Q.2a. Differentiate the analogies between magnetic circuit and DC resistive circuit with suitable units (7)

Answer:

DC Resistive Circuit	**Magnetic Circuit**
Current I (Amperes)	Flux ϕ (Wb)
Voltage V (V)	Magneto motive force (mmf) $\Im$ (AT)
Resistance $R = l/\sigma A$ (Ω)	Reluctance $\Re = l/\mu A$ (H^{-1})
Conductivity σ (S/m)	Permeability μ (H/m)
Conductance $G = 1/R$ (S)	Permeance $\mathbf{P} = 1/\Re$ (H)
$I = V/R$	$\phi = \Im / \Re$
Current density $J = I/A$ (A/m^2)	Flux density $B = \phi/A$(Wb/m^2)

b. In the magnetic circuit shown in Fig.3 the coil of 500 turns carries a current of 4 A. The air-gap lengths are $g_1 = g_2 = 0.25$ cm and $g_3 = 0.4$ cm. The cross-sectional areas are related such that $A_1 = A_2 = 0.5A_3$. The permeability of iron may be assumed to be infinite. Determine the flux densities B_1, B_2, and B_3 in the gaps g_1, g_2, and g_3, respectively. Neglect leakage and fringing. (9)

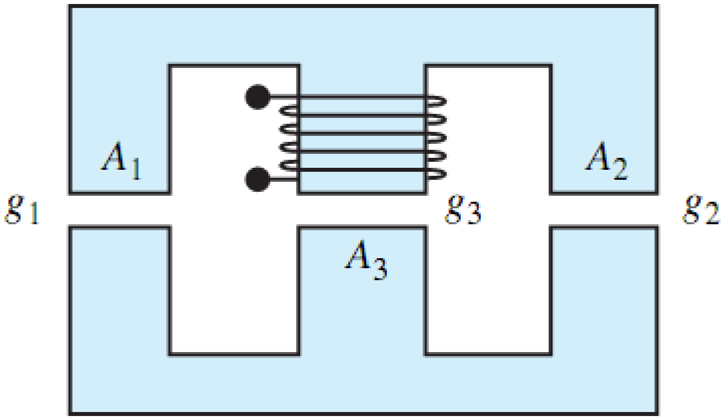
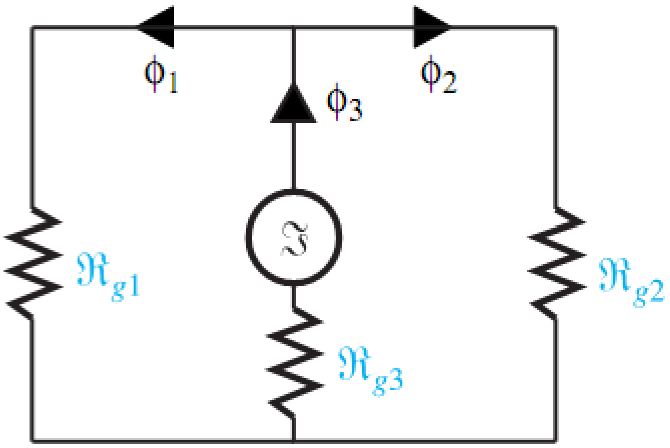


Fig.3

Answer:

The reluctance of the iron is negligible, the equivalent magnetic circuit is shown below



(up to equivalent circuit 4M)

Given $\Im = NI = 500 \times 4 = 2000$ At, $g_1 = g_2 = 0.25$ cm, and $g_3 = 0.4$ cm; $A_1 = A_2 = 0.5A_3$; $\mu_1 = \mu_2 = \mu_3 = \mu_0$; $\phi_3 = \phi_1 + \phi_2$; and $H_1g_1 + H_3g_3 = H_2g_2 + H_3g_3 = \Im = 2000$, we have

$$H_1g_1 = H_2g_2 \qquad \text{or} \qquad H_1 = H_2$$

since $g_1 = g_2$. Thus, $B_1 = B_2$, since $\mu_1 = \mu_2 = \mu_0$. Because $A_1 = A_2$, it follows that $\phi_1 = \phi_2$ and $\phi_3 = 2\phi_1$. But $B_3 = \phi_3/2A_1 = B_1$. Thus,

$$B_1 = B_2 = B_3; \qquad H_1 = H_2 = H_3$$

We had $H_1g_1 + H_3g_3 = 2000$, which is rewritten as

$$\frac{B_1}{\mu_0}(0.25 \times 10^{-2}) + \frac{B_1}{\mu_0}(0.4 \times 10^{-2}) = 2000$$

or,

$$B_1 = \frac{2000 \times 4\pi \times 10^{-7}}{0.65 \times 10^{-2}} = 0.387 \text{ T} = B_2 = B_3$$

(For calculations 5M given)

Q.3 a. Differentiate between core and shell-type transformers with neat sketches.(6)

Answer:

(For two diagrams 2M and theory given 4M)

Two types of cores are commonly employed in practice—*core-type* and *shell-type*. In core-type construction shown in Fig. 4(a) the windings are wound around the two legs of a rectangular magnetic core, while in shell-type construction of Fig. 4(b), the windings are wound on the central leg of a three-legged core. Though most of the flux is confined to a high permeability core, some flux always leaks through the core and embraces paths which partially lie in the air surrounding the core legs on which the coils are wound. This flux which links one of the windings without linking the other, though small in magnitude, has a significant effect on the transformer behaviour. Leakage is reduced by bringing the two coils closer. In a core-type transformer this is achieved by winding half low-voltage (LV) and half high-voltage (HV) winding on each limb of the core as shown in Fig. 4(a). The LV winding is wound on the inside and HV on outside to reduce the amount of insulation needed. Insulation between the core and the inner winding is then stressed to low voltage. The two windings are arranged as *concentric* coils. In shell-type construction leakage is reduced by subdividing each winding into subsections (wound as *pancake* coils) and interleaving LV and HV windings as shown in Fig. 4 (b).

The core-type construction has a longer mean length of core and a shorter mean length of coil turn. This type is better suited for EHV (extra high voltage) requirement since there is better scope for insulation. The shell-type construction has better mechanical support and good provision for bracing the windings. The shell-type transformer requires more specialized fabrication facilities than core-type, while the latter offers the additional advantage of permitting visual inspection of coils in the case of a fault and ease of repair at substation site. For these reasons, the present practice is to use the core-type transformers in large high-voltage installations.

Transformer windings are made of solid or stranded copper or aluminium strip conductors. For electronic transformers, “magnet wire” is normally used as conductor. Magnet wire is classified by an insulation class symbol, A, B, C, F and H, which is indication of the safe operating temperature at which the conductor can be used. Typical figures are the lowest 105 °C for class-A and highest 180 °C for class-H.

The windings of huge power transformers use conductors with heavier insulation (cloth, paper, etc.) and are assembled with greater mechanical support and the winding layers are insulated from each other—this is known as *minor insulation* for which pressed board or varnished cloth is used. *Major insulation*, insulating cylinders made of specially selected pressed board or synthetic resin bounded cylinders, is used between LV and core and LV and HV. Insulating barriers are inserted between adjacent limbs when necessary and between coils and core yokes.

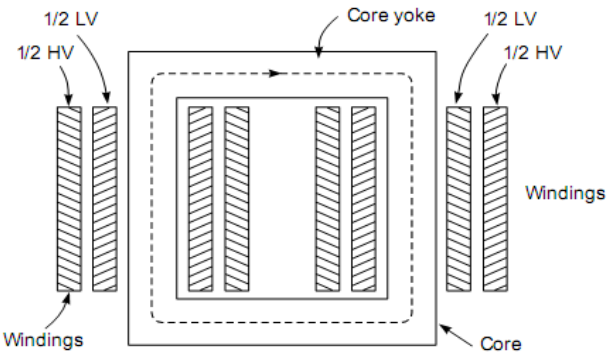


Fig.4(a) core type transformer

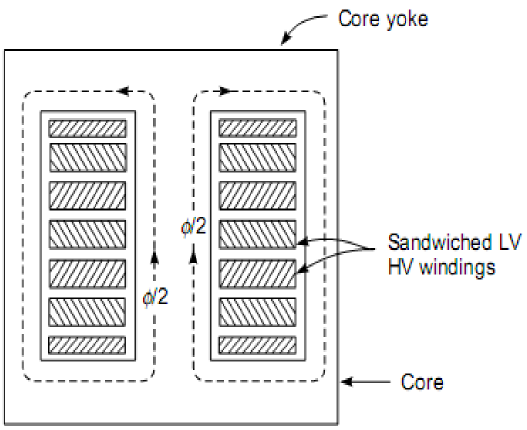


Fig4(b) shell type transformer

b. A 20-KVA, 50-Hz, 2000/200-V distribution transformer has a leakage impedance of 0.42 + j 0.52 Ω in the high-voltage (HV) winding and 0.004 + j 0.05 Ω in the low-voltage (LV) winding. When seen from the LV side, the shunt branch admittance Y_0 is (0.002 – j 0.015) mho (at rated voltage and frequency). Draw the equivalent circuit referred to

(i) HV side

(ii) LV side, indicating all impedances on the circuit. (10)

Answer:

The HV side will be referred as 1 and LV side as 2.

