Q.2 a. Explain the following terms
(i) Graph of a network
(ii) tree of a graph. **(5)**

Answer:

Q.2(a) The diagram drawn such that each passive element irrespective of its type and value is simply represented by a line with small circle at the ends, the active sources are assumed to be zero is known as graph of a network. 3 marks

A tree of a network graph is the part of a network graph in which all the junction nodes are present but no closed loops are present. 2 marks

b. Using source transformation technique find the equivalent voltage source between the points A and B for the network as shown in Fig.2 **(4)**

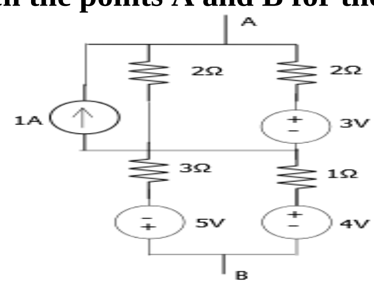


Fig.2

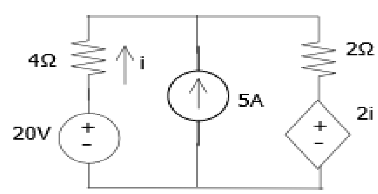
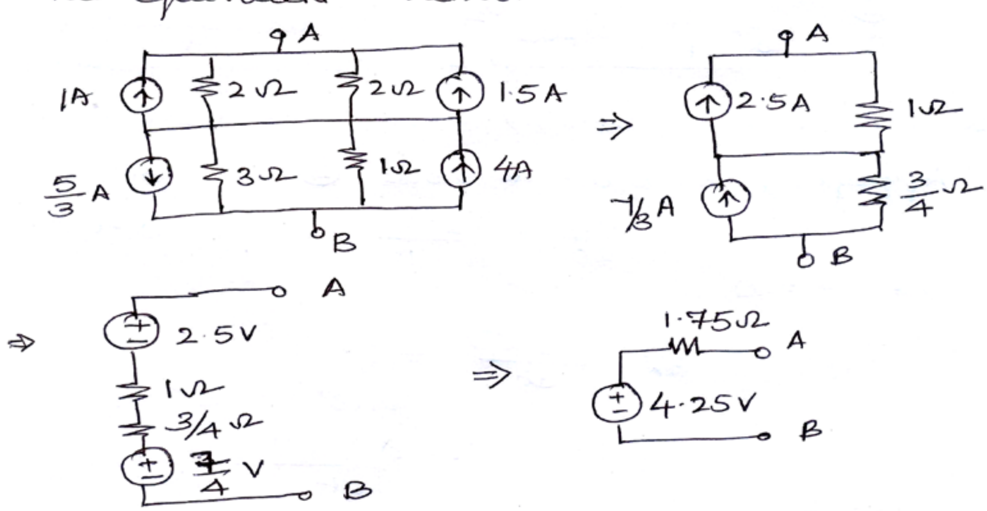


Fig.3

Answer:

(b) Converting voltage sources into current sources the equivalent network is as shown below

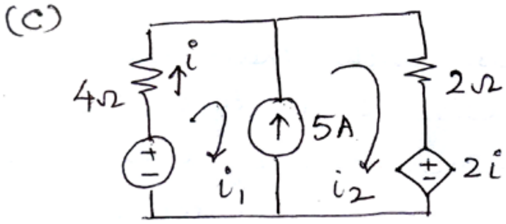


4 marks (each step 1 mark)

c. **Determine the current 'i' using mesh analysis for the network as shown in Fig.3** **(7)**

Answer:

(c)



Due to ideal Current source between the loops the Super-mesh equation is

$$20 - 4i_1 - 2i_2 - 2i = 0 \quad -(1) \quad \ldots\ldots 2 \text{ Marks}$$

$$i = i_1 \quad \text{and} \quad i_2 - i_1 = 5 \quad -(2) \quad \ldots\ldots 2 \text{ Marks}$$

