Q.2a. Consider the following propositions concerned with a certain triangle ABC. (8)

p: **ABC is isosceles**
q: **ABC is equilateral**
r: **ABC is equiangular**

Write down the following propositions in words:

(i) $p \wedge (\sim q)$ **(ii)** $(\sim p) \vee q$
(iii) $p \to q$ **(iv)** $q \to p$
(v) $(\sim r) \to (\sim q)$ **(vi)** $p \leftrightarrow (\sim q)$
(vii) $r \to q$ **(viii)** $(q \wedge r) \to p$

Answer:

Q2(a)(1) ABC is isosceles and is not equilateral
(2) ABC is not isosceles or ABC is equilateral
(3) If ABC is isosceles, then it is equilateral
(4) If ABC is equilateral, then it is isosceles
(5) If ABC is not equiangular then it is not equilateral
(6) If ABC is isosceles then it is not equilateral and if ABC is not equilateral then it is isocles.
(7) If ABC is equiangular then it is equilateral
(8) If ABC is equilateral and it is also equiangular then it is isosceles.

b. Is the following arguments valid?
If two sides of a triangle are equal,
then opposite angles are equal
Two sides of a triangle are not equal
$\therefore$ **The opposite angles are not equal** **(8)**

Answer:

In symbolic form the argument is

$p \rightarrow q$

$\sim p$

$\therefore \sim q$

The truth table is

Premises				Conclusion	
p	q	$p \rightarrow q$	$\sim p$	$\sim q$	
T	T	T	F	F	
T	F	F	F	T	
F	T	T	T	F	→ critical row
F	F	T	T	T	→ critical row

In this case the third and fourth rows are critical rows. But in the third row, the conclusion is false. Hence the argument is a fallacy.

Q.3 a. If *m* = 2, *n* = 5 and

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Determine the group code $e_H : B^2 \rightarrow B^5$ **(8)**

Answer:

we have $B^2 = \{00, 01, 10, 11\}$ Then

$e(00) = 00x_1x_2x_3$

Where $x_1 = 0.1 + 0.0 = 0$

$x_2 = 0.1 + 0.1 = 0$

$x_3 = 0.0 + 0.1 = 0$

$\therefore e(00) = 00000$

Now $e(01) = 01x_1x_2x_3$

Where $x_1 = 0.1 + 1.0 = 0$

$x_2 = 0.1 + 1.1 = 1$

$x_3 = 0.0 + 1.1 = 1$

$\therefore e(01) = 01011$

Next $e(10) = 10x_1x_2x_3$

$x_1 = 1.1 + 0.0 = 1$

$x_2 = 1.1 + 1.0 = 1$

$x_3 = 1.0 + 0.1 = 0$

$\therefore e(10) = 10110$

Similarly $e(11) = 11101$

b. Define group code. Show that (2,5) encoding function $e: B^2 \to B^5$ **defined by** $e(00) = 0000,\ e(10) = 10101,\ e(01) = 01110,\ e(11) = 11011$ **is a group code. (8)**

Answer:

An (m, n) encoding function $e : B^m \to B^n$ is called a group code if range of e is a sub-group of B^n

ie $(Ran(e), \oplus)$ is a group.

$\oplus$	0000	01110	10101	11011
0000	0000	01110	10101	11011
01110	01110	0000	11011	10101
10101	10101	11011	0000	01110
11011	11011	10101	01110	0000

Since closure property satisfied. It is a group code.

Q.4 a. In a group (G,0), prove that **(8)**

$(a_0 b)^{-1} = b^{-1}{}_0 a^{-1} \quad \forall a, b \in G$

Answer:

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Let $a, b \in G$, Then $a^{-1}, b^{-1}, a \circ b, b^{-1}, a^{-1} \in G$

$$\therefore (b^{-1} \circ a^{-1}) \circ (a \circ b) = [b^{-1} \circ (a^{-1} \circ a)] \circ b$$

(since o is associative)

$$= (b^{-1} \circ e_G) \circ b$$

$$= b^{-1} \circ b = e_G$$

Also $(a \circ b) \circ (b^{-1} \circ a^{-1}) = [a \circ (b \circ b^{-1})] \circ a^{-1}$

(o is associative)

$$= (a \circ e_G) \circ a^{-1} = a \circ a^{-1}$$

$$= e_G$$

$$\therefore (b^{-1} \circ a^{-1}) \circ (a \circ b) = (a \circ b) \circ (b^{-1} \circ a^{-1}) = e_G$$

$$\therefore (a \circ b)^{-1} = b^{-1} \circ a^{-1}$$

b. What do you mean by addition modulo *m*? Show that the set $G = \{0,1,2,\ldots,m-1\}$ of first m non-negative integers is an Abelian group under the composition addition modulo *m*. (8)

Answer:

Addition modulo m: The addition modulo m of any two integers a and b is denoted by $a +_m b$ and defined by $a +_m b = r$, $(0 \leq r < m)$ where r is the least non-negative remainder when a + b is divisible by m

(i) Let $a, b \in G$ and $a +_m b = r$ $\quad 0 \le r < m-1$ 135/14

$\therefore r \in G$ i.e $a +_m b \in G$, $a, b \in G$

$\therefore +_m$ is closed.