Transformation ratio,

$$a = \frac{N_1}{N_2} = \frac{2000}{200} = 10$$

(2M for transformation ratio)

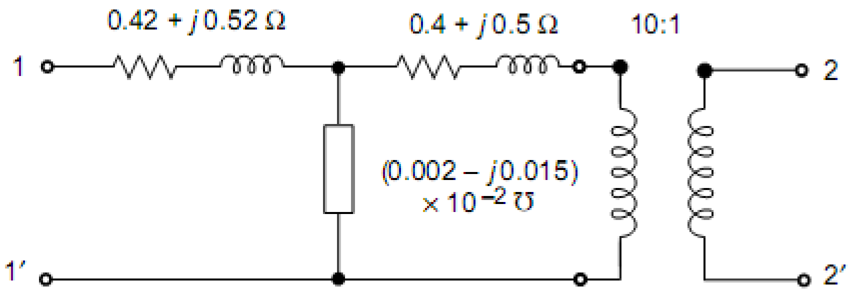
(a) Equivalent circuit referred to HV side (side 1)

$$\bar{Z}_2' = (10)^2 (0.004 + j\,0.005) = 0.4 + j\,0.5\ \Omega$$

$$\bar{Y}_0' = \frac{1}{(10)^2}(0.002 - j\,0.015)$$ (Notice that in transforming admittance is divided by a^2)

(Impedence and admittance 4M)

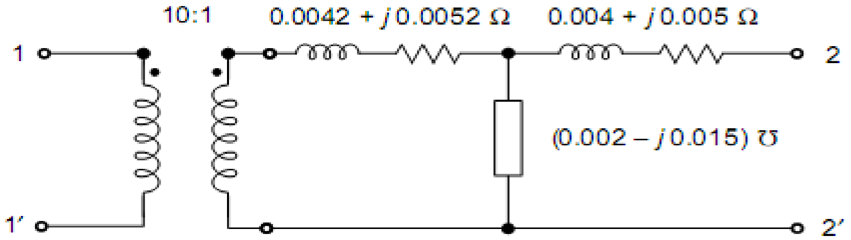
The equivalent circuit is drawn below



Equivalent circuit referred to LV side (side 2).

$$\bar{Z}_1' = \frac{1}{(10)^2}(0.42 + j\,0.52) = 0.0042 + j\,0.0052$$

The equivalent circuit is drawn below

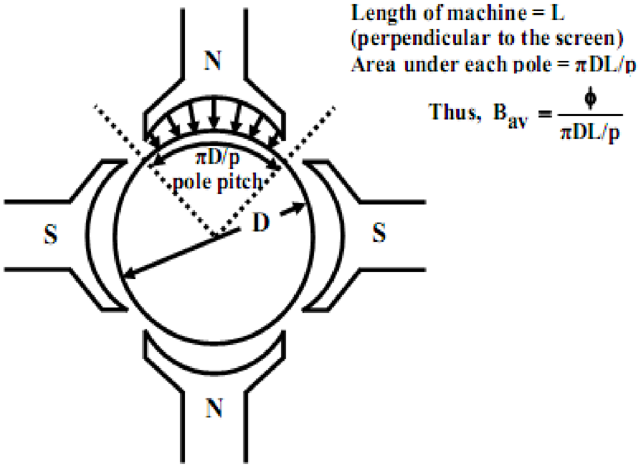


(Each equivalent circuit carries 3M each)

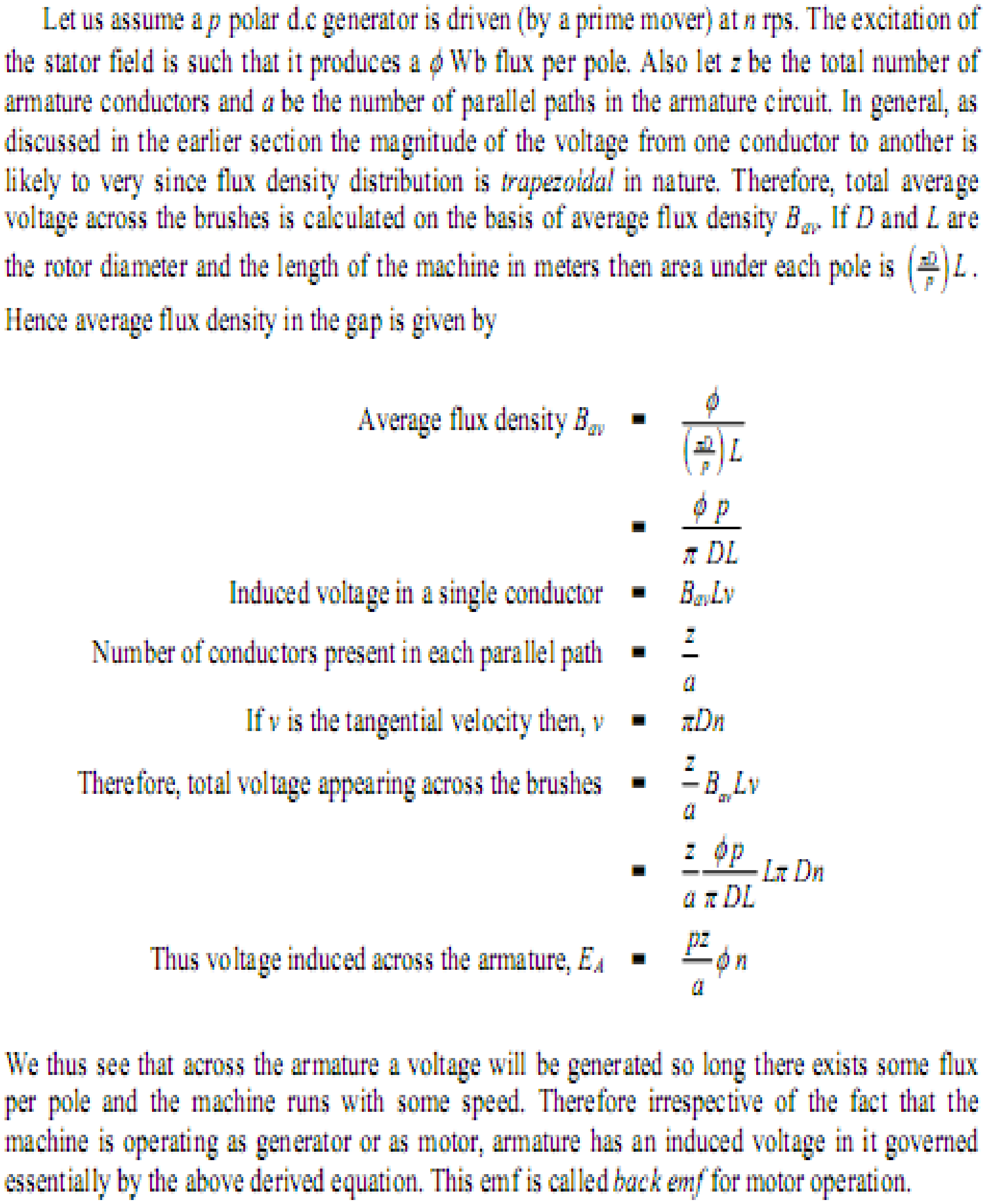
Q.4 a. Derive the expression for the induced emf of a dc machine from fundamental principles. (6)

Answer:

Consider a D.C generator whose field coil is excited to produce a flux density distribution along the air gap and the armature is driven by a prime mover at constant speed as shown in figure below.



