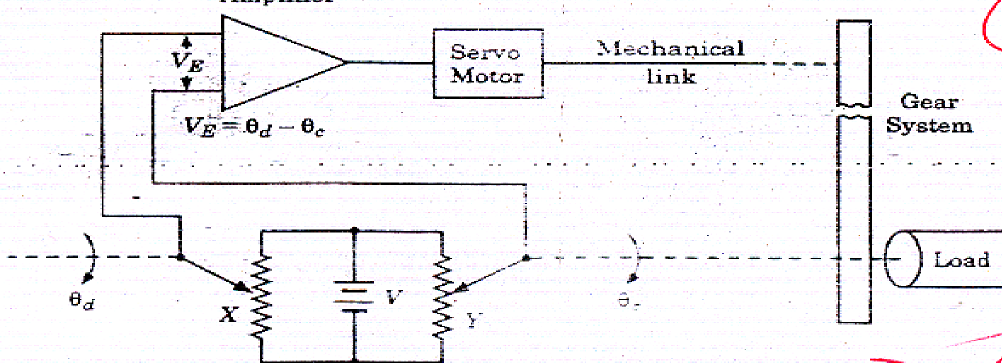
Q.2 a. Explain servomechanism (Position control system). (8)

Answer:

Servomechanism (Position Control System)

Position Control Servomechanism is a feed-back control system in which the controlled variable is a mechanical position or the time derivative of position *i.e.* $\frac{dx}{dt}, \frac{d^2x}{dt^2}$ e.g. velocity and acceleration. A system used to change the position as shown in Fig. 1.9. The position (θ_c) of load is sensed, which positions the slider arm Y of potentiometer. The desired position (θ_d) is given at arm X of potentiometer. The error voltage proportional to position ($\theta_d - \theta_c$) is amplified by an amplifier. The amplified signal is fed to the servomotor which in turn brings the load to the desired position by use of Gear System.

2



2

~~Fig. 1.10~~ Position Feed-back Control System.

Actual movements of the slides on the machine is achieved through **servo drive.** The amount of movement and the rate of movement are controlled by the Programmable system depending upon the type of system used *i.e.* closed loop or open loop feed-back system.

2

In open loop system, the programmable system sends output signal for movement but doesnot check wheather actual movement is taking place as shown in fig. 1.11 (*a*). Close loop system is characterised by Presence of feedback. In this, the programmable system sends the command of movement and the result is continuously monitored by the system through various feedback devices as shown in Fig. 1.11 (*b*). There are generally two types of feedback of the system Position Feedback and velocity feedback.

2

The examples discussed above are self-explanatory and are used in Industrial Processes. More Sophisticated and complex models are available in various Engineering and Non-Engineering Fields. In the light of above discussion the Reader is advised to identify, study and analyse systems which occur in our over day lives *e.g.* operating a toilet flush, eating, driving while Looking etc.

b. Write differential equation for mechanical translational system shown in fig.2. Also draw analogous system for this using force-voltage analogy. (8)

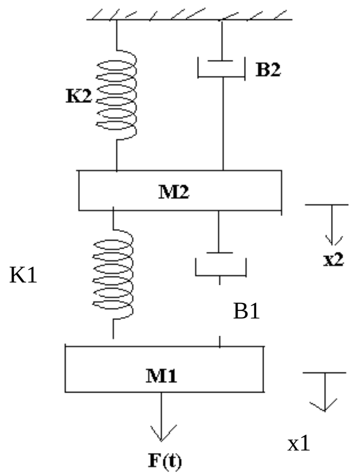
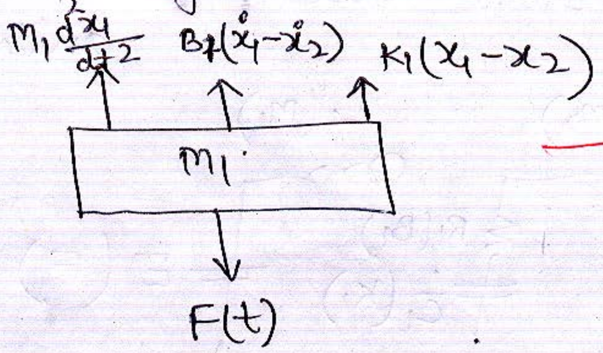


Fig.2

Answer:

11½/14

By free body diagram.

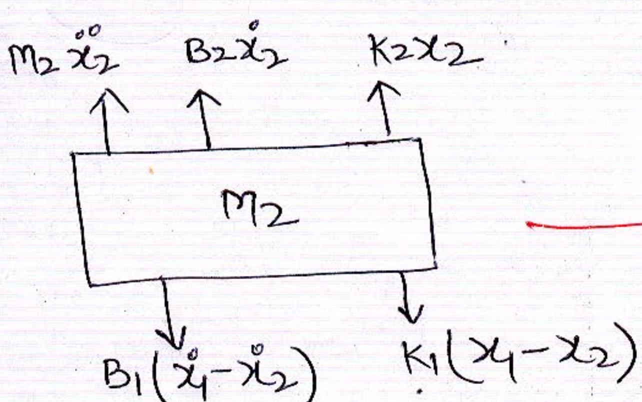


— 2

so differential equation is

$$F(t) = M_1\frac{d^2x_1}{dt^2} + B_1\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) + K_1(x_1 - x_2)$$

and



— 2

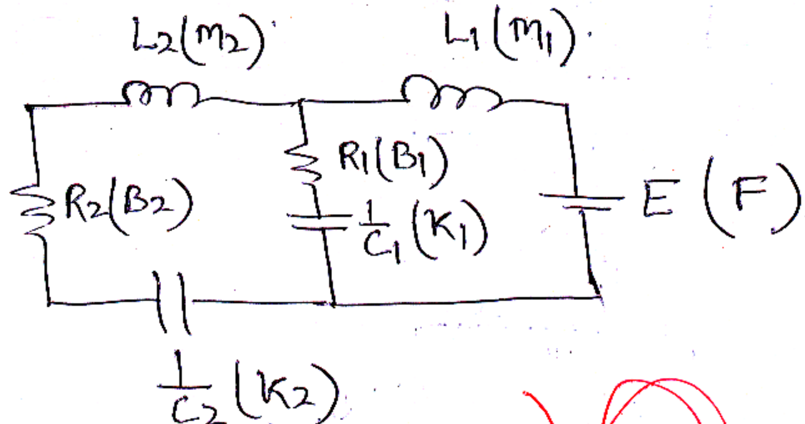
so differential equation is

$$B_1(\dot{x}_1 - \dot{x}_2) + K_1(x_1 - x_2) = M_2\ddot{x}_2 + B_2\dot{x}_2 + K_2x_2$$

or $$M_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + K_2x_2 = B_1\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) + K_1(x_1 - x_2)$$

2

Analogous circuit based on Force-Voltage analogy.



