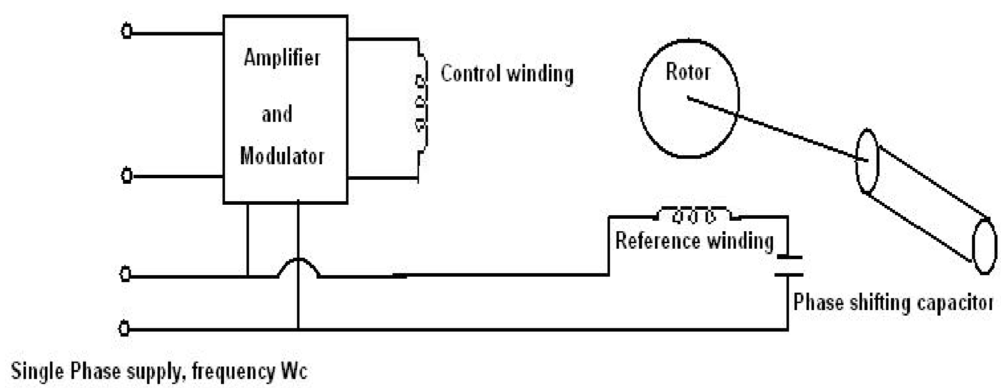
Q.2a. Compare open loop & closed loop control systems using suitable example.

Ans : Article 1.3 of Text book I

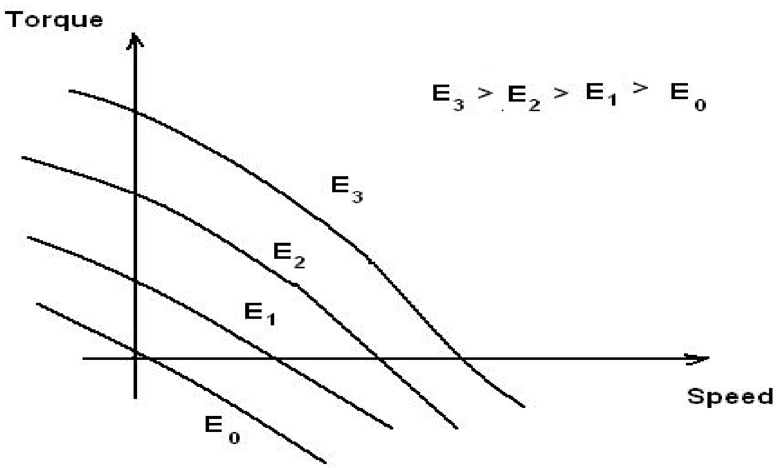
b. Describe a two phase a.c. servomotor and derive its transfer function.

Working of AC Servomotor:
The symbolic representation of an AC servomotor as a control system is shown in figure. The reference winding is excited by a constant voltage source with a frequency of the range 50 to 1000Hz. By using frequency of 400Hz or higher, the system can be made less susceptible to low frequency noise. Due to this feature, ac drives are extensively used in aircraft and missile control system in which the noise and disturbance often create problems



Symbolic Representation of AC servo motor

The control winding is excited by the modulated control signal and this voltage is of variable magnitude and polarity. The control signal of the servo loop (or the system) dictates the magnitude and polarity of this voltage.The control phase voltage is supplied from a servo amplifier and it has a variable magnitude and polarity (+ or –90^o phase angle w.r.to the reference phase). The direction of rotation of the motor reverses as the polarity of the control phase signal changes sign.



Servo Characteristics

Note that the curve for zero control voltage goes through the origin and the motor develops a decelerating torque.

From the torque speed characteristic shown above we can write

$$T = -k_n \frac{d\theta}{dt} + k_c e_c \qquad (1)$$

Where

T = torque

k_n = a positive constant = -ve of the slope of the torque-speed curve

k_c = a positive constant = torque per unit control voltage at zero speed

θ = angular displacement

Further, for motor we have

$$T = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} \qquad (2)$$

Where J = moment of inertia of motor and load reffered to motor shaft

f = viscous friction coefficient of the motor and load referred to the motor shaft

form eqs.(1) and (2) we have

$$J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} = -k_n\frac{d\theta}{dt} + k_c e_c$$

$$J\frac{d^2\theta}{dt^2}+(f+k_n)\frac{d\theta}{dt}=k_c e_c \qquad (3)$$

Taking the laplace transform on both sides, putting initial conditions zero and simplifying we get

$$\frac{\theta(s)}{E_c(s)}=\frac{k_c}{Js^2+(f+k_n)s}=\frac{k_m}{s(\tau_m s+1)} \qquad (4)$$

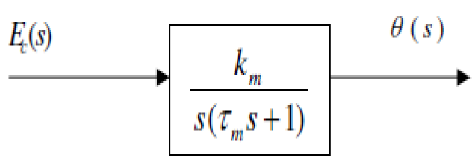
Where

$$k_m=\frac{k_c}{(f+k_n)}=\text{motor gain constant}$$

If the moment of inertia J is small. Then τ_m is small and for the frequency range of relevance to ac servometer $|\tau_m s|<<1$,then from eq (4) we can write the transfer function as

$$\frac{\theta(s)