(1Marks)



Let us assume a p polar d.c generator is driven (by a prime mover) at n rps. The excitation of the stator field is such that it produces a ϕ Wb flux per pole. Also let z be the total number of armature conductors and a be the number of parallel paths in the armature circuit. In general, as discussed in the earlier section the magnitude of the voltage from one conductor to another is likely to very since flux density distribution is *trapezoidal* in nature. Therefore, total average voltage across the brushes is calculated on the basis of average flux density B_{av}. If D and L are the rotor diameter and the length of the machine in meters then area under each pole is $\left(\frac{\pi D}{p}\right)L$.

Hence average flux density in the gap is given by

$$\text{Average flux density } B_{av} = \frac{\phi}{\left(\frac{\pi D}{p}\right)L}$$

$$= \frac{\phi\, p}{\pi\, DL}$$

$$\text{Induced voltage in a single conductor} = B_{av}Lv$$

$$\text{Number of conductors present in each parallel path} = \frac{z}{a}$$

$$\text{If } v \text{ is the tangential velocity then, } v = \pi Dn$$

$$\text{Therefore, total voltage appearing across the brushes} = \frac{z}{a}B_{av}Lv$$

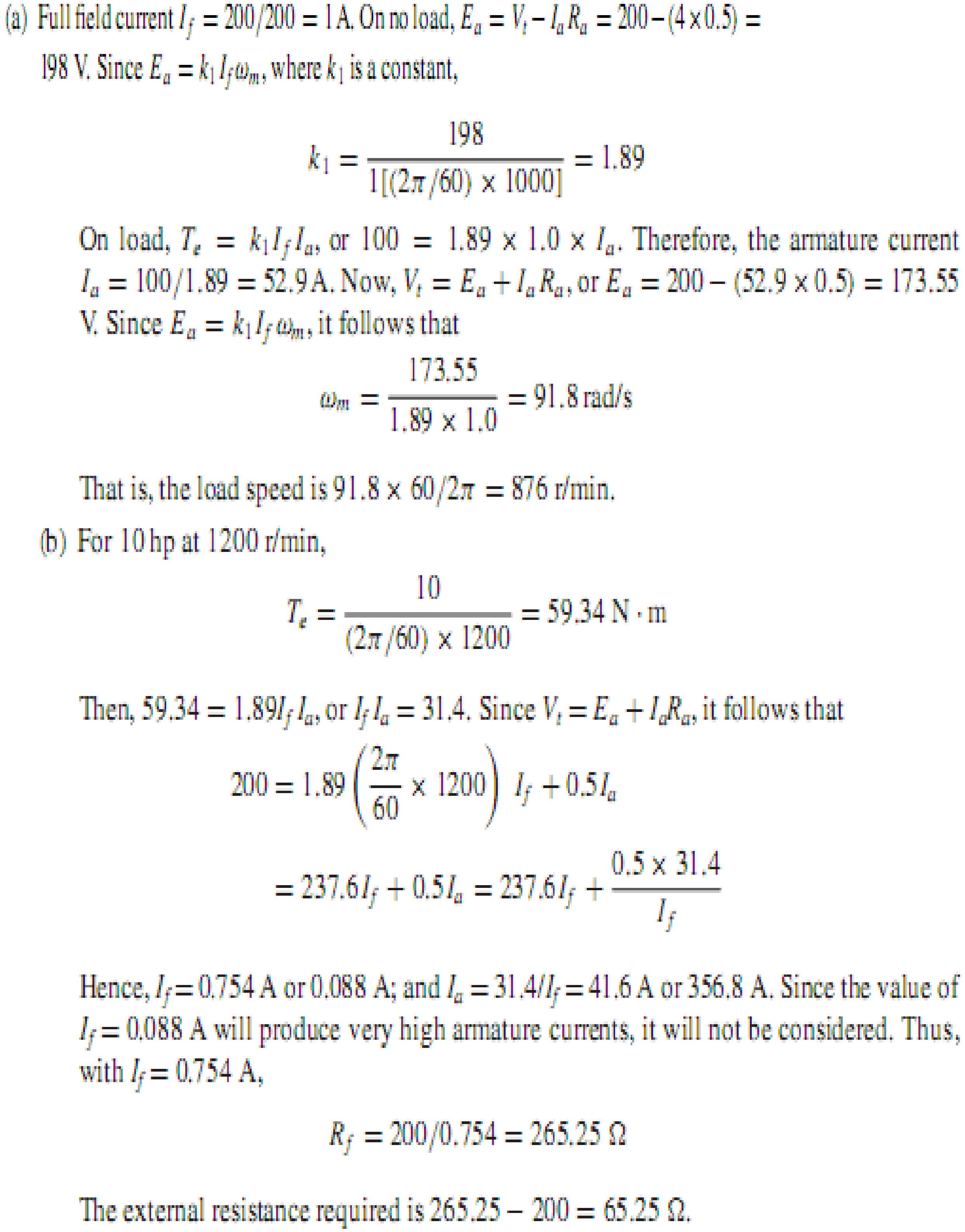
$$= \frac{z}{a}\frac{\phi p}{\pi DL}L\pi Dn$$

$$\text{Thus voltage induced across the armature, } E_A = \frac{pz}{a}\phi\, n$$

We thus see that across the armature a voltage will be generated so long there exists some flux per pole and the machine runs with some speed. Therefore irrespective of the fact that the machine is operating as generator or as motor, armature has an induced voltage in it governed essentially by the above derived equation. This emf is called *back emf* for motor operation.

b. A 200-V dc shunt motor has a field resistance of 200Ω and an armature resistance of 0.5Ω. On no load, the machine operates with full field flux at a speed of 1000 r/min with an armature current of 4 A. Neglect magnetic saturation and armature reaction.
(i) If the motor drives a load requiring a torque of 100 N ·m, find the armature current and speed of the motor.
(ii) If the motor is required to develop 10 hp at 1200 r/min, compute the required value of external series resistance in the field circuit. (10)

Answer:



(a) Full field current $I_f = 200/200 = 1$ A. On no load, $E_a = V_t - I_a R_a = 200 - (4 \times 0.5) =$ 198 V. Since $E_a = k_1 I_f \omega_m$, where k_1 is a constant,

$$k_1 = \frac{198}{1[(2\pi/60) \times 1000]} = 1.89$$

On load, $T_e = k_1 I_f I_a$, or $100 = 1.89 \times 1.0 \times I_a$. Therefore, the armature current $I_a = 100/1.89 = 52.9$ A. Now, $V_t = E_a + I_a R_a$, or $E_a = 200 - (52.9 \times 0.5) = 173.55$ V. Since $E_a = k_1 I_f \omega_m$, it follows that

$$\omega_m = \frac{173.55}{1.89 \times 1.0} = 91.8 \text{ rad/s}$$

That is, the load speed is $91.8 \times 60/2\pi = 876$ r/min.

(b) For 10 hp at 1200 r/min,

$$T_e = \frac{10}{(2\pi/60) \times 1200} = 59.34 \text{ N} \cdot \text{m}$$

Then, $59.34 = 1.89 I_f I_a$, or $I_f I_a = 31.4$. Since $V_t = E_a + I_a R_a$, it follows that

$$200 = 1.89 \left(\frac{2\pi}{60} \times 1200 \right) I_f + 0.5 I_a$$

$$= 237.6 I_f + 0.5 I_a = 237.6 I_f + \frac{0.5 \times 31.4}{I_f}$$

Hence, $I_f = 0.754$ A or 0.088 A; and $I_a = 31.4/I_f = 41.6$ A or 356.8 A. Since the value of $I_f = 0.088$ A will produce very high armature currents, it will not be considered. Thus, with $I_f = 0.754$ A,

$$R_f = 200/0.754 = 265.25\ \Omega$$

The external resistance required is $265.25 - 200 = 65.25\ \Omega$.