Q.3 a. Find transfer function $\frac{C(s)}{R(s)}$ of the system shown in fig.3 by block diagram reduction method. (8)

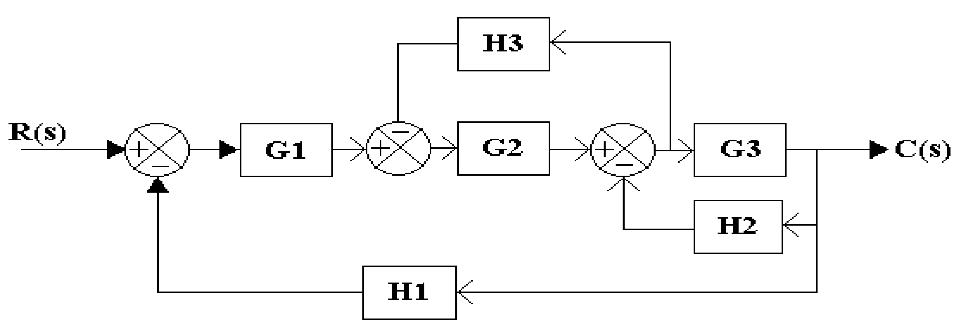
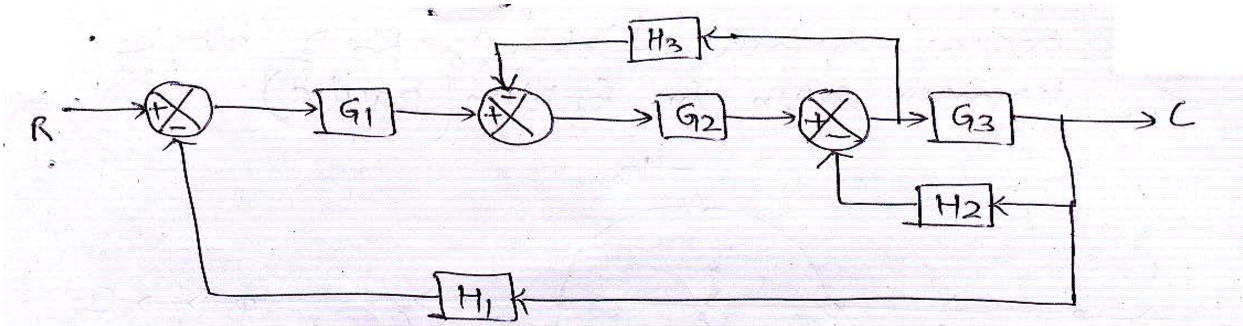
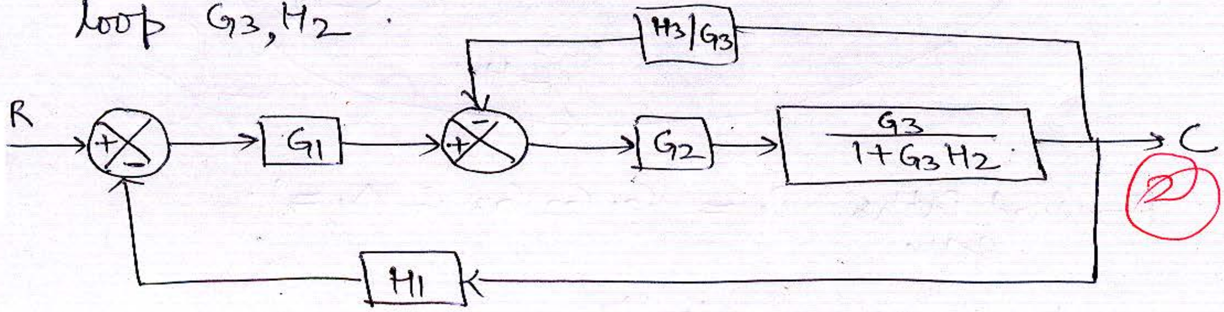


Fig.3

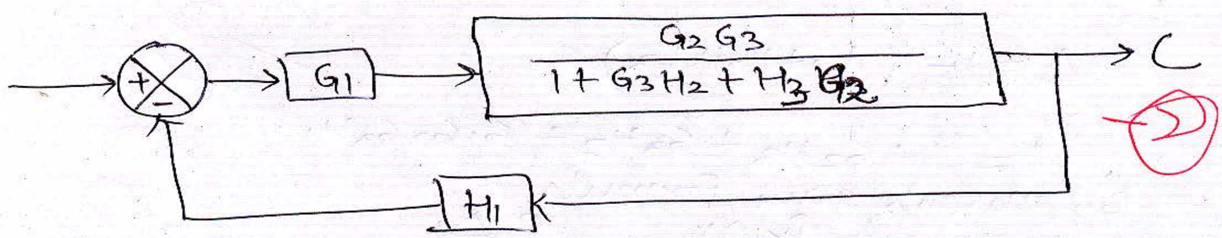
Answer:



shift take off Point after G_3 & reduced feedback loop G_3, H_2.

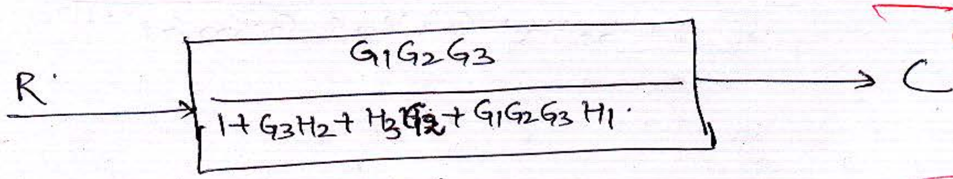


use cascading rule for G_2 & $\frac{G_3}{1+G_3H_2}$ & resolve Feed back loop



use cascading rule & resolve feedback rule.

2



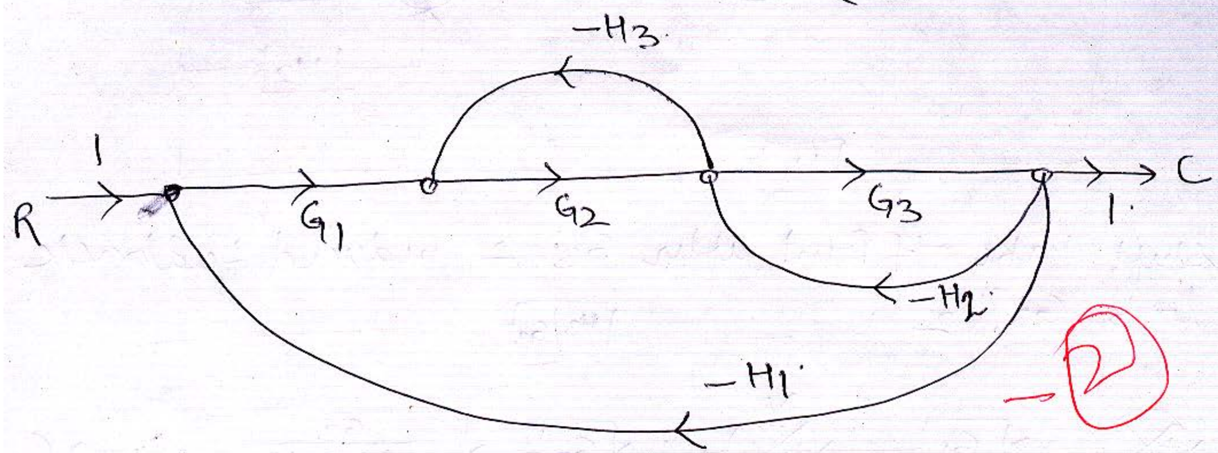
$$T.F = \frac{C}{R} = \frac{G_1G_2G_3}{1+G_3H_2+H_3G_2+G_1G_2G_3H_1}$$

2

b. Find transfer function $\frac{C(s)}{R(s)}$ of the system given in part (a) using Mason's gain formula. **(8)**

Answer:

Draw signal flow graph of block diagram given in Problem (part a).