Substituting (2) in (1), we get

$$20 - 4i - 2(5+i) - 2i = 0 \ldots\ldots 1 \text{ Mark}$$

$$10 - 8i = 0$$

$$i = \frac{10}{8} = 1.25\text{ A} \quad \Big\} \ 2 \text{ Marks}$$

Q.3 a. In a network shown in Fig.4, $v_1(t) = e^{-t} \quad for \quad t \geq 0$ **and is zero for all t<0. If the capacitor is initially uncharged, determine the value of** $\frac{d^2 v_2}{dt^2} \quad at \quad t = 0^+$ **(10)**

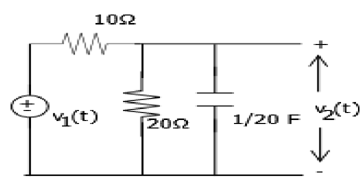
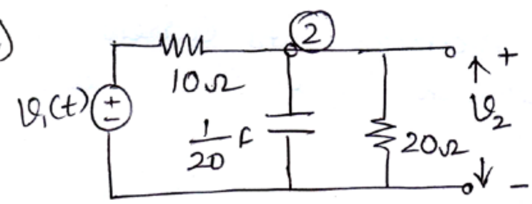


Fig.4

Answer:

Q.3 a)



For $t<0$, $v_1(t) = 0$, ∴ the Capacitor is initially uncharged   1 Mark

and hence $v_C(0^-) = v_C(0^+) = v_2(0^+) = 0$

Applying KCL at node ②, we get

$$\frac{v_2}{20} + \frac{v_2 - v_1}{10} + C\frac{dv_2}{dt} = 0 \quad -(1) \ \ldots\ 2 \text{ Mark}$$

at $t = 0^+$

$$-\frac{v_1(0^+)}{10} + \frac{1}{20}\frac{dv_2(0^+)}{dt} = 0 \quad \ldots\ldots\ldots \ 1 \text{ Mark}$$

$$v_1(0^+) = e^{-0} = 1 \ \ldots\ 1 \text{ mark}$$

$$\therefore \frac{dv_2(0^+)}{dt} = \frac{1}{10} \times 20 = 2V/sec \text{ -- 1 mark}$$

Differentiating equation (1) with respective 't',

$$\frac{dv_2}{dt}\left[\frac{1}{20}+\frac{1}{10}\right] - \frac{1}{10}\frac{dv_2(t)}{dt} + \frac{1}{20}\frac{d^2v_2(t)}{dt^2} = 0 \quad \text{.... 1 Mark} \quad (2)$$

at $t = 0^+$

$$\frac{dv_2(0^+)}{dt}\left[\frac{1}{20}+\frac{1}{10}\right] - \frac{1}{10}\frac{dv_1(0^+)}{dt} + \frac{1}{20}\frac{d^2v_2(0^+)}{dt^2} = 0$$

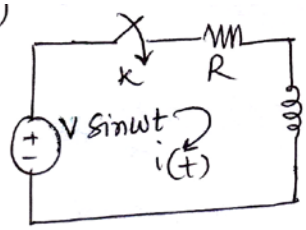
$$\left.\begin{aligned} v_1(t) = e^{-t} \quad \therefore \frac{dv_1(t)}{dt} &= -e^{-t} \\ \frac{dv_1(0^+)}{dt} &= -e^{0} = -1 \end{aligned}\right\} \text{1 Mark}$$

$$\therefore \frac{d^2v_2(0^+)}{dt^2} = -\frac{4}{10} \times 20 = -8V/sec^2 \text{ -- 2 Mark}$$

b. **A series RL circuit is driven by a sinusoidal voltage source $V\sin\omega t$. Find the expression for current by solving differential equation.** **(6)**

Answer:

(b)



The differential equation by applying KVL

$$L\frac{di}{dt} + Ri = V\sin\omega t \text{ — 1Mark}$$

The Complementary function is obtained by considering zero on the right-hand side of the differential equation

$$\left.\begin{aligned} L\frac{di}{dt} + Ri &= 0 \\ s + R/L &= 0 \\ s &= -R/L \\ \therefore i_c &= Ke^{-R/L\,t} \end{aligned}\right\} \text{2 Marks}$$