Q.5 a. Show power flow in a DC motor with the help of diagram. What are the various losses that occur in DC motor? Give the condition for maximum motor efficiency. (8)

Answer:

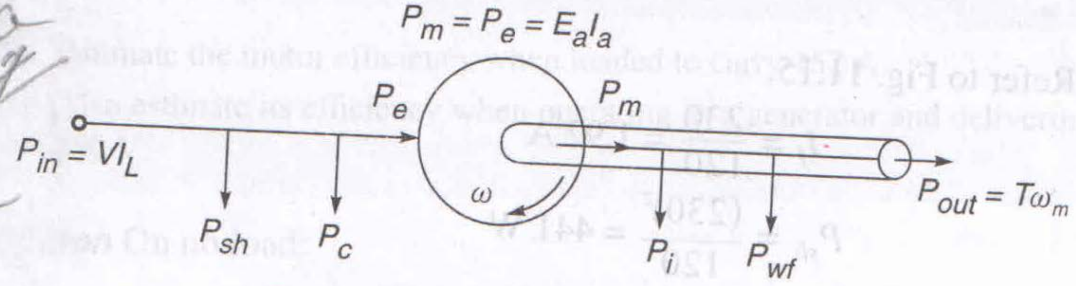


Fig. 10.33 Power flow in a dc motor

onstant Loss

P_i = core loss (including stray load core loss)
P_{wf} = windage and friction loss (at specified speed)
P_{sh} = shunt field copper loss (in a shunt machine)
$P_k = P_i + P_{wf} + P_{sh}$ = total constant loss

Variable Loss

$P_c = I_a^2 R_a$
= Copper loss (inclusive of copper loss in series winding in a series motor, and also inclusive of stray-load copper loss)

Total motor loss, $P_L = P_k + P_c$

$$P_m = P_e = E_a I_a$$

The motor efficiency is given by

$$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{VI_L - P_k - I_a^2 R_a}{VI_L} \tag{10.41}$$

Under load, $I_L \approx I_a$; I_f is small in a shunt motor.

Then

$$\eta = 1 - \frac{1}{V}\left(\frac{P_k}{I_a} + I_a R_a\right) \tag{10.42}$$

The maximum motor efficiency occurs at

$$\frac{P_k}{I_a} = I_a R_a \tag{10.43}$$

or $I_a^2 R_a = P_k$

or Variable loss = constant loss

b. What is a damper winding? What is the function of it and where it is located? (8)

Answer:

Damper Winding

Additional damping is provided in the salient pole synchronous machine by means of damper bars located in the main poles of the machine and short-circuited through round rings at both ends. As the rotor oscillates, the damper bars have a relative movement with respect to the air-gap flux pattern which causes induction of emfs and flow of currents in these bars. The torque created by the bar currents as per Lenz's law always opposes the relative motion. This is how a positive damping term is brought into play so that the oscillatory motion of the rotor about the operating point is considerably reduced in amplitude and the rotor quickly returns to the steady position. These short-circuited bars are known as *damper winding* or *ammortisseur winding*. These act like a squirrel cage induction motor thereby providing a starting torque for the motor which otherwise being of synchronous kind is not self-starting. Therefore, the damper winding serves the dual purpose.

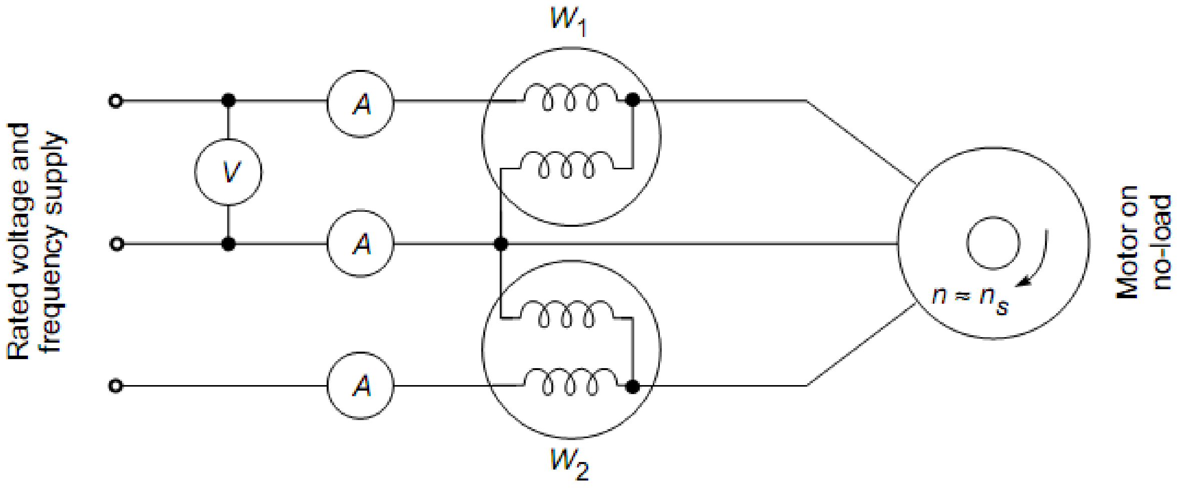
Q.6 a. Explain the No-load test with connection diagram to determine the parameters of circuit model of induction motor. (8)

Answer:

circuit 2M rest calculations 6M

As the circuit model of an induction motor is similar to that of a transformer, the parameters of the model can be obtained by means of non loading tests as in the case of the transformer—no-load test (corresponding to the OC test on the transformer)

In this test the motor is run on no-load at rated voltage and frequency. The applied voltage and current and power input to motor are measured by the metering as per the following figure.



Let the meter readings be Power input = P_0 (3-phase)
Current = I_0 (average of the three meter readings)
Voltage = V_0 (Line-Line rated voltage)

Power input at no-load (P_0) provides losses only as the shaft output is zero. These losses comprise,

$$P_0 \text{ (no-load loss)} = P_{c1} \text{ (stator copper loss)} + [P_i \text{ (iron/core loss)} + P_{wf} \text{ (windage and friction loss)} = \text{Rotational loss}]$$

wherein core loss occurs only in the stator as the slip is extremely low (of the order of 0.001) and so the frequency of rotor current is as low as 0.05 Hz.

The magnitude of no-load current in an induction motor is about 30-40% of full-load current because of the air-gap. So the stator copper loss at no-load needs to be accounted for. This can be estimated by measuring dc stator resistance and correcting to ac value (50 Hz) and corrected for temperature (°C).

The mechanical power developed corresponds to P_{wf} only and so, as already mentioned above the slip is very low and the output resistance

$$R_2'(1/s_0 - 1) = \text{very large}$$

Also $R_2'/s_0 >> X_2'$ and so X_2' can be ignored. The corresponding no-load circuit model is drawn in Fig. 9.18(a) wherein R_2'/s_0 appears in parallel to R_i. By combining the parallel shunt resistances, the final circuit at no-load is as given in Fig. Here R_{iwf} accounts for rotational loss, i.e., core loss and windage and friction loss. Magnitude-wise $R_{iwf} >> X_m$.

R_1, the stator resistance, is found by dc testing of the stator winding and correcting the value to ac operation (at 50 Hz). X_1, the stator leakage reactance, will be found from the blocked-rotor test which follows. We can then find X_m and R_{iwf} from the no-load (NL) test data. By simplification of the circuit of Fig.

$$R_0 = R_1 + \frac{X_m^2/R_{ifw}}{1 + (X_m/R_{iwf})^2}$$

$$X_0 = X_1 + \frac{X_m}{1 + (X_m/R_{iwf})^2}$$

The equivalent circuit is drawn in Fig.

It can be justifiably assumed that $(X_m/R_{iwf})^2 = 0$, so we get from the above equations

$$R_{iwf} = \frac{X_m^2}{R_0 - R_1}$$

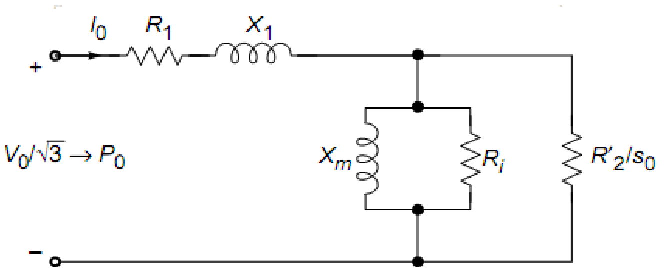
$$X_m = X_0 - X_1$$

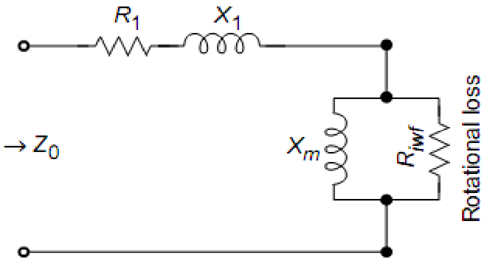
From the NL test data (V_0, I_0, P_0) we can find from the circuit of Fig.

