confined to the region |x| < w/2. With this assumption, the conducting attenuation constant of a wide microstrip line is given by

$$\alpha_c \simeq \frac{8.686R_s}{Z_0 w}$$
 dB/cm for  $\frac{w}{h} > 1$  (11-1-21)

![](_page_22_Figure_2.jpeg)

Figure 11-1-6 Dielectric attenuation factor of microstrip as a function of w/h for silicon and gallium arsenide substrates. (After R. A. Pucel et al. [10]; reprinted by permission of IEEE, Inc.)

![](_page_22_Figure_4.jpeg)

Figure 11-1-7 Current distribution on microstrip conductors. (After R. A. Pucel et al. [10]; reprinted by permission of IEEE, Inc.)

where  $R_s = \sqrt{\frac{\pi f \mu}{\sigma}}$  is the surface skin resistance in  $\Omega$ /square,

$$R_s = \frac{1}{\delta\sigma}$$
 is  $\Omega$ /square  
 $\delta = \sqrt{\frac{1}{\pi f\mu\sigma}}$  is the skin depth in cm

For a narrow microstrip line with w/h < 1, however, Eq. (11-1-21) is not applicable. The reason is that the current distribution in the conductor is not uniform, as assumed. Pucel and his coworkers [10, 11] derived the following three formulas from the results of Wheeler's work [3]:

-

$$\frac{\alpha_{c} Z_{0} h}{R_{s}} = \frac{8.68}{2\pi} \left[ 1 - \left(\frac{w}{4h}\right)^{2} \right] \left[ 1 + \frac{h}{w'} + \frac{h}{\pi w'} \left( \ln \frac{4\pi w}{t} + \frac{t}{w} \right) \right]$$
for  $\frac{w}{h} \le \frac{1}{2\pi}$  (11-1-22)
$$\frac{a_{c} Z_{0} h}{R_{s}} = \frac{8.68}{2\pi} \left[ 1 - \left(\frac{w'}{4h}\right)^{2} \right] \left[ 1 + \frac{h}{w'} + \frac{h}{w'} \left( \ln \frac{2h}{t} - \frac{t}{h} \right) \right]$$
for  $\frac{1}{2\pi} < \frac{w}{h} \le 2$  (11-1-23)

and

$$\frac{\alpha_{c} Z_{0} h}{R_{s}} = \frac{8.68}{\left\{\frac{w'}{h} + \frac{2}{\pi} \ln\left[2\pi e\left(\frac{w'}{2h} + 0.94\right)\right]\right\}^{2}} \left[\frac{w'}{h} + \frac{w'/(\pi h)}{\frac{w'}{2h} + 0.94}\right] \times \left[1 + \frac{h}{w'} + \frac{h}{\pi w'} \left(\ln\frac{2h}{t} - \frac{t}{h}\right)\right] \quad \text{for } 2 \le \frac{w}{h} \quad (11-1-24)$$

where  $\alpha_c$  is expressed in dB/cm and

$$e = 2.718$$

$$w' = w + \Delta w \tag{11-1-25}$$

$$\Delta w = \frac{t}{\pi} \left( \ln \frac{4\pi w}{t} + 1 \right) \qquad \text{for } \frac{2t}{h} < \frac{w}{h} \le \frac{\pi}{2} \tag{11-1-26}$$

$$\Delta w = \frac{t}{\pi} \left( \ln \frac{2h}{t} + 1 \right) \qquad \text{for } \frac{w}{h} \ge \frac{\pi}{2} \tag{11-1-27}$$

The values of  $\alpha_c$  obtained from solving Eqs. (11-1-22) through (11-1-24) are plotted in Fig. 11-1-8. For purposes of comparison, values of  $\alpha_c$  based on Assadourian and Rimai's Eq. (11-1-21) are also shown.

b. Explain (i) Distributed Parameters (ii) Characteristic Impedance and (iii) Attenuation Losses in a parallel strip-lines. (8)

Answer:

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### 11-2-1 Distributed Parameters

In a microwave integrated circuit a strip line can be easily fabricated on a dielectric substrate by using printed-circuit techniques. A parallel stripline is similar to a two-conductor transmission line, so it can support a quasi-TEM mode. Consider a TEM-mode wave propagating in the positive z direction in a lossless strip line (R = G = 0). The electric field is in the y direction, and the magnetic field is in the x direction. If the width w is much larger than the separation distance d, the fringing capacitance is negligible. Thus the equation for the inductance along the two conducting strips can be written as

$$L = \frac{\mu_c d}{w} \qquad \text{H/m} \qquad (11-2-1)$$

where  $\mu_c$  is the permeability of the conductor. The capacitance between the two conducting strips can be expressed as

$$C = \frac{\epsilon_d w}{d} \qquad \text{F/m} \tag{11-2-2}$$

where  $\epsilon_d$  is the permittivity of the dielectric slab.