The particular integral depends on source since the given i/p is sine, i_p is also sine wave

$$i_p = \frac{V}{\sqrt{R^2+\omega^2L^2}}\sin\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right) \text{ -- (2 marks)}$$

$$i = i_c + i_p = K e^{-R/L\, t} + \frac{V}{\sqrt{R^2+\omega^2L^2}} \sin\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

Sub $t = 0$, $i = 0$, Substituting in the above equation, we get

$$\frac{V}{\sqrt{R^2+\omega^2L^2}} \sin\left(0 - \tan^{-1}\frac{\omega L}{R}\right) + K = 0$$

$$\therefore K = \frac{V}{\sqrt{R^2+\omega^2L^2}} \sin\left(\tan^{-1}\frac{\omega L}{R}\right) \quad \text{1 Mark}$$

Q.4 a. Obtain the Laplace transform of the function $e^{-at}\sin\omega t$ from the definition of Laplace transform. (4)

Answer:

Q.4 (a)

$$X(s) = \int_{0^-}^{\infty} x(t)\, e^{-st}\, dt = \int_0^{\infty} e^{-at} \sin\omega t\, e^{-st}\, dt \quad \text{-- 1 Mark}$$

$$= \int_0^{\infty} \frac{e^{j\omega t} - e^{-j\omega t}}{2j}\, e^{-(a+s)t}\, dt \quad \text{------ 1 Mark}$$

$$= \frac{1}{2j}\int_0^{\infty}\left[e^{-(a+s-j\omega)t} - e^{-(a+s+j\omega)t}\right] dt$$

$$= \frac{1}{2j}\left[\frac{1}{s+a-j\omega} - \frac{1}{s+a+j\omega}\right]$$

$$= \frac{1}{2j}\,\frac{[\cancel{s}+\cancel{a}+j\omega - \cancel{s}-\cancel{a}+j\omega]}{(s+a)^2+\omega^2}$$

$$= \frac{\omega}{(s+a)^2+\omega^2}$$

2 marks

b. Using partial fraction expansion find the inverse Laplace transform of

$$F(s) = \frac{s}{(s+1)^2(s+3)} \quad \textbf{(6)}$$

Answer:

(b)

$$F(s) = \frac{A}{(s+1)^2} + \frac{B}{(s+1)} + \frac{C}{(s+3)} \quad \text{-------- 1 Mark}$$

$$A = (s+1)^2 \cdot F(s) = \cancel{(s+1)^2} \frac{s}{\cancel{(s+1)^2}(s+3)}\Bigg|_{s=-1} = \frac{-1}{-1+3} = -\frac{1}{2} \quad \text{- 1 Mark}$$

$$B = \left[\frac{d}{ds}(s+1)^2 F(s)\right] = \frac{d}{ds}\left[\frac{s}{s+3}\right] = \frac{3}{(s+3)^2}\Bigg|_{s=-1} = \frac{3}{(-1+3)^2} = 3/4 \quad \text{2 Marks}$$

$$C = (s+3)F(s) = \cancel{(s+3)} \frac{s}{(s+1)^2\cancel{(s+3)}}\Bigg|_{s=-3} = \frac{-3}{(-3+1)^2} = -3/4 \quad \text{1 Mark}$$

$$F(s) = \frac{-1/2}{(s+1)^2} + \frac{3/4}{(s+1)} - \frac{3/4}{(s+3)}$$

taking inverse Laplace transform

$$f(t) = -\tfrac{1}{2} t e^{-t} + \frac{3}{4} e^{-t} - \frac{3}{4} e^{-3t} \quad \text{2 Marks}$$

c. For the waveform shown in Fig.5, find the Laplace transform of the signal. (6)