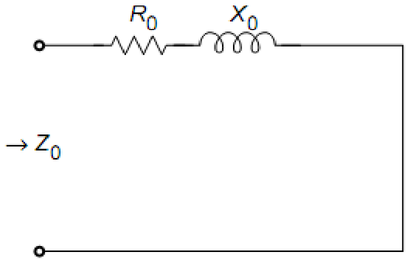
$$Z_0 = \frac{V_0/\sqrt{3}}{I_0}$$

$$R_0 = \frac{P_0/3}{I_0^2}$$

$$X_0 = (Z_0^2 - R_0^2)^{1/2}$$







b. Write a short note on isolated induction generator and explain how the slip should be negative. (8)

Answer:

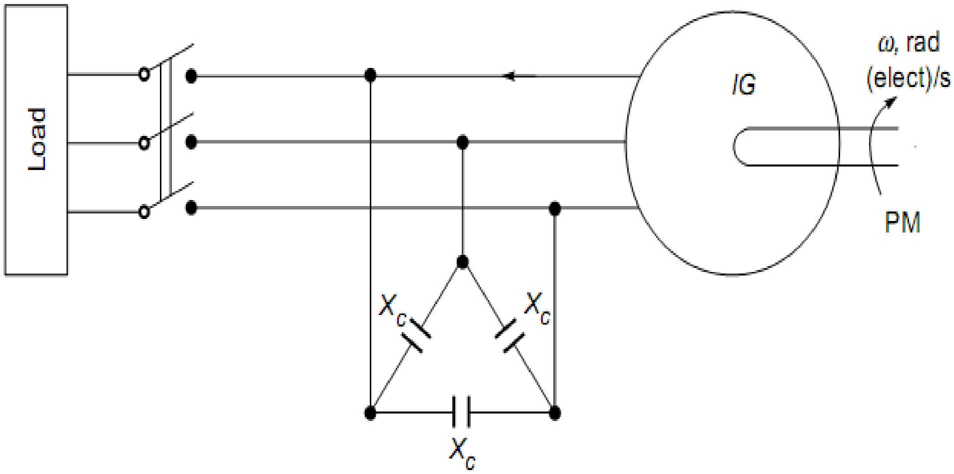
An isolated induction generator feeding a load is shown in Fig. The delta-connected capacitors across the generator terminals provide the magnetizing current necessary to excite the isolated generator. The voltage build-up will be explained later in this section. As the generator is loaded, the operating frequency depends primarily upon rotor speed but is affected by the load, while the voltage is mainly decided by capacitor reactance (X_C) at the operating frequency.

Let ω_0 = rated frequency

ω_s = operating frequency (stator)

ω_r = stator frequency corresponding to rotor speed

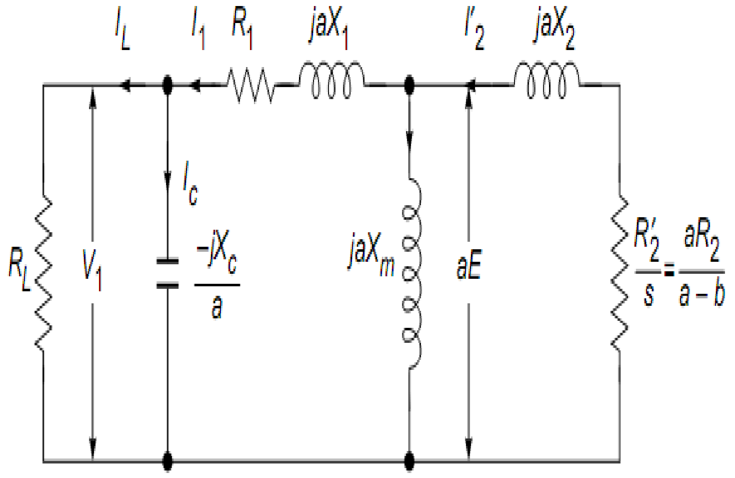
$a = \omega_s/\omega_0$ and $b = \omega_r/\omega_0$



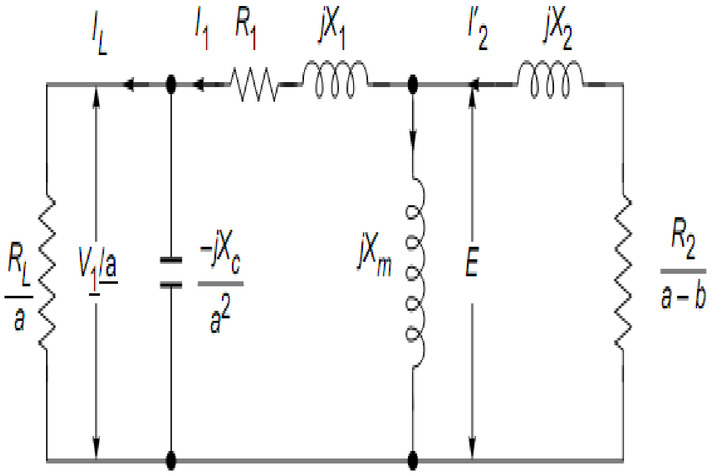
The machine slip (which should be negative) can be expressed as

$$s = (\omega_s - \omega_r)/a\omega_0$$
$$= (a\omega_0 - b\omega_0)/a\omega_0 = (a - b)/a;\ b < a$$

Assuming that the machine reactances, excitation reactances and machine induced emf correspond to the rated frequency (ω_0), the per phase equivalent circuit of the system at operating frequency $\omega_s = a\omega_0$ is drawn in Fig. (a) with load considered as purely resistive. Dividing throughout by 'a', the circuit is reduced to the rated (fixed) frequency and is drawn in Fig. (b).



(a)



(b)

Q.7 a. A universal motor (ac-operated) has a 2-pole armature with 960 conductors. At a certain load the motor speed is 5000 rpm and the armature current is 4.6 A; the armature terminal voltage and input are respectively 100 V and 300 W. Compute the following, assuming an armature resistance of 3.5 W.

(i) Effective armature reactance

(ii) Maximum value of useful flux/pole (8)

Answer:

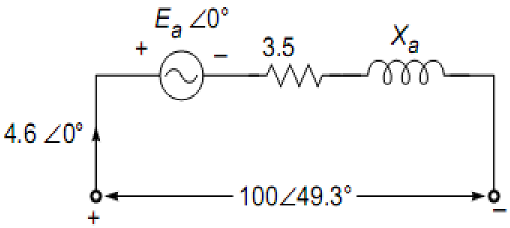
For circuit 2M and for(a) and (b) 3M each

The operating conditions in terms of voltage and current of the armature circuit are shown in Fig.

$$100 \times 4.6 \cos \phi = 330 \text{ W}$$

or $$\phi = 49.3^\circ$$

(lagging because of reactive nature of the circuit).



(a) From the circuit the following can be written

$$\frac{100\angle 49.3^\circ - E_a \angle 0^\circ}{3.5 + jX_a} = 4.6\angle 0^\circ$$

$\bar{E}_a$ is in-phase with $\bar{I}_a$

or $$65.2 + j75.8 - E_a = 16.1 + j4.6\,X_a$$

Equating real and imaginary parts,

$$E_a = 65.2 - 16.1 = 49.1 \text{ V}$$

$$X_a = \frac{75.8}{4.6} = 165\ \Omega$$

(b) $$E_a = \frac{1}{\sqrt{2}} \cdot \frac{\Phi n Z}{60}\left(\frac{P}{A}\right)$$

$$\Phi = \frac{\sqrt{2} \times 49.1 \times 60}{5000 \times 960} = 0.868 \text{ mWb}$$

b. Give the typical applications of : **(2x4)**

(i) Resistance Split-phase Motor
(ii) Capacitor-start Motor
(iii) Permanent-capacitor Motor
(iv) Two-value Capacitor Motor

Answer:

(2M for each) ***(8M)***

(i)Resistance Split-phase Motor It has a low starting current and moderate starting torque. It is used for easily started loads and typical applications include fans, saws, grinders, blowers, centrifugal pumps, office equipment, washing machines, etc. These are usually available in the range of 1/20 to 1/2 kW.