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Forward Paths gain $P_1 = G_1 G_2 G_3$ & $\Delta_1 = 1$.

loops $L_1 = -G_2 H_3$

$L_2 = -G_3 H_2$

$L_3 = -G_1 G_2 G_3 H_1$

2

there is No Nontouching loop.

$\Delta = 1 - (L_1 + L_2 + L_3)$

2

$= 1 + G_2 H_3 + G_3 H_2 + G_1 G_2 G_3 H_1$

By Mason's gain formula

$$T.F = \frac{P_K \Delta_K}{\Delta} = \frac{P_1 \Delta_1}{\Delta}$$

2

$$= \frac{G_1 G_2 G_3}{1 + G_2 H_3 + G_3 H_2 + G_1 G_2 G_3 H_1}$$

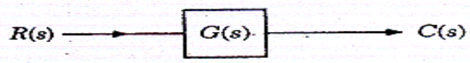
Q.4 a. Discuss effect of parameter variation in:
(i) Open loop system
(ii) Closed loop system **(8)**

Answer:

~~4.1~~ PARAMETER VARIATION IN CONTROL SYSTEM

The Parameter of a System vary in different manners (*e.g.* Environmental Condition, age, etc.) Let us study the effects in open Loop-Control System and Close-Loop Control System.

(*a*) *Effect of Parameter Variation in Open-loop Control System:* Consider an Open-loop Control System as shown in Fig. 4.1



4

Fig. 4.1

The Output of the System is given by

$$C(s) = G(s)\,R(s) \qquad \text{...(4.1)}$$

Suppose $G(s)$ changes to $[G(s) + \Delta G(s)]$ due to parameter variation, where $\Delta G(s)$ is very small. This corresponds to change in Output from $C(s)$ to $[C(s) + \Delta C(s)]$,

$$\therefore \quad C(s) + \Delta\, C(s) = [G(s) + \Delta G(s)]\; R(s)$$

Referring eq. 4.1, the above equation reduced to

$$\Delta C(s) = \Delta G(s) \cdot R(s) \qquad \text{...(4.2)}$$

The equation 4.2 gives the effect of change in Output due to parameter variation.

(*b*) *Effect of parameter variation in Close-Loop Control System:* Consider a Close-Loop Control System as shown in Fig. 4.2. The overall Transfer Function is given by

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)} \qquad \text{...(4.3)}$$

4

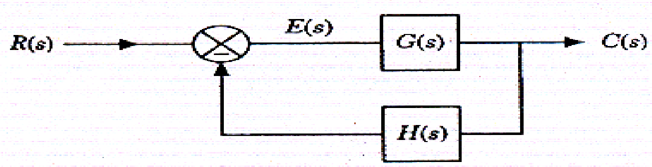


Fig. 4.2

Suppose $G(s)$ changes to $G(s) + \Delta G(s)$ due to parameter variation where $G(s) >> \Delta G(s)$. The corresponding change in Output is

$$C(s) + \Delta C(s) = \frac{[G(s) + \Delta G(s)]\, R(s)}{1 + [G(s) + \Delta G(s)]\, H(s)}$$

The term $\Delta G(s)\, H(s)$ is negligible an compared to $1 + G(s)\, H(s)$ So neglecting this term, we have

$$C(s) + \Delta\, C(s) = \frac{[G(s) + \Delta G(s)]\, R(s)}{1 + G(s)\, H(s)}$$

By use of eq. 4.3, we have

$$\Delta C(s) = \frac{\Delta\, G(s) \cdot R(s)}{1 + G(s)\, H(s)} \qquad \text{...(4.4)}$$

The eq. 4.4 gives the change in Output due to parameter variation.

From eq. 4.2 and eq. 4.4, it is observed that in a Close-Loop Control System, the change in Output due to parameter variation in $G(s)$ is reduced by a factor of $[1 + G(s)\; H(s)]$ which does not exist in an Open-Loop Control System due to absence of feed back.

b. Explain effect of feedback on disturbances in forward path of control system **(8)**

Answer:

(a) *Disturbance in Forward Path:* Let us assume that there is a Disturbance Signal $T_d(s)$ present in the Forward Path of a Control System as shown in Fig. 4.6.

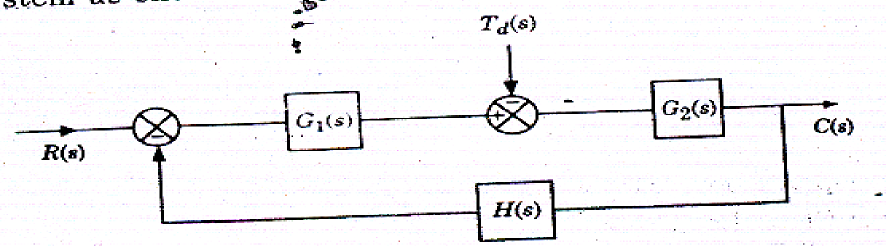


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Fig. 4.6

The ratio of Output $C(s)$ and Disturbance Signal $T_d(s)$, when $R(s) = 0$, is obtained by applying block reduction technique. The System becomes:

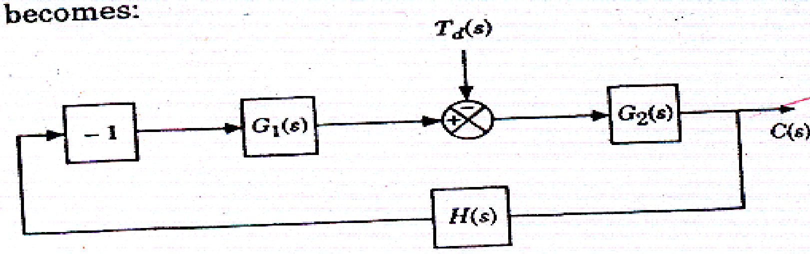

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Fig. 4.7

System have Forward Gain $G(s) = G_2(s)$
and Feed-back Gain $H'(s) = -G_1(s)\,H(s)$

$\therefore$ Ratio of $C(s)/-T_d(s)$ is $\dfrac{-G(s)}{1 - G(s)\,H'(s)}$

4

Putting $G(s)$ and $H(s)$

$$\therefore \quad \frac{C(s)}{T_d(s)} = \frac{-G_2(s)}{1 + G_1(s)\,G_2(s)\,H_1(s)}$$

or

$$C(s) = \frac{-T_d(s)\,G_2(s)}{1 + G_1(s)\,G_2(s)\,H_1(s)}$$

Assume that $G_1(s)\,G_2(s)\,H_1(s) >> 1$, hence we get

$$C(s) = \frac{-T_d(s)}{G_1(s)\,H(s)} \qquad \text{...(4.12)}$$

Therefore it is seen that effect of disturbance on the Output can be made small by selecting $G_1(s)$ as large as possible. Thus effect of disturbance can be decreased by Feed-back.