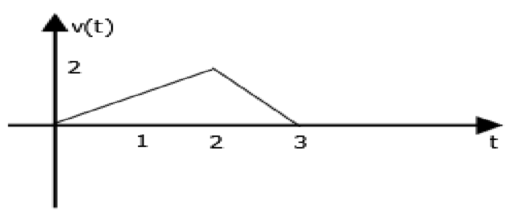
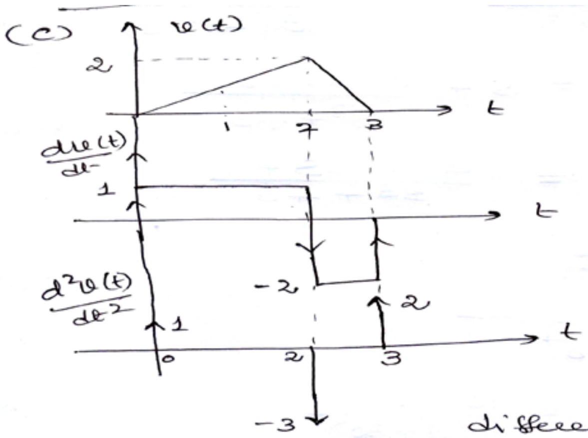


Fig.5

Answer:

(c)



waveforms — 2 Marks

The given waveform can be mathematically represented as

$$v(t) = \begin{cases} t, & 0 \le t \le 2 \\ 6-2t, & 2 \le t \le 3 \end{cases} \quad \text{2 Marks}$$

differentiating v(t) we get

$$\frac{dv(t)}{dt} = \begin{cases} 1, & 0 \le t \le 2 \\ -2, & 2 \le t \le 3 \end{cases} \quad \text{- 1 Mark}$$

differentiating once again

we get

$$\frac{d^2 v(t)}{dt} = \delta(t) - 3\delta(t-2) + 2\delta(t-3)$$

Taking Laplace transform on bothsides

$$S^2 V(s) = 1 - 3e^{-2s} + 2e^{-3s} \quad \text{or} \quad V(s) = \frac{1-3e^{-2s}+2e^{-3s}}{s^2}$$ 1 Mark

Q.5 a. For the LC network shown in Fig.6, find the transform impedance Z(s). **(7)**

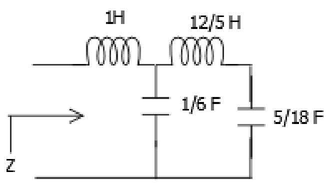


Fig.6

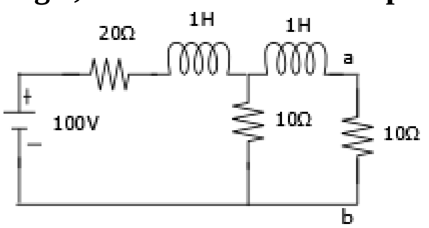
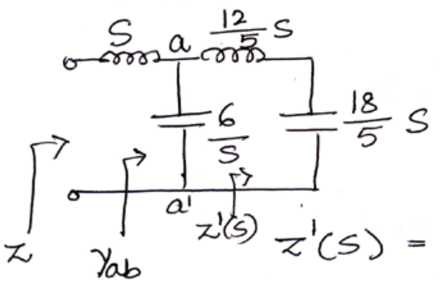


Fig.7

Answer:

Q.5 a)



———— 2 Marks

$$z'(s) = \frac{12}{5}s + \frac{18}{5}s = \frac{12s^2+8}{5s}$$ 1 Mark

$$Y_{ab}(s) = \frac{1}{z'(s)} + \frac{s}{6} = \frac{5s}{12s^2+8} + \frac{s}{6}$$

$$= \frac{5s + s(12s^2+3)}{6(2s^2+3)} = \frac{s^3+4s}{3(2s^2+3)}$$ – 2 Marks

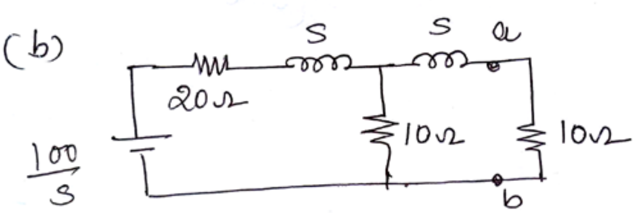
$$Z(s) = s + \frac{1}{Y_{ab}(s)} = s + \frac{3(2s^2+3)}{s^3+4s}$$

$$= \frac{s^4+10s^2+9}{s^3+4s}$$ —— 2 Marks

b. In the network shown in Fig.7, find the voltage across R_L=10Ω using Thevinin's theorem. **(9)**