(ii) Let $a, b, c \in G$

$(a +_m b) +_m c = (a+b) +_m c$

$[\because a +_m b = a+b \pmod m]$

= least non-negative remainder when $a+b+c$ is divided by m

$= a +_m (b +_m c)$

$+_m$ is associative.

(iii) $0 \in G$, $0 +_m a = a +_m 0 = a \ \forall a \in G$

0 is the identity element.

(iv) Let $r \neq 0 \in G$ $\quad (m-r) +_m r = 0$

$= r +_m (m-r)$

$\therefore (m-r)$ is the inverse of r

$0 +_m 0 = 0$, 0 inverse is 0

(v) Let $a, b \in G$

$a +_m b$ = least non-negative integer when $a+b$ is divisible by m

$= b +_m a$

$\therefore (G, +_m)$ is a finite Abelian Group.

Q.5 a. What do you mean by recurrence relation? Solve the following recurrence relation (8)

$a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 0$

Answer:

(a) (135/14) (6)

Recurrence Relation: A recurrence relation for the sequence $\{S_n\}$ is an equation that relates S_n in terms of one or more of previous terms of the sequence.

The given recurrence relation is

$$a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 0$$

Let $a_n = r^n$ be a solution of the given ~~eq~~ recurrence relation, Then the characteristic equation is

$$r^3 - 8r^2 + 21r - 18 = 0$$

$$r = 2, 3, 3$$

The solution of the recurrence relation is

$$a_n = (b_1 + b_2 n)3^n + b_3 2^n$$

where b_1, b_2, b_3 are constant.

b. Solve the recurrence relation $f_n = f_{n-1} + f_{n-2}$, $n \geq 2$ with initial conditions $f_0 = 1,\ f_1 = 1$. (8)

Answer:

Let $a_n = r^n$ be a solution of the given recurrence relation.

Then the characteristic equation is $r^2 - r - 1 = 0$

$$\therefore r_1 = \frac{1+\sqrt{5}}{2}, \quad r_2 = \frac{1-\sqrt{5}}{2}$$

$\therefore$ The general solution is

$$f_n = b_1\left(\frac{1+\sqrt{5}}{2}\right)^n + b_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

Again $a_0 = 1 \Rightarrow b_1 + b_2 = 1$

$a_1 = 1 \Rightarrow b_1\left(\frac{1+\sqrt{5}}{2}\right) + b_2\left(\frac{1-\sqrt{5}}{2}\right) = 1$

$$\therefore b_1 = \frac{\sqrt{5}+1}{2\sqrt{5}}, \quad b_2 = \frac{\sqrt{5}-1}{2\sqrt{5}}$$

$$\therefore f_n = \left(\frac{\sqrt{5}+1}{2\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{\sqrt{5}-1}{2\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n$$

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Q.6 a. Show that $A \cap (B - C) = (A \cap B) - (A \cap C)$ notations are usuals. (8)

Answer:

$$R.H.S = (A \cap B) - (A \cap C)$$

$$= (A \cap B) \cap (A \cap C)' \quad (\because P - Q = P \cap Q')$$

$$= (A \cap B) \cap (A' \cup C') \quad \text{(Using DeMorgan Law)}$$

$$= (A \cap B \cap A') \cup (A \cap B \cap C') \quad \text{(Using Distributive Law)}$$

$$= (A \cap A' \cap B) \cup (A \cap B \cap C') \quad \text{(Using Commutative law)}$$

$$= (\varphi \cap B) \cup (A \cap B \cap C') \quad \text{(Using Inverse law)}$$

$$= \varphi \cup (A \cap B \cap C') \quad \text{(Using Identity law)}$$

$$= A \cap B \cap C' \quad \text{(Using Identity law)}$$

$$= A \cap (B \cap C')$$

$$= A \cap (B - C) = L.H.S$$

b. In a city three daily newspapers, X, Y, Z are published. 65% of the people of the city read X, 54% read Y, 45% read Z, 38% read X and Y, 32% read Y and Z, 28% read X and Z, 12% do not read any of the three papers. If 1000000 persons live in the city, find the number of the persons who read all the three newspapers. (8)

Answer:

Here $n(X)=65$, $n(Y)=54$, $n(Z)=45$

$n(X\cap Y)=38$ $n(Y\cap Z)=32$, $n(X\cap Z)=28$

$n(X\cup Y\cup Z)'=12$ $n(X\cap Y\cap Z)=?$

$n(X\cup Y\cup Z)=1000000-n(X\cup Y\cup Z)'$

$=88\%$

The numbers of persons who read all the three newspapers:

$$n(X\cup Y\cup Z)=n(X)+n(Y)+n(Z)-n(X\cap Y)-n(Y\cap Z)-n(X\cap Z)+n(X\cap Y\cap Z)$$

$$\therefore\ 88=65+54+45-38-32-28+n(X\cap Y\cap Z)$$

$$\therefore\ n(X\cap Y\cap Z)=22\%$$

$$=\frac{22}{100}\times 1000000$$

$$=220000$$

Q.7 a. Using rules of inference, show that $s\vee r$ **is tautologically implied by** $p\vee q$, $p\to r, q\to s$ **(8)**

Answer:

Sl. no	Proposition	Explanation
1.	$p \vee q$	Rule p
2.	$7p \to q$	Equivalence on (1)
3.	$q \to s$	Rule p
4.	$7p \to s$	chain Rule (2), (3)
5.	$p \to r$	Rule p
6.	$s \to p$	Equivalence on (4)
7.	$7s \to r$	chain rule 6, 5
8.	$s \vee r$	Equivalence on (7)

b. Give direct and indirect proof of $p \to q,\ q \to r,\ 7(p \wedge q),\ p \vee r \Rightarrow r$ **(8)**

Answer:

b. Direct proof

1. $p \rightarrow q$
2. $q \rightarrow r$
3. $p \rightarrow r$
4. $p \vee r$
5. $\neg p \vee r$
6. $(p \vee r) \wedge (\neg p \vee r)$
7. $(p \vee \neg p) \vee r$
8. $F \vee r$
9. r

Indirect proof:

1. $\neg p$
2. $p \vee r$
3. p
4. $p \rightarrow q$
5. q
6. $q \rightarrow r$
7. r
8. F

Q.8 a. If $a = \{1,2,3,4,5,6,7\}$ and a relation R defined as $R = \{(x, y), \|x - y\| = 2\}$. Is R is an equivalence relation? (8)

Answer:

$R = \{(1,3), (2,4), (3,5) (4,6), (5,7), (6,4) (5,3) (4,2), (3,1), (7,5)\}$

R is not reflexive $\because (x,x) \notin R$

R is Symmetric $(x,y) \in R$ and $(y,x) \in R$

R is not transitive $(x,y), (y,z) \in R$
but $(x,z) \notin R$

$\therefore$ R is not equivalence relation.

b. If $(L, \leq)$ **be a Lattice and** $a, b, c \in L$ **prove that if** $a \leq c, b \leq c$, **(8)**

(i) $a \vee b = a \oplus b \leq c$

(ii) $a \wedge b = a * b \leq c$

Answer:

(i) Given $a \leq c$, $b \leq c$

c is the upper bound of a and b

w.r.t $a \vee b$ is the least upper bound of a & b

$a \vee b \leq a = c$ & $a \vee b \leq b = c$

$\therefore a \vee b \leq c$

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b (ii) Let $x = a * b = a \wedge b$

$\Rightarrow a \wedge b$ is glb of a & b

$\Rightarrow a \wedge b$ is the lower bound of a & b

$\Rightarrow a \wedge b \leq a$, $a \wedge b \leq b$

$\Rightarrow a \leq c$

$\therefore a \wedge b \leq c$

Q.9 a. Check whether the function $f(x) = x^2 - 11$ **from R to R is one-one onto or both. Justify. (8)**

Answer:

Given that $f(x) = x^2 - 11 \ \forall x \in R$

For one-one $f(x) = f(y)$

$\Rightarrow x = y$

$\therefore x^2 - 11 = y^2 - 11$

$\therefore x^2 = y^2 \Rightarrow x = \pm y$

f is not one-one

For onto $\forall y \in R \ \exists f(x) = y$

$\therefore x^2 - 11 = y, \ x^2 = y + 11$

$\therefore x = \sqrt{y+11} \notin R$

$\therefore$ f is not onto.

b. If $f, g : R \to R$ **where** $f(x) = ax + b, \ g(x) = 1 - x + x^2$ **and** $(g_0 f)(x) = 9x^2 - 9x + 3$**, then find the values of "a" and "b".** **(8)**

Answer:

b Given $(g_o f) f(x) = 9x^2 - 9x + 3$

$g[f(x)] = 9x^2 - 9x + 3$

$g(ax+b) = 9x^2 - 9x + 3$

$1 - (ax+b) + (ax+b)^2 = 9x^2 - 9x + 3$

Equating coefficient of x^2, $a^2 = 9 \Rightarrow a = \pm 3$

" " x $-a + 2ab = -9$

" " constant term $1 - b + b^2 = 3$

$b^2 - b - 2 = 0$

$b = \frac{1 \pm 3}{2} = 2, -1$

$\therefore a = 3, -3$ $b = 2, -1$

<u>Text Book</u>

<u>D</u>iscrete Mathematical Structures, D S Chandrasekharaiah, Prism Books Pvt. Ltd., 2011