If the two parallel strips have some surface resistance and the dielectric substrate has some shunt conductance, however, the parallel stripline would have some losses. The series resistance for both strips is given by

$$R = \frac{2R_s}{w} = \frac{2}{w} \sqrt{\frac{\pi f \mu_c}{\sigma_c}} \qquad \Omega/m \qquad (11-2-3)$$

where  $R_s = \sqrt{(\pi f \mu_c)/\sigma_c}$  is the conductor surface resistance in  $\Omega$ /square and  $\sigma_c$  is the conductor conductivity in  $\mathcal{O}/m$ . The shunt conductance of the strip line is

$$G = \frac{\sigma_d w}{d} \qquad \text{U/m} \qquad (11-2-4)$$

where  $\sigma_d$  is the conductivity of the dielectric substrate.

#### 11-2-2 Characteristic Impedance

The characteristic impedance of a lossless parallel strip line is

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{d}{w} \sqrt{\frac{\mu_d}{\epsilon_d}} = \frac{377}{\sqrt{\epsilon_{rd}}} \frac{d}{w} \quad \text{for } w \gg d \quad (11-2-5)$$

The phase velocity along a parallel strip line is

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_d \epsilon_d}} = \frac{c}{\sqrt{\epsilon_{rd}}}$$
 m/s for  $\mu_c = \mu_0$  (11-2-6)

The characteristic impedance of a lossy parallel strip line at microwave frequencies  $(R \ll \omega L \text{ and } G \ll \omega C)$  can be approximated as

....

$$Z_0 \simeq \sqrt{\frac{L}{C}} = \frac{377}{\sqrt{\epsilon_{rd}}} \frac{d}{w} \qquad \text{for } w \ge d \qquad (11-2-7)$$

## 11-2-3 Attenuation Losses

The propagation constant of a parallel strip line at microwave frequencies can be ex-

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad \text{for } R \ll \omega L \text{ and } G \ll \omega C$$
$$\approx \frac{1}{2} \left( R \sqrt{\frac{C}{L}} + G \sqrt{\frac{L}{C}} \right) + j\omega \sqrt{LC} \quad (11-2-8)$$

Thus the attenuation and phase constants are

 $\alpha = \frac{1}{2} \left( R \sqrt{\frac{C}{L}} + G \sqrt{\frac{L}{C}} \right)$ Np/m (11-2-9)

and

$$\beta = \omega \sqrt{LC} \quad \text{rad/m} \qquad (11-2-10)$$

Substitution of the distributed parameters of a parallel strip line into Eq.(11-2-9) yields the attenuation constants for the conductor and dielectric losses:

$$\alpha_c = \frac{1}{2}R\sqrt{\frac{C}{L}} = \frac{1}{d}\sqrt{\frac{\pi f\epsilon_d}{\sigma_c}} \qquad \text{Np/m} \qquad (11-2-11)$$

and

$$\alpha_d = \frac{1}{2}G \sqrt{\frac{L}{C}} = \frac{188\sigma_d}{\sqrt{\epsilon_{rd}}} \qquad \text{Np/m} \qquad (11-2-12)$$

Q.9 a. What is a substrate? What are the characteristics of an ideal substrate material? (6)

#### Answer:

#### Substrate Materials

A substrate of monolithic microwave integrated circuits is a piece of substance on which electronic devices are built. The ideal substrate materials should have the following characteristics [2]:

- 1. High dielectric constant (9 or higher)
- 2. Low dissipation factor or loss tangent
- 3. Dielectric constant should remain constant over the frequency range of interest and over the temperature range of interest
- 4. High purity and constant thickness
- 5. High surface smoothness
- 6. High resistivity and dielectric strength
- 7. High thermal conductivity
- b. What for resistive materials are used and write the properties of a good microwave resistor? (4)

#### Answer:

### 12-1-4 Resistive Materials

Resistive materials are used in monolithic microwave integrated circuits for bias networks, terminations, and attenuators. The properties required for a good microwave resistor are similar to those required for low-frequency resistors and should be [5]

- 1. Good stability
- 2. Low temperature coefficient of resistance (TCR)
- 3. Adequate dissipation capability
- 4. Sheet resistivities in the range of 10 to 1000  $\Omega$  per square
- c. Explain thin film formation.

Answer:

### 2-4 THIN-FILM FORMATION

The choice of lumped or distributed elements for amplifier matching networks depends on the operating frequency. When the frequency is up to X band, its wavelength is very short, and a smaller lumped element exhibits a negligible phase shift. Because of the advanced thin-film technology, the size of lumped elements can be greatly reduced and their operating frequencies can reach up to 20 GHz. Beyond that distributed elements are preferred. In monolithic microwave integrated circuits (MMICs), lumped resistors are very useful in thin-film resistive terminations for couplers, lumped capacitors are absolutely essential for bias bypass applications, and planar inductors are extremely useful for matching purposes, especially at lower microwave frequencies where stub inductors are physically too large [11].

- (i) Planar Resistor Film
- (ii) Planar Inductor Film
- (iii) Planar Capacitor Film

(6)