Answer:

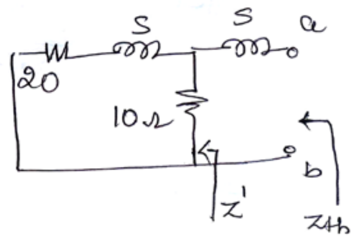
(b)



– – – – – – 1 Mark

$$V_{th} = \frac{\frac{100}{S} \times 10}{30+S} = \frac{10^4}{S(S+30)} \quad \text{———— 1 Mark}$$

$$Z_{th} = Z' + S$$



$$Y' = \frac{1}{Z'} = \frac{1}{10} + \frac{1}{20+S}$$

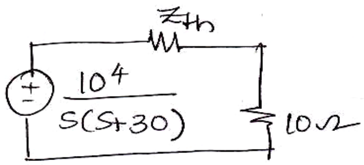
$$Y' = \frac{30+S}{10(20+S)}$$

$$Z' = \frac{10(20+S)}{30+S} \quad \& \quad Z_{th} = S + \frac{10(20+S)}{30+S}$$

(4)

$$= \frac{S^2+40S+200}{30+S}$$

4 Marks



$$V_L = \frac{\frac{10^4}{S(S+30)} \times 10}{10 + \frac{S^2+40S+200}{(30+S)}}$$

$$V_L = \frac{10^5}{10S[S+30] + S[S^2+40S+200]}$$

$$= \frac{10^5}{S^3 + 50S^2 + 500S}$$

3 Mark

Q.6 a. Explain the voltage and admittance transfer functions for a two port network.(4)

Answer:

6 a) Voltage transfer function

The voltage transfer function is the ratio of the Laplace transform of the voltage at one port to the Laplace transform of the voltage at other port, neglecting the initial conditions.

$$G_{12}(S) = \frac{V_1(S)}{V_2(S)}, \text{ inverse voltage transfer function}$$

$$G_{21}(S) = \frac{V_2(S)}{V_1(S)}$$

Transfer admittance function

It is the ratio of Laplace transform of the current at one port to the Laplace transform of the voltage at other port, neglecting the initial conditions.

$$Y_{12}(S) = \frac{I_1(S)}{V_2(S)} \quad \& \quad Y_{21}(S) = \frac{I_2(S)}{V_1(S)}$$

b. Determine the voltage transfer function and driving point impedance of the network shown in Fig.8 **(5)**

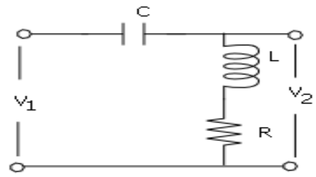
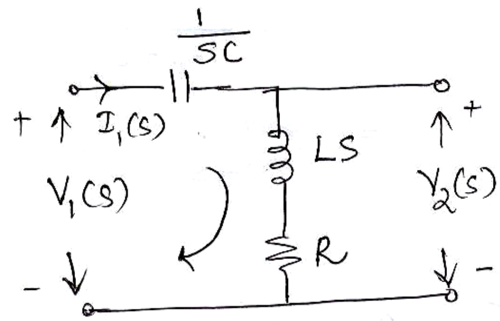


Fig.8

Answer:

6(b)