Q.5 a. Determine error constants & corresponding steady state error for a system with

$$\textbf{G(s)} = \frac{100}{s(1+2s)(1+0.01s)} \;\&\; H(s) = 1 \qquad \textbf{(8)}$$

Answer:

$$G(s)H(s) = \frac{100}{s(1+2s)(1+0.01s)}$$

$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{100}{s(1+2s)(1+0.01s)}$$

$$= \infty$$

$$e_{ss} = \frac{1}{1+K_p} = 0 \quad 2$$

$$K_v = \lim_{s\to 0} s\,G(s)H(s) = \lim_{s\to 0} \frac{100s}{s(1+2s)(1+0.01s)}$$

$$= \lim_{s\to 0} \frac{100}{(1+2s)(1+0.01s)} = 100$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{100} = 0.01 \quad 4$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s) = \lim_{s\to 0} \frac{100s}{(1+2s)(1+0.01s)}$$

$$= 0$$

$$e_{ss} = \frac{1}{K_a} = \infty \quad 2$$

b. **A unity feedback system is shown in Fig.4** **(8)**

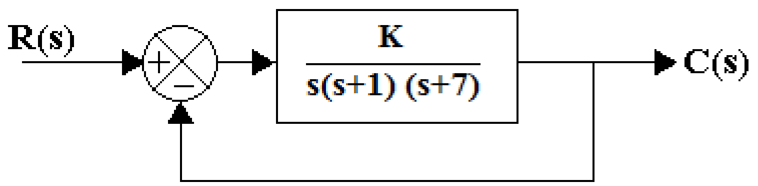


Fig.4

(i) Determine range of K for stable system
(ii) Value of K when roots of system lie on jw axis
(iii) Frequency of sustained oscillations

Answer:

$1 + G(s)H(s) = 0$

$1 + \frac{K}{s(s+1)(s+7)} = 0$

$s^3 + 8s^2 + 7s + K = 0$ — 1

by Routh's array

s^3	1	7
s^2	8	K
s^1	$\frac{56-K}{8}$	0
s^0	K	0

— 2

(i) for stable system

$K > 0$

$\frac{56-K}{8} > 0$ — 1

$\boxed{0 < K < 56}$

It is range of K for stable system

(ii) At $K = 56$ roots of system lie on $j\omega$ axis. — 1

(iii) for sustained oscillations

$8s^2 + K = 0$

$s^2 = -\frac{K}{8} = -7$ — 2

$s = \sqrt{-7} = \pm j\sqrt{7}$

$\sqrt{7}$ = rad/sec is frequency of oscillations

Q.6 **Draw root locus as k varied from 0 to ∞ for unity feedback system has an open-loop transfer function** $G(s)H(s) = \dfrac{K}{s(s+3)(s^2+2s+2)}$ **(16)**

Answer:

open loop zeros (Z) = 0.

open loop Poles (P) = 4.

$(0, -3, -1+j1, -1-j1)$.

No. of root locus branches = 4 (1)

No. of branches terminates at ∞ = No. of Asymptotes = P − Z = 4.

angle of Asymptotes $\phi_a = \dfrac{(2q+1)180^\circ}{P-Z}$ (1)

if P − Z = 4, $\phi_a = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

Centroid of Asymptotes

$$\sigma_a = \frac{\Sigma \text{ real part of Poles} - \Sigma \text{ real part of zeros}}{P-Z}$$

$$= -1.25$$ (2)

Breakaway point on real axis:

$$1 + G(s)H(s) = 0$$

$$s^4 + 5s^3 + 8s^2 + 6s + K = 0$$

so $K = -(s^4 + 5s^3 + 8s^2 + 6s)$

$$\frac{dK}{ds} = 0 = -(4s^3 + 15s^2 + 16s + 6)$$

roots of polynomial are

$s = -2.3$, $-.725 + j0.365$

& $-0.725 - j0.365$

Only $s = -2.3$ lie on root locus (2)

so it is Break-away point.

By Routh's array

$$s^4 + 5s^3 + 8s^2 + 6s + K = 0$$

$$\begin{array}{c|ccc} s^4 & 1 & 8 & K \\ s^3 & 5 & 6 & 0 \\ s^2 & \frac{34}{5} & K & 0 \\ s^1 & \left(\frac{\frac{204}{5} - 5K}{34/5}\right) & 0 & 0 \\ s^0 & K & 0 & 0 \end{array}$$