(ii)Capacitor-start Motor This motor has a high starting torque and therefore is used for hard starting loads, such as compressors, conveyors, pumps, certain machine tools, refrigeration and air-conditioning equipment, etc. This is the most commonly used induction motor and is available up to sizes as large as 6 kW.

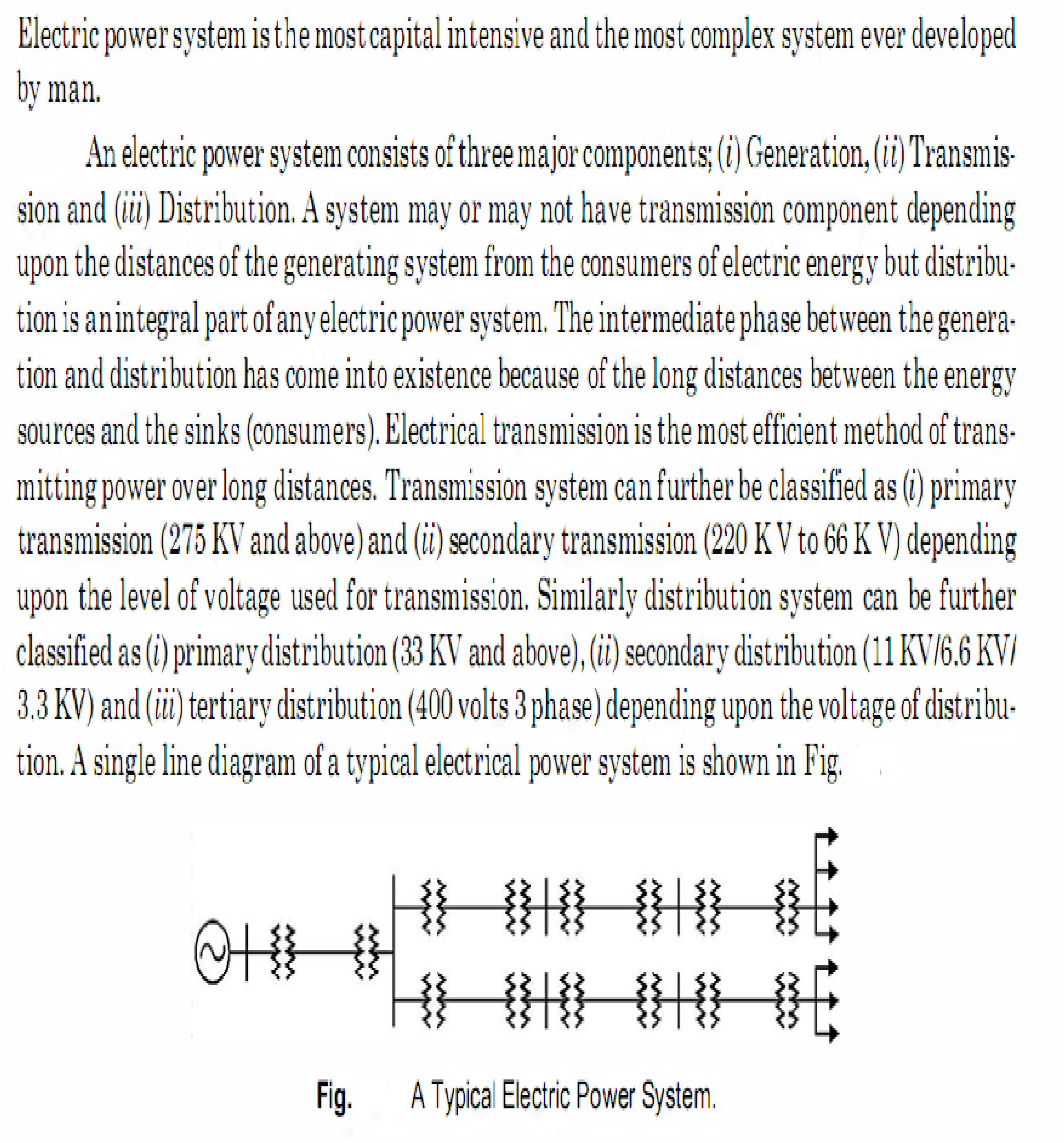
(iii)Permanent-capacitor Motor It has a high starting torque but slightly lower than that of the capacitor-start motor as a result of the compromise between starting and running performances and the capacitor cost. Because of the permanent capacitor it has a better running power factor and efficiency and a quieter and smoother operation. It is used for both easy and hard to start loads. In fact in modern practice ceiling fans, air-circulators and blowers use this type of motor.

(iv)Two-value Capacitor Motor It combines the advantages of capacitor-start and permanent-capacitor motors and is used for hard to start loads. At the same time it gives a high power factor and efficiency under running conditions. Typical applications are refrigerators, compressors and stockers.

Q.8 a. Draw single line diagram of a power system. Label all the major components of the system. (8)

Answer:

For single line diagram 4M and rest 4M



Electric power system is the most capital intensive and the most complex system ever developed by man.

An electric power system consists of three major components; (*i*) Generation, (*ii*) Transmission and (*iii*) Distribution. A system may or may not have transmission component depending upon the distances of the generating system from the consumers of electric energy but distribution is an integral part of any electric power system. The intermediate phase between the generation and distribution has come into existence because of the long distances between the energy sources and the sinks (consumers). Electrical transmission is the most efficient method of transmitting power over long distances. Transmission system can further be classified as (*i*) primary transmission (275 KV and above) and (*ii*) secondary transmission (220 K V to 66 K V) depending upon the level of voltage used for transmission. Similarly distribution system can be further classified as (*i*) primary distribution (33 KV and above), (*ii*) secondary distribution (11 KV/6.6 KV/3.3 KV) and (*iii*) tertiary distribution (400 volts 3 phase) depending upon the voltage of distribution. A single line diagram of a typical electrical power system is shown in Fig.

Fig. A Typical Electric Power System.

b. Discuss the advantages of ring main distribution system over the radial system. (8)

Answer: Each system carries 4M

Depending upon the type of supply, distribution system can be classified as follows :

(*i*) a.c. single phase (single phase loads only);

(*ii*) 3-phase, 3-wire (3 phase loads only); and

(*iii*) 3-phase, 4-wire (all types of loads).

The distribution system can also be classified depending upon the connections. These are the following two systems :

(*i*) Radial system and

(*ii*) Ring Mains system.

The Radial Systems

The electric energy distribution originally was through radial system. A typical radial distribution system is shown in Fig. 10.3.