1 Mark

By KVL, we can write

$$\frac{1}{SC} I_1(S) + (SL+R) I_1(S) - V_1(S) = 0$$

Or $$V_1(S) = \left[R + SL + \frac{1}{SC}\right] I_1(S)$$

$$V_2(S) = (R+SL)\ I_1(S)$$

2 Marks

$$G_{21}(s) = \frac{V_2(S)}{V_1(S)} = \frac{R+SL}{R+SL+\frac{1}{SC}}$$ ——— 1 Mark

$$Z_{11}(S) = \frac{V_1(S)}{I_1(S)} = R + SL + \frac{1}{SC}$$ ——— 1 Mark

c. Find the range of k in $F(s)$ so that $F(s) = 2s^4 + s^3 + ks^2 + s + 2$ is Hurwitz. **(7)**

Answer:

6(c)

$$M(S) = 2S^4 + KS^2 + 2$$

$$N(S) = S^3 + S$$

——— 1 Mark

$$
\begin{array}{l}
s^3+s)\; 2s^4+Ks^2+2\; (2s \\
\quad\; 2s^4+2s^2 \\
\hline
(K-2)s^2+2)\; s^3+s\; \left(\frac{s}{K-2}\right. \\
\qquad\qquad\quad s^3+\frac{2s}{K-2} \\
\hline
\left(\frac{K-4}{K-2}\right)s\;)\;(K-2)s^2+2\;\left(\frac{(K-2)^2}{(K-4)}s\right. \\
\qquad\qquad\quad (K-2)s^2 \\
\hline
\qquad 2)\;\frac{(K-4)}{(K-2)}s\;\left(\frac{K-4}{K-2}\times\frac{s}{2}\right. \\
\qquad\quad\;\; \frac{(K-4)}{(K-2)}s \\
\hline
\end{array}
$$

4 Marks

For Hurwitz, all the quotients of continued fraction expansion must be positive

$\therefore$ $K-2>2$ or $K>2$

2 Marks

$K-4>0$ or $K>4$

$\therefore$ K must be greater than 4

Q.7 a. Express the h-parameters in terms of Z-parameters. (7)

Answer:

Q.7 a) Network equations interms of Z-parameters given by

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad -(1)$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad -(2)$$

1 Mark

N/w eqns. interms of h-parameters given by

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad -(3)$$
$$I_2 = h_{21}I_1 + h_{22}V_2 \quad -(4)$$

1 Mark

To compare equations (1) & (2) with (3) & (4) rearrange eqns. (1) & (2) interms of V_1 & I_2

from (2)

$$I_2 = \frac{V_2}{Z_{22}} - \frac{Z_{21} I_1}{Z_{22}} \quad -(5) \quad \ldots\ldots \quad 1 \text{ Mark}$$

Substituting (5) in (1)

$$V_1 = Z_{11} I_1 + Z_{12}\left[\frac{V_2}{Z_{22}} - \frac{Z_{21}}{Z_{22}} I_1\right]$$

$$V_1 = \left[Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}}\right] I_1 + \frac{Z_{12}}{Z_{22}} V_2 \quad -(6)$$

2 marks

Comparing (4) and (5)

$$h_{21} = \frac{-Z_{21}}{Z_{22}} \quad \& \quad h_{22} = \frac{1}{Z_{22}} \quad \ldots\ldots \quad 1 \text{ Mark}$$

Comparing (3) and (6)

$$h_{11} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}} = \frac{\Delta Z}{Z_{22}}, \text{ where } \Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

1 Mark

b. For the network shown in Fig.9, find the transmission parameters. (9)

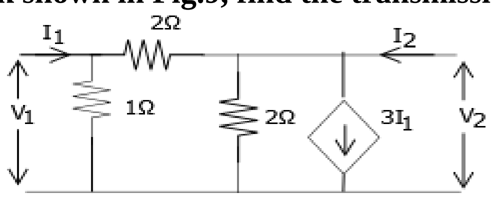
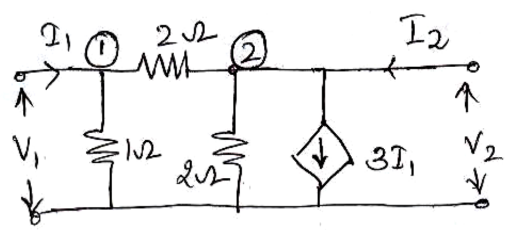