2

for Intersection on $j\omega$ axis

$$\frac{204}{5} - 5K = 0$$

$$\boxed{K = 8.16}$$

2

$$\frac{34}{5}s^2 + 8.16 = 0$$

$$s = \pm j1.1$$

angle of departure for Complex Poles

$$\phi_d = 180 - \phi_p + \phi_z$$

here $\phi_z = 0$

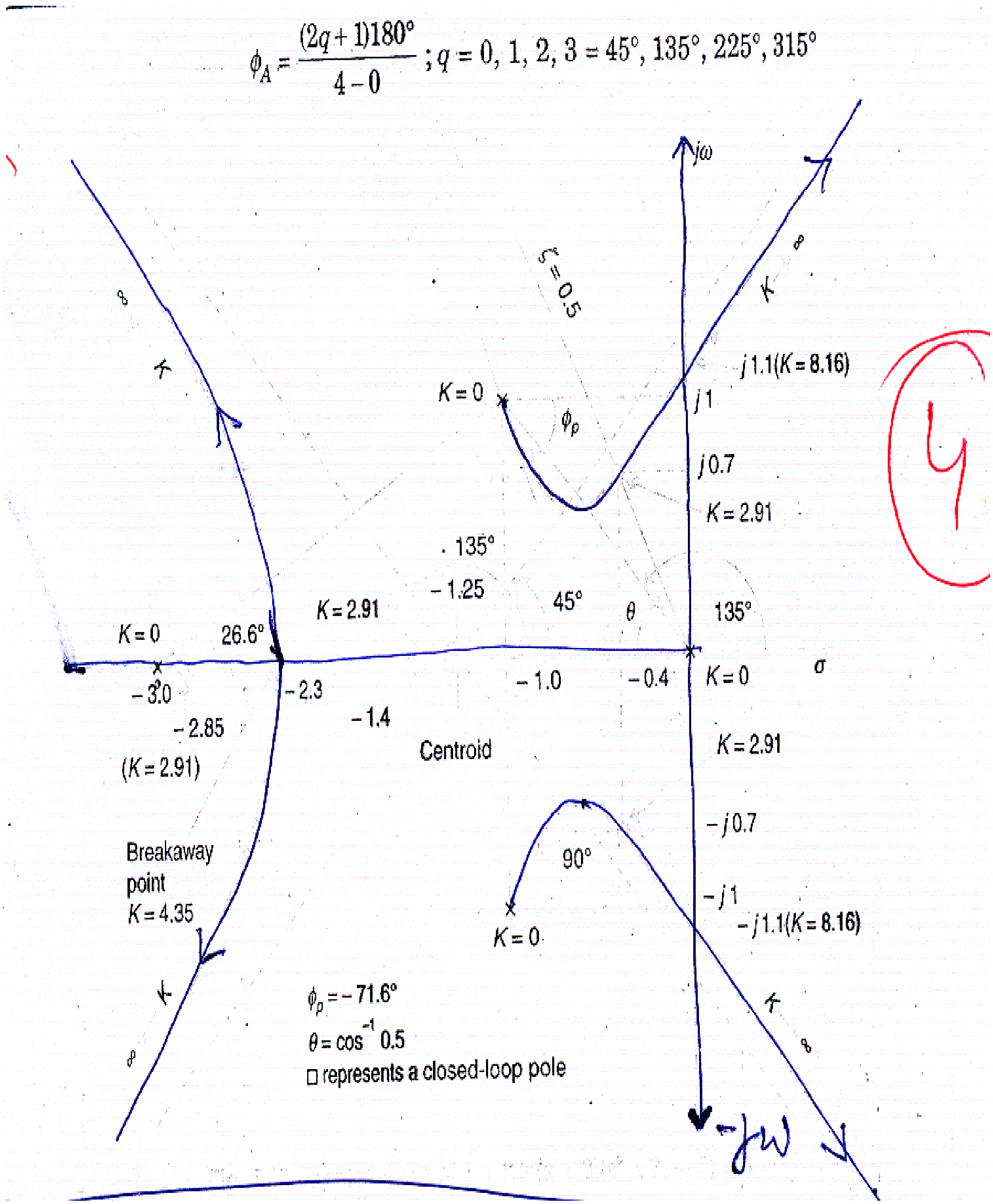
$$\phi_p = +136 + 90 + 26.6$$

$$= 251.6^\circ$$

2

$$\phi_d = 180 - 251.6$$

$$= -71.6^\circ$$



Q.7 a. For a unity feedback system $G(s) = \dfrac{800(s+2)}{s^2(s+10)(s+40)}$ **(8)**

Draw the Bode plot. Find out W_{gc}, W_{pc}, GM and PM. Comment on stability.

Answer:

For a unity feed back system $G(s) = \dfrac{800(s+2)}{s^2(s+10)(s+40)}$.

Draw the Bode Plot. Find out ω_{gc}, ω_{pc}, GM and PM. Comment on stabillity.

Solution: Step 1: $G(s) = \dfrac{800(s+2)}{s^2(s+10)(s+40)}$ and $H(s) = 1$

$$\therefore \quad G(s)\,H(s) = \frac{800(s+2)}{s^2(s+10)(s+40)} = \frac{800(2)\left(\frac{s}{2}+1\right)}{s^2(10)\left(\frac{s}{10}+1\right)(40)\left(\frac{s}{40}+1\right)}$$

Replace s by $j\omega$

$$\therefore \quad G(j\omega)H(j\omega) = \frac{4\left(1+\frac{j\omega}{2}\right)}{(j\omega)^2\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{40}\right)}$$

Step 2: The basic factors are:—

(*i*) $K = 4$

(*ii*) Two poles at origin i.e. $\dfrac{1}{(j\omega)^2}$

(*iii*) $+\dfrac{j\omega}{2}$ with $T_1 = \dfrac{1}{2}$ $\quad \therefore \quad \omega_{c_1} = \dfrac{1}{T_1} = 2$

(*iv*) $\dfrac{1}{1+\frac{j\omega}{10}}$ with $T_2 = \dfrac{1}{10}$ $\quad \therefore \quad \omega_{c_2} = \dfrac{1}{T_2} = 10$

(*v*) $\dfrac{1}{1+\frac{j\omega}{40}}$ with $T_3 = \dfrac{1}{40}$ $\quad \therefore \quad \omega_{c_3} = \dfrac{1}{40} = 40$

Step 3: Magnitude Plot Analysis: (*i*) As $K = 4$ so we have log magnitude is 20 log 4 = 12 dB, which is a straight line parallel to x-axis.

(*ii*) The two poles at origin i.e. $\dfrac{1}{(j\omega)^2}$ will contribute a straight line of slope of – 40 dB/decade passing through intersection point of $\omega = 1$ and 0 dB. The resultant of $K = 4$ and $\dfrac{1}{(j\omega)^2}$ is a straight line with slope of – 40 dB/decade passing through intersection point of $\omega = 1$ and 14 dB till first corner frequency comes..

(*iii*) Due to basic factor $1 + \frac{j\omega}{2}$ corner frequency is $\omega_{c_1} = 2$ and this factor will contribute + 20 dB/decade at $\omega = \omega_{c_1}$. Hence, resultant slope from 2 rad/sec. onwards become $-40 + 20 = -20$ rad/sec. till next corner frequency occurs.