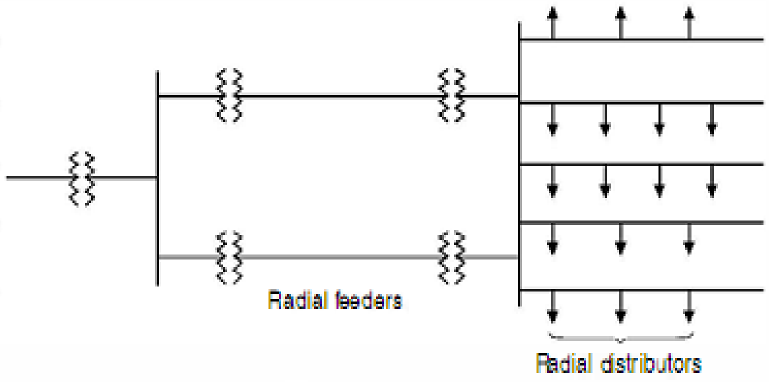
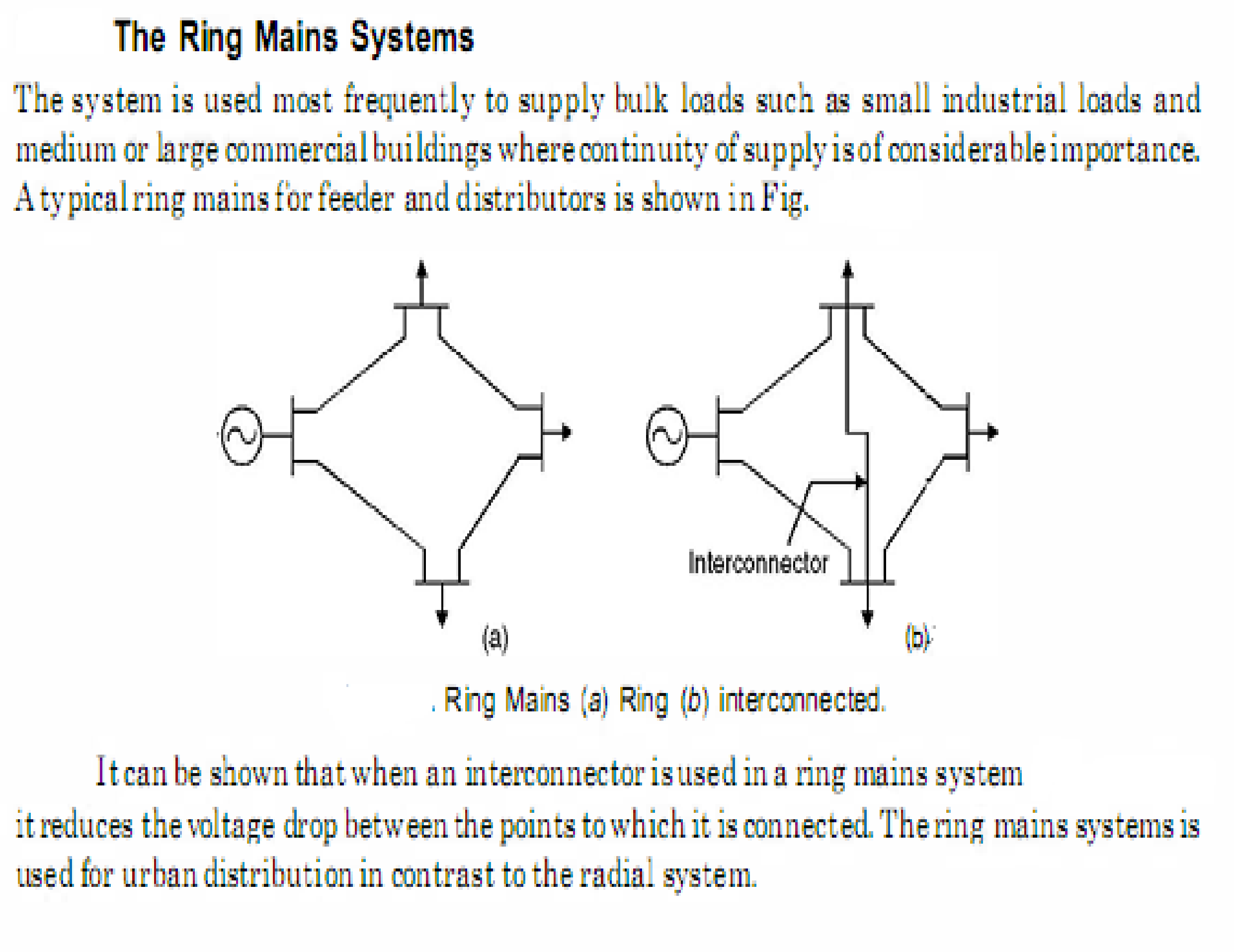


Fig. Radial Distributors.

The advantages of radial system are its simplicity and low cost, which result from a straight forward circuit arrangement where a single or radial path is provided between the consumer and the source or bulk power supply. With such an arrangement, the amount of switching equipment required is small and the protective relaying is simple. The major disadvantage of radial system is its lack of security of supply. When a fault occurs on any section of the feeder; a number of consumers will be without supply for a considerable period. The radial system is normally used for rural distribution these days.



The Ring Mains Systems

The system is used most frequently to supply bulk loads such as small industrial loads and medium or large commercial buildings where continuity of supply is of considerable importance. A typical ring mains for feeder and distributors is shown in Fig.

. Ring Mains (*a*) Ring (*b*) interconnected.

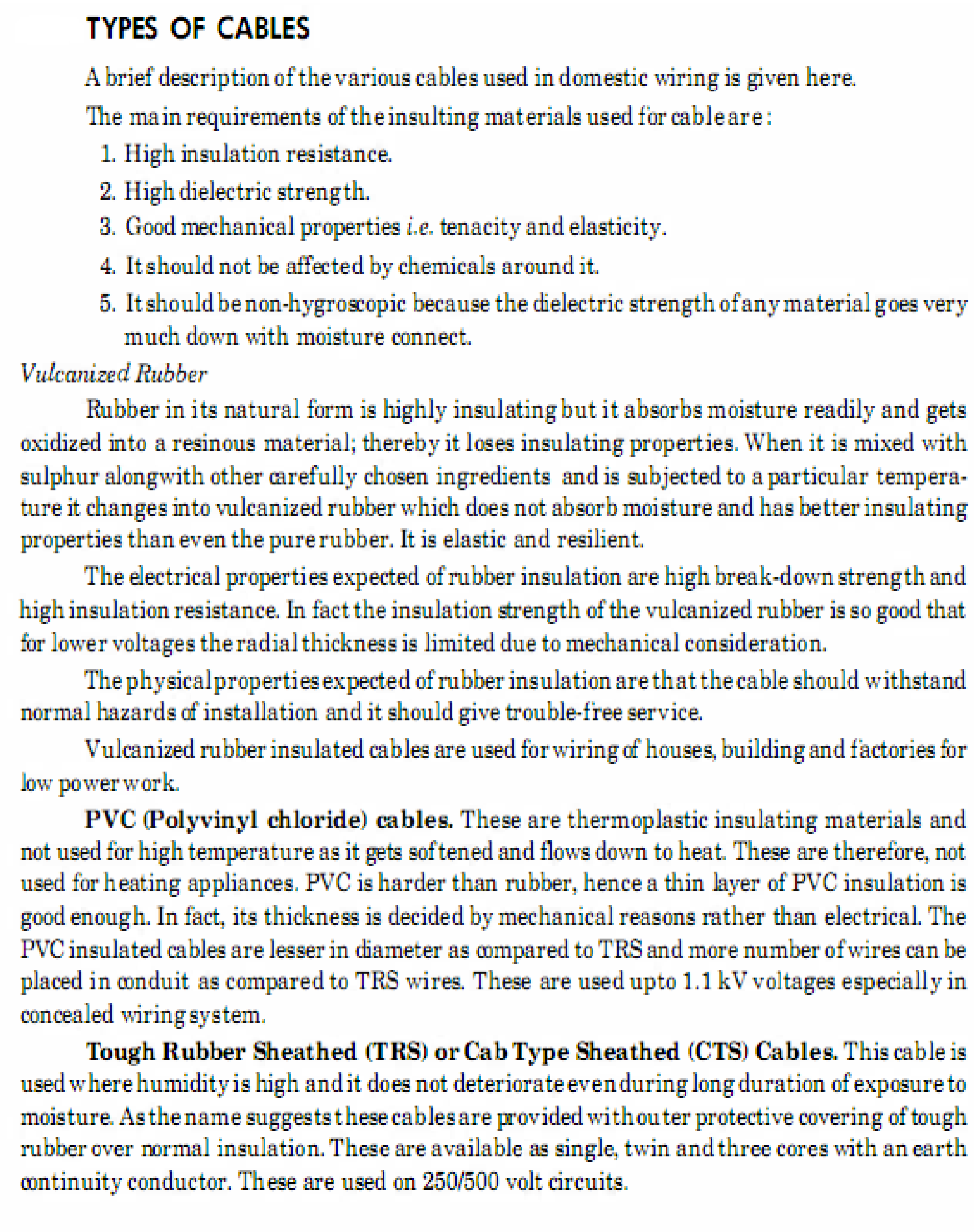
It can be shown that when an interconnector is used in a ring mains system it reduces the voltage drop between the points to which it is connected. The ring mains systems is used for urban distribution in contrast to the radial system.

Q.9 a. Write a short notes on: (2x3)

(i) Vulcanized India Rubber cables
(ii) PVC (Polyvinyl chloride) cables
(iii) Tough Rubber Sheathed (TRS) cables used in domestic wiring

Answer:

For types of cables 2M, VR,PVC,TRS carries 2M each



TYPES OF CABLES

A brief description of the various cables used in domestic wiring is given here.

The main requirements of the insulting materials used for cable are :

1. High insulation resistance.
2. High dielectric strength.
3. Good mechanical properties *i.e.* tenacity and elasticity.
4. It should not be affected by chemicals around it.
5. It should be non-hygroscopic because the dielectric strength of any material goes very much down with moisture connect.