Fig.9

Answer:

(b)



The network equations in terms of transmission Parameters are

$$V_1 = A V_2 - B I_2 \qquad \text{where } A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0}, \; B = \left.\frac{V_1}{I_2}\right|_{V_2 = 0}$$

$$I_1 = C V_2 - D I_2 \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0}, \; D = \left.\frac{I_1}{I_2}\right|_{V_2 = 0}$$

1 Mark

Applying KCL at node 1 and 2, we get

$$I_1 = \frac{V_1}{1} + \frac{V_1 - V_2}{2}$$

$$I_1 = 1.5 V_1 - 0.5 V_2 \quad -(1)$$ . . . . . . 1 Mark

$$I_2 = 3 I_1 + \frac{V_2}{2} + \frac{V_2 - V_1}{2}$$

$$I_2 = 3 I_1 + V_2 - 0.5 I_1 \quad -(2)$$ . . . . 1 Mark

Substituting equation (1) in (2)

$$I_2 = 3[1.5 V_1 - 0.5 V_2] + V_2 - 0.5 V_1$$

$$I_2 = 4 V_1 + 0.5 V_2 \quad \text{or} \quad V_1 = \frac{-0.5}{4} V_2 + \frac{1}{4} I_2 \quad -(3)$$ 3 Marks

$\therefore \; A = -1/8 \; \& \; B = -1/4 \; \Omega$

Substituting eqn. (3) in (1), we get

$$I_1 = \left[-\frac{1}{8} V_2 + \frac{1}{4} I_2\right] \times 1.5 - 0.5 V_2$$

$$I_1 = \frac{-11}{16} V_2 + 3/8 \; I_2$$ 2 Marks

$C = -\frac{11}{16}$ ℧ and $D = \frac{-3}{8}$ —— 1 Mark

Q.8 a. Represent the admittance function $Y(s) = \dfrac{4(s+1)(s+3)}{s(s+2)}$ **in Foster form and hence synthesize the Network. (10)**

Answer:

8(a) The singularity at the origin is a pole and singularity nearest to ∞ is a zero ∴ It must be RL admittance function 2 Marks

$$Y(s) = \frac{4s^2 + 16s + 12}{s^2 + 2s} = 4 + \frac{8s + 12}{s^2 + 2s}$$ - - - - - 2 Marks

$$s^2+2s \overline{)\, 4s^2 + 16s + 12 \,}(4$$
$$\underline{4s^2 + 8s}$$
$$8s + 12$$

$$Y(s) = 4 + \frac{A}{s} + \frac{B}{(s+2)}$$

$$Y(s) = 4 + \frac{6}{s} + \frac{2}{s+2}$$

$$Y(s) = Y_1 + Y_2 + Y_3$$

$$Y_1 = 4\,\mho \quad \therefore \; R_1 = 1/4\ \Omega$$

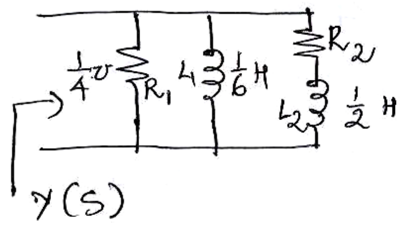
$$Y_2 = \frac{6}{s} \qquad Z_2 = \frac{s}{6} = L_1 s \;, \; \therefore L_1 = \frac{1}{6}\,H$$

$$Y_3 = \frac{2}{s+2} = \frac{1}{\frac{s}{2}+1} = \frac{1}{Z' + Z''}$$

$$Z' = s/2 = L_2 s \;\; \& \;\; L_2 = \frac{1}{2}\,H$$

$$Z'' = 1\ \Omega \quad , \therefore R_2 = 1\,\Omega$$

4 Marks



b. Indicate the following functions are either RC, RL or LC impedance functions with appropriate reasons. (6)

(i) $Z(s) = \dfrac{s^3 + 2s}{s^4 + 4s^2 + 3}$ **(ii)** $Z(s) = \dfrac{s^2 + 4s + 3}{s^2 + 6s + 8}$