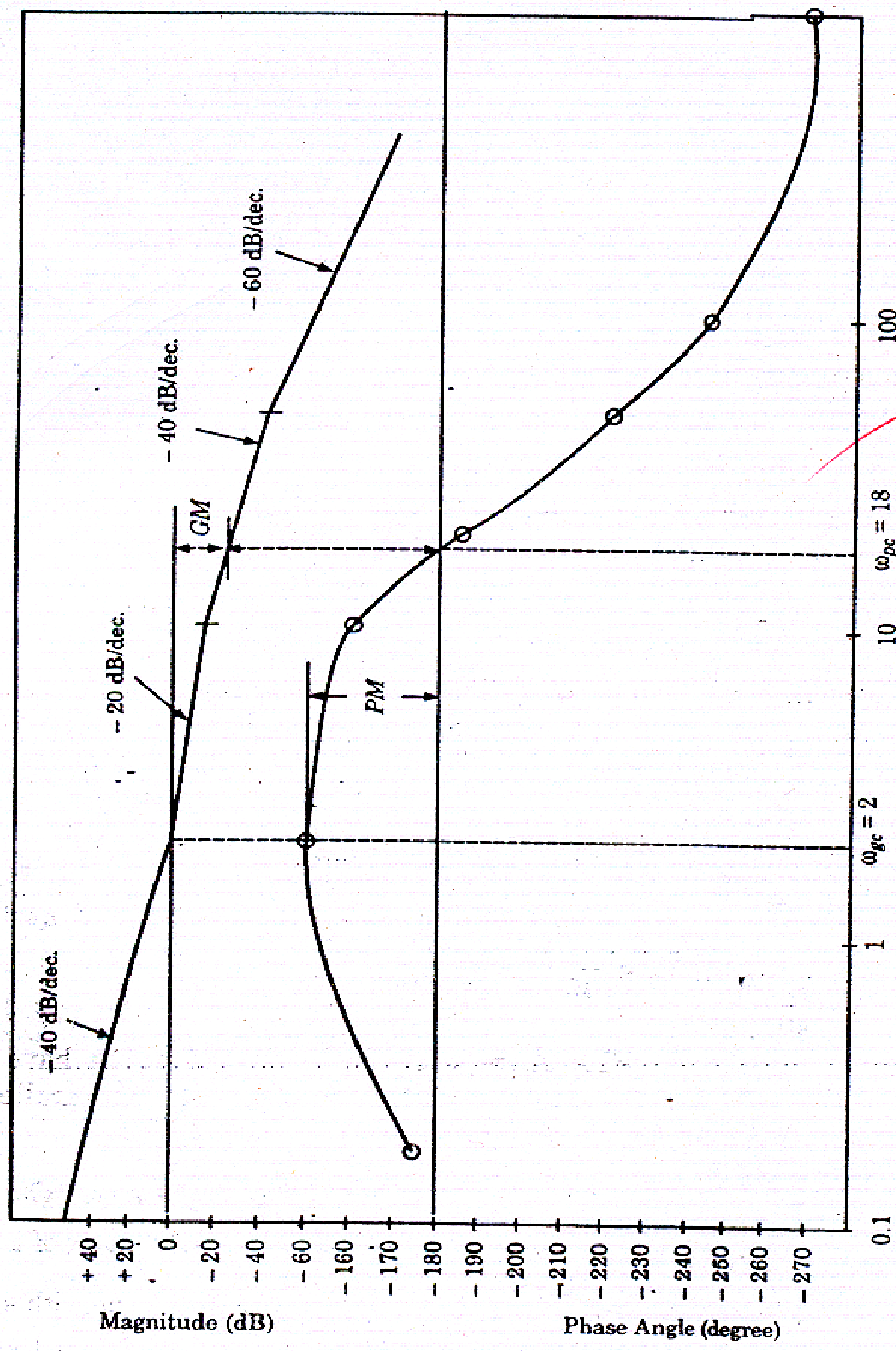


Fig. 9.49

(*iv*) The next corner frequency is $\omega_{c_2} = 10$ which is due to $\frac{1}{1+\frac{j\omega}{10}}$. This factor will contribute – 20 dB/decade individually and hence resultant slope from 10 rad/sec. onwards becomes – 20 – 20 = – 40 rad/sec. till next corner frequenjcy comes.

(*v*) Due to factor $\frac{1}{1+\frac{j\omega}{40}}$ we have corner frequency $\omega_{c_3} = 40$, which will contribute slope of – 20 dB/decade at $\omega = \omega_{c_3}$. hence, the resultant slope becomes – 40 – 20 = – 60 rad/sec. till infinity. The magnitude plot is shown in fig. 9.49.

Step 4: Phase Angle Plot: The resultant phase angle equation can be written from the individual phase angles associated with different basic factors is

$$\phi_R = -180° + \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{40}$$

The Resultant Phase angles with different increasing frequencies are

Frequency (ω)	0.2	2	10	20	50	100	∞
Phase angle (ϕ_R)	– 175.7°	– 149.16°	– 160.4°	– 185.7°	– 222°	– 243.5°	– 270°

The phase angle curve is shown in Fig. 9.49.

From the Bode Plot we have $\omega_{gc} = 2$ rad/sec, $\omega_{pc} = 18$ rad/sec., *GM* = + 24 dB and *PM* = 31°. Since *PM* and *GM* both are positive so system is STABLE.

b. Discuss the stability of system using nyquist plot for G(s)H(s) = $\frac{20}{s(s+4)(s-2)}$ **(8)**

Answer:

M = $$|G(j\omega)H(j\omega)| = \frac{20}{\omega\sqrt{\omega^2+(4)^2}\sqrt{\omega^2+(2)^2}}$$

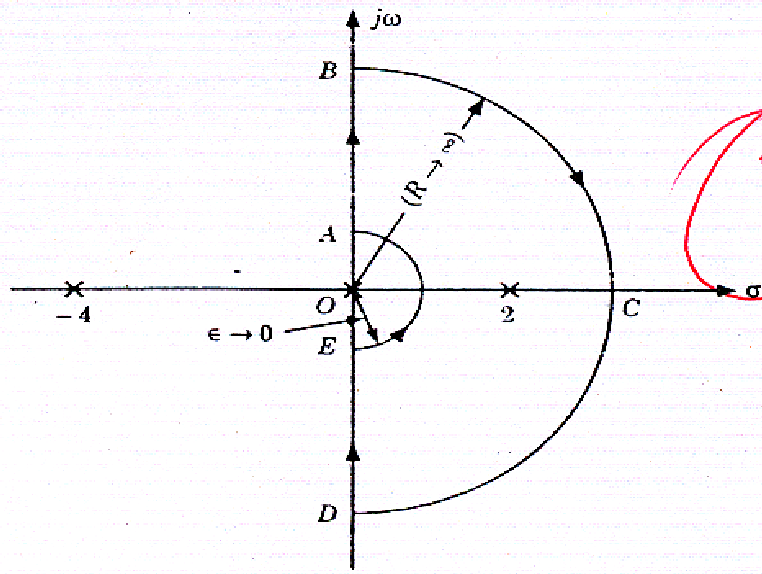
and $$\phi = -90^\circ - \tan^{-1}\frac{\omega}{4} - 180^\circ + \tan^{-1}\frac{\omega}{2}$$

$$= -270 - \tan^{-1}\frac{\omega}{4} + \tan^{-1}\frac{\omega}{2}$$

1

At $\omega = 0$, $|G(j\omega)H(j\omega)| = \infty$, $\phi = -270^\circ$

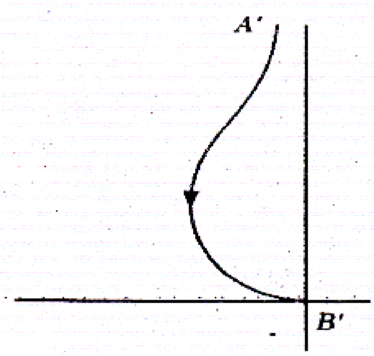
$\omega = \infty$, $|G(j\omega)H(j\omega)| = 0$, $\phi = -270^\circ$



1

Fig. 9.50

The plot for the section AB is shown in fig. 9.51 as $A'B'$



2

Fig. 9.51

(*ii*) Section *BCD*: For this section, we have $s = Re^{j\theta}$

where R approaches to infinity

θ varies from $+90^\circ$ to -90° through 0°.

$$\therefore \quad G(s)H(s) = \frac{20}{Re^{j\theta}(Re^{j\theta}+4)(Re^{j\theta}-2)} = \frac{20}{R^3e^{j3\theta}} \quad (\text{As } R \to \infty)$$

$$= 0e^{-j3\theta}$$

Therefore, the BCD maps into a point $B'C'D'$ as the radius approches to zero.

(iii) Section DE: For this section, $s = -j\omega$. So the mapping of section DE is mirror image of section AB.