Vulcanized Rubber

Rubber in its natural form is highly insulating but it absorbs moisture readily and gets oxidized into a resinous material; thereby it loses insulating properties. When it is mixed with sulphur alongwith other carefully chosen ingredients and is subjected to a particular temperature it changes into vulcanized rubber which does not absorb moisture and has better insulating properties than even the pure rubber. It is elastic and resilient.

The electrical properties expected of rubber insulation are high break-down strength and high insulation resistance. In fact the insulation strength of the vulcanized rubber is so good that for lower voltages the radial thickness is limited due to mechanical consideration.

The physical properties expected of rubber insulation are that the cable should withstand normal hazards of installation and it should give trouble-free service.

Vulcanized rubber insulated cables are used for wiring of houses, building and factories for low power work.

PVC (Polyvinyl chloride) cables. These are thermoplastic insulating materials and not used for high temperature as it gets softened and flows down to heat. These are therefore, not used for heating appliances. PVC is harder than rubber, hence a thin layer of PVC insulation is good enough. In fact, its thickness is decided by mechanical reasons rather than electrical. The PVC insulated cables are lesser in diameter as compared to TRS and more number of wires can be placed in conduit as compared to TRS wires. These are used upto 1.1 kV voltages especially in concealed wiring system.

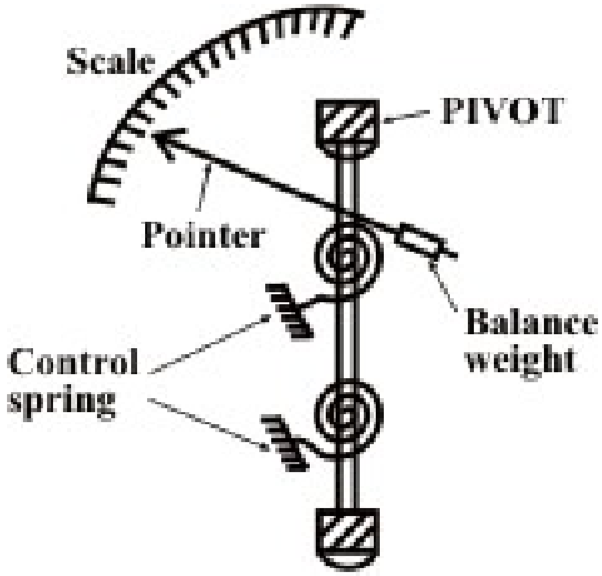
Tough Rubber Sheathed (TRS) or Cab Type Sheathed (CTS) Cables. This cable is used where humidity is high and it does not deteriorate even during long duration of exposure to moisture. As the name suggests these cables are provided with outer protective covering of tough rubber over normal insulation. These are available as single, twin and three cores with an earth continuity conductor. These are used on 250/500 volt circuits.

b. Explain the basic theory and construction of a permanent magnet moving coil (PMMC) instrument and its operation. (10)

Answer:

For diagram 4M, three torques carries 2M each

The basic theory of moving-coil instruments may be dealt with considering a rectangular coil of N turns, free to rotate about a vertical axis.



The above figure shows the basic construction of a PMMC instrument. A moving coil instrument consists basically of a permanent magnet to provide a magnetic field and a small lightweight coil is wound on a rectangular soft iron core that is free to rotate around its vertical axis. When a current is passed through the coil windings, a torque is developed on the coil by the interaction of the magnetic field and the field set up by the current in the coil. The aluminum pointer attached to rotating coil and the pointer moves around the calibrated scale indicates the deflection of the coil. To reduce parallax error a mirror is usually placed along with the scale the scale. A balance weight is also attached to the pointer to counteract its weight. To use PMMC device as a meter, two problems must be solved. First, a way must be found to return the coil to its original position when there is no current through the coil. Second, a method is needed to indicate the amount of coil movement. The first problem is solved by the use of hairsprings attached to each end of the coil as shown in figure. These hairsprings are not only supplying a restoring torque but also provide an electric connection to the rotating coil. With the use of hairsprings, the coil will return to its initial position when no current is flowing though the coil. The springs will also resist the movement of coil when there is current through coil. When the developing force between the magnetic fields (from permanent magnet and electro magnet) is exactly equal to the force of the springs, the coil rotation will stop. The coil set up is supported on jeweled bearings in order to achieve free movement. Two other features are considered to increase the accuracy and efficiency of this meter movement. First, an iron core is placed inside the coil to concentrate the magnetic fields. Second, the curved pole faces ensure the turning force on the coil increases as the current increases.

It is assumed that the coil sides are situated in a uniform radial magnetic field of flux density B , let the length of a coil side (within the magnetic field) be l (meter), and the distance from each coil side to the axis be r(meter).
It has been mentioned that the interaction between the induced field and the field produced by the permanent magnet causes a deflecting torque, which results in rotation of the coil. The deflecting torque produced is described below in mathematical form:
Deflecting Torque: If the coil is carrying a current of , the force on a coil side = $.i$ *amp*
$B\ il\ N$ (newton, N).
∴Torque due to both coil sides = $(2r)(BilN)\ (Nm) = (Gi\ Nm)$
where is the Galvanometer constant and it is expressed as $G\ 2GrBlN= (Nm/\ amp)$ = ($NBA\ Nm\ /amp$). (note $A = 2rl$ = area of the coil.)

N = no. of turns of the coil.
B = flux density in Wb/m^2 Wb/m^2.
l = length of the vertical side of the coil, m.
$2r$ = breadth of the coil, m
i = current in ampere.
$A = 2rl$ = area, m^2

Controlling Torque: The value of control torque depends on the mechanical design of the control device. For spiral springs and strip suspensions, the controlling torque is directly proportional to the angle of deflection of the coil.
Control torque = $C\ \theta$

where, θ = deflection angle in radians and spring constant = $C\ Nm/\ rad$
Damping Torque: It is provided by the induced currents in a metal former or core on which the coil is wound or in the circuit of the coil itself. As the coil moves in the field of the permanent magnet, eddy currents are set up in the metal former or core. The magnetic field produced by the eddy currents opposes the motion of the coil. The pointer will therefore swing more slowly to its proper position and come to rest quickly with very little oscillation. Electromagnetic damping is caused by the induced effects in the moving coil as it rotates in magnetic field, provided the coil forms part of closed electric circuit.

Let the velocity of the coil is $\omega(t) = \frac{d\theta}{dt} rad./\sec.$, and let the resistance of the coil circuit with N turns be $R\Omega$. Then the velocity of a coil side $v(t)= r\frac{d\theta}{dt}\ (m/\sec.)$

$\therefore$ E.m.f induced in each turn of the coil = $2Blv = 2Blr\frac{d\theta}{dt}\ volt.$ (note both the sides of the coil having same e.m.fs but they are additive in nature).

$\therefore$ Induced current across N turns of coil = $\frac{2BlN\, r}{R}\frac{d\theta}{dt} = \frac{G}{R}\frac{d\theta}{dt}\ amps.$(R = resistance of coil)

By Lenz's Law, torque produced = $Gi = G\frac{G}{R}\frac{d\theta}{dt} = \frac{G^2}{R}\frac{d\theta}{dt} = D\frac{d\theta}{dt}\ (Nm)$= opposing torque. Note, $D = \frac{G^2}{R}$ is the damping constant for the induced currents in the coil due to its motion. This damping torque is active when the coil posses a change in deflection. A metal former or core may be considered as a single-turn coil and if its dimensions are $l_1\, and\, r_1$ and its resistance R_1. Similarly, damping torque for the former or core can be computed as

Damping torque (for the core or former) = $D_1\frac{d\theta}{dt}$ (Nm) where D_1 = damping constant due to induced currents in the core or former. In addition to the induced current damping, there will be a small damping torque due to air friction. Air damping torque $(D_2\frac{d\theta}{dt})$ may be assumed to be proportional to the angular velocity of the coil.

Equation of motion: The resulting torque in a coil or motion of a coil in a magnetic field is due to the combined effect of deflecting torque (T_d), controlling torque ($C\theta$), damping torque ($D\frac{d\theta}{dt}$) and it is expressed mathematically as

$$J\frac{d^2\theta}{dt^2} = Gi - C\theta - D\frac{d\theta}{dt} \Rightarrow J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + C\theta = Gi$$

where J is the moment of inertia of the moving parts. One can easily study the dynamic behavior of the above second order system by solving the differential equation.