Answer:

8(b) (i) $\dfrac{s(s^2+2)}{(s^2+1)(s^2+3)} = \dfrac{s^3 + 2s}{s^4 + 4s^2 + 3}$

represents LC driving point impedance 3 Marks
because (i) poles and zeros are located on the imaginary axis and alternative (ii) highest and lowest power in both numerator and denominator differ by one

(ii) $\dfrac{(s+1)(s+3)}{(s+2)(s+4)}$

Poles & zeros lie on negative real axis and alternative and singularity nearest to the origin is a zero & nearest to $s=\infty$ is a pole therefore it is a RL impedance function

Q.9 a. Obtain the zeros of transmission for the network shown in Fig.10 **(8)**

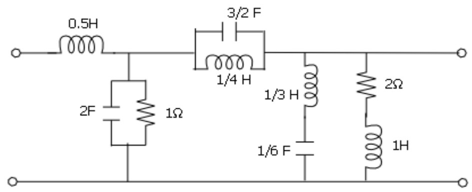


Fig.10

Answer:

9(a) The impedance due to 0.5 H

$Z_1(s) = sL = 0.5s$, for the zero transmission $Z_1(s) = \infty$ i.e, at $s = \infty$ } 1 Mark

The impedance due to parallel combination of 2F and 1Ω

$Z_2(s) = \frac{1}{2s} \| 1 = \frac{1}{1+2s}$ } 1 Mark

For the zero of transmission $Z_2(s) = 0$ i.e, at $s = \infty$

The impedance due to parallel combination of $\frac{1}{4}$ H & $\frac{3}{2}$ F

$Z_3(s) = \frac{s}{4} \| \frac{2}{3s} = \frac{2s}{3s^2+8}$

For the zero of transmission $Z_3(s) = \infty$

i.e at $3s^2 + 8 = 0$ or $s = \pm j\sqrt{8/3}$ } 2 Marks

The impedance due to series combination of $\frac{1}{3}$ H and $\frac{1}{6}$ F

$Z_4(s) = \frac{s}{3} + \frac{6}{s} = \frac{s^2+18}{3s}$

For the zero transmission $Z_5(s) = 0$

i.e at $s^2 + 18 = 0$ or $s = \pm j\sqrt{18}$ } 2 Marks

The impedance due to series combination of 2Ω and 1H

$Z_5(s) = 2 + s$

For the zero transmission $Z_6(s) = 0$ i.e, $s + 2 = 0$

Or $s = -2$ } 1 Mark

Therefore, three zeros ~~are~~ occur at

$s = \pm j\sqrt{8/3}$, $\pm j\sqrt{18}$, -2

and two zeros are at $s = \infty$ } 1 Mark

b. Synthesize the network function $Z_{21}(s) = \dfrac{2}{s^3 + 2s^2 + 4s + 2}$ **into an LC network terminated with** 1Ω**.** **(8)**

Answer:

9 b) The numerator is constant & considered as even; so divide by the odd part of the denominator polynomial as

$$Z_{21}(s) = \frac{P(s)}{Q(s)} = \frac{P(s)}{M(s)+N(s)} = \frac{P(s)/N(s)}{1+\frac{M(s)}{N(s)}}$$

$$= \frac{z_{21}'}{1+z_{22}}$$

$$z_{21} = \frac{P(s)}{N(s)} = \frac{2}{s^3+4s}$$

$$z_{22} = \frac{M(s)}{N(s)} = \frac{3s^2+2}{s^3+4s}$$

2 Mark

The continued fraction expansion of z_{22} is given by

$$s^3+4s\,)\,3s^2+2\,(\frac{3}{s}$$
$$3s^2+12$$
$$-10\,)\,s^3+4\,(-\frac{s^3}{10}$$
$$s^3$$

$\therefore$ we got a negative quotient

Consider $Y_{22} = \dfrac{1}{z_{22}} = \dfrac{s^3+4s}{3s^2+2}$

2 Marks