(iv) Section EFA: For section EFA, we have $s = \varepsilon e^{j\theta}$

where ε approaches to zero

θ varies from $-90°$ to $+90°$ through $0°$

$$\therefore \quad G(s)H(s) = \frac{20}{\varepsilon e^{j\theta}(\varepsilon e^{j\theta}+4)(\varepsilon e^{j\theta}-2)} = \frac{20}{\varepsilon e^{j\theta}(4)(-2)} \quad (\text{As } \varepsilon \to 0)$$

$$= \infty \; e^{-j(\theta + 180°)}$$

So mapping of this part is $E'F'A'$ with semicircle of radius approach to infinity which varies from $-90°$ to $-270°$ through $-180°$ as shown in fig. 9.52.

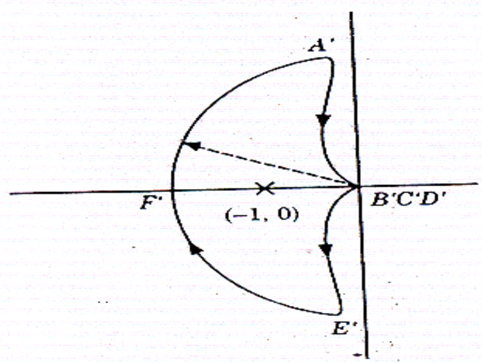


Fig. 9.52

Step 4: The complete Nyquist plot is shown in fig. 9.52.

Step 5: From Nyquist Plot it is observed that critical point $(-1, 0)$ is encircle once in clockwise direction. Therefore, $N = -1$. From the given transfer function $G(s)H(s)$ it is clear that one pole lie on RHS of S-plane. Therefore $P = 1$.

We know that $N = P - Z$

$$\therefore \quad Z = P - N = 1 - (-1) = 2$$
$$\neq 0$$

Since $Z \neq 0$, so system is UNSTABLE.

Q.8 **A system has open loop transfer function G(s) = $\frac{4}{s(2s+1)}$. It is desired to have the phase margin as 40°. Design a lead compensator to meet desired specifications using Bode plot.** **(16)**

Answer:

A system has open loop transfer function $G(s) = \frac{4}{s(2s+1)}$. It is desired to have the phase margin as 40°. Design a lead compensator to meet desire specification.

Solution: **Step 1:** As the value of $K = 4$ $\therefore$ we have

$$G(s) = \frac{4}{s(2s+1)}$$

Step 2: Sketch the Bode Plot of $G(j\omega) = \dfrac{4}{j\omega(2j\omega+1)}$ which is shown with dotted line in Fig. 10.8. The uncompensated system has a phase margin of 20° and gain cross over frequency of ω_g = 1.4 rad/sec.

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Step 3: Additional lead required is given as

$$\phi_m = \phi_S - \phi_P + \in$$

we have $\phi_S = 40°$, $\phi_P = 20°$ and $\in = 5°$ (Assume)

$$\therefore \quad \phi_m = 40° - 20 + 5° = 25°$$

Step 4: α Parameter of lead compensator is;

2

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m} = \frac{1-\sin 25°}{1+\sin 25°}$$

$$= 0.406$$

Step 5: The uncompensated system has gain of $-20 \log \dfrac{1}{\sqrt{\alpha}}$

2

$$= -10 \log\frac{1}{\alpha} = -10 \log\left(\frac{1}{0.406}\right)$$

$$= -4 \text{ dB}$$

Refer Fig.10.8 and find frequency corresponding to – 4 dB. This is ω_g' = 1.75 rad/sec. The difference in *PM* of $G(j\omega)$ at 1.4 and 1.75 rad/sec. is less than 5°. So go to next step.

Step 6: Set $\omega_g' = \omega_m \;\therefore\; \omega_m$ = 1.75 rad/sec. Now, we obtain the *T* parameter of lead compensator ;

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \quad \therefore \frac{1}{T} = 1.12$$

Step 7: Two corner frequencies are

$$\omega_{C1} = \frac{1}{T} = 1.12 \quad \text{or} \quad T = 0.90$$

4

$$\omega_{C2} = \frac{1}{\alpha T} = 2.8 \quad \text{or} \quad \alpha T = 0.36$$

$\therefore$ The lead compensator is

$$G_C(s) = \frac{1+0.90s}{1+0.36s}$$

and compensated open-loop transfer function is

$$G(s)\,G_C(s) = \frac{4(1+0.90s)}{s(2s+1)(1+0.36s)}$$

Sketch the bode plot of compensated transfer function which is shown in Fig. 10.8. From the diagram we observe that phase margin is increased from 20° to 41°. Then the compensator satisfies the desirability.

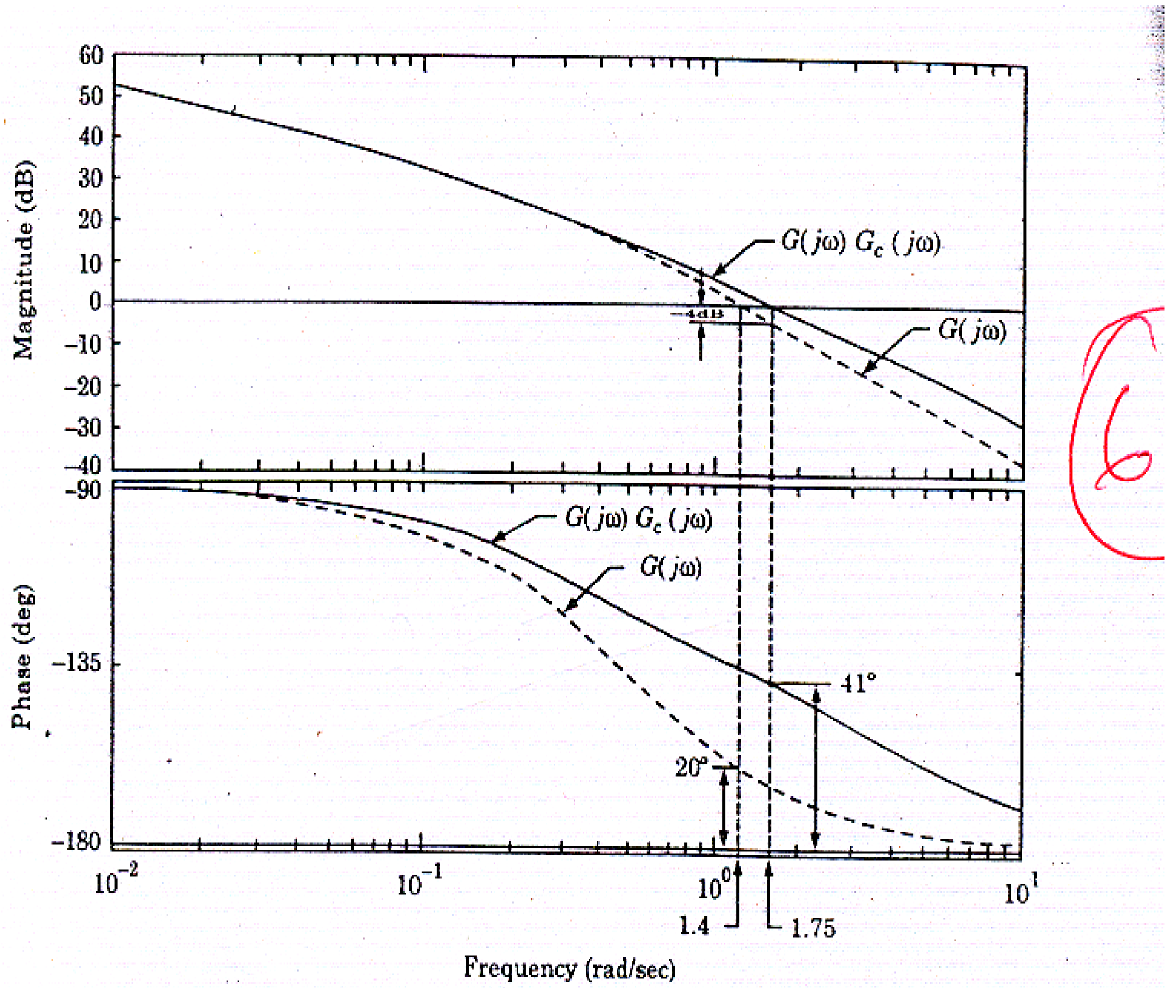

Fig. 10.8

Q.9 a. Explain direct method of Liapunov for linear system. (8)

Answer:

In case of linear systems, the direct method of Liapunov provides a simple approach to stability analysis. It must be emphasized here that compared to the results presented in Chapter 6, no new results are obtained by the use of the direct method for the stability analysis of linear systems. However, the study of linear systems using the direct method is quite useful because it extends our thinking to nonlinear systems.

2

Consider a linear autonomous system described by the state equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \quad \text{...(13.9)}$$

The linear system is asymptotically stable in-the-large at the origin if and only if given any symmetric, positive definite matrix **Q** (see Appendix II), there exists a symmetric positive definite matrix **P** which is the unique solution

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q} \quad \text{...(13.10)}$$

2

Proof

To prove the sufficiency of the reult of above theorem, let us assume that a symmetric positive definite matrix **P** exists which is the unique solution of eqn. (13.11). Consider the scalar function (Appendix II),

$$V(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}$$

Note that $V(\mathbf{x}) > \mathbf{0}$ for $\mathbf{x} \neq \mathbf{0}$

and $V(\mathbf{0}) = \mathbf{0}$

The time derivative of $V(\mathbf{x})$ is

$$\dot{V}(\mathbf{x}) = \dot{\mathbf{x}}^T\mathbf{P}\mathbf{x} + \mathbf{x}^T\mathbf{P}\dot{\mathbf{x}}$$

Using eqns. (13.9) and (13.10) we get

$$\dot{V}(\mathbf{x}) = \mathbf{x}^T\mathbf{A}^T\mathbf{P}\mathbf{x} + \mathbf{x}^T\mathbf{P}\mathbf{A}\mathbf{x}$$

2

$$= \mathbf{x}^T(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A})\mathbf{x}$$

$$= -\mathbf{x}^T\mathbf{Q}\mathbf{x}$$

Since **Q** is positive definite, $\dot{V}(\mathbf{x})$ is negative definite. Norm of **x** may be defined as (Appendix II)

$$\|\mathbf{x}\| = (\mathbf{x}^T\mathbf{P}\mathbf{x})^{1/2}$$

Then $V(\mathbf{x}) = \|\mathbf{x}\|^2$

$$V(\mathbf{x}) \to \infty \text{ as } \|\mathbf{x}\| \to \infty$$

The system is therefore asymptotically stable in-the large at the origin (refer Theorem 3).

In order to show that the result is also necessary, suppose that the system is asymptotically stable and **P** is negative definite, consider the scalar function

$$V(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x} \quad \text{...(13.11)}$$

2

Therefore $\dot{V}(x) = -[\dot{\mathbf{x}}^T\mathbf{P}\mathbf{x} + \mathbf{x}^T\mathbf{P}\dot{\mathbf{x}}]$

$$= \mathbf{x}^T\mathbf{Q}\mathbf{x}$$

$$> 0$$

There is contradiction since $V(\mathbf{x})$ given by eqn. (13.11) satisfies instability theorem (refer Theorem 4)

Thus the conditions for the positive definiteness of **P** are necessary and sufficient for asymptotic stability of the system of eqn. (13.9).

b. Obtain state space model of electric network shown in Fig.5. (8)

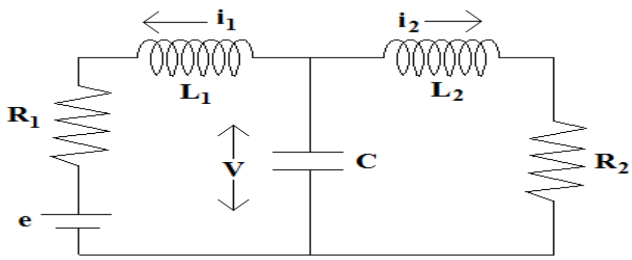

Fig.5

Assume Voltage & Current in R_2 as Output Variables.

Answer:

Let $v = x_1$ & $\dot{x}_1 = \frac{dv}{dt}$

$i_1 = x_2$ & $\dot{x}_2 = \frac{di_1}{dt}$ 1

& $i_2 = x_3$ & $\dot{x}_3 = \frac{di_2}{dt}$

By KVL & KCL in electrical network.

$$i_1 + i_2 + C\frac{dv}{dt} = 0$$

$$L_1\frac{di_1}{dt} + R_1 i_1 + e - v = 0$$ 1

$$L_2\frac{di_2}{dt} + R_2 i_2 - v = 0$$

so we get

$$\frac{dv}{dt} = -\frac{1}{C}i_1 - \frac{1}{C}i_2.$$

or $\dot{x}_1 = -\frac{1}{C}x_2 - \frac{1}{C}x_3.$

$$\frac{di_1}{dt} = \frac{1}{L_1}v - \frac{R_1}{L_1}i_1 - \frac{1}{L_1}e$$ 2

$(e = u)$.

or $\dot{x}_2 = \frac{1}{L_1}x_1 - \frac{R_1}{L_1}x_2 - \frac{1}{L_1}u$

$$\frac{di_2}{dt} = \frac{1}{L_2}v - \frac{R_2}{L_2}i_2$$

$$\dot{x}_3 = \frac{1}{L_2}x_1 - \frac{R_2}{L_2}x_3.$$ 2

. we get

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} & -\frac{1}{C} \\ -\frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} v \\ i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{L_1} \\ 0 \end{bmatrix} u$$

Let $y_1 = i_2 R_2 = R_2 x_3$

& $y_2 = i_2 = x_3$.

So output equation

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & R_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

2

TEXT-BOOK

I. Control Systems Engineering, I.J. Nagrath and M. Gopal, Fifth Edition (2007 New Age International Pvt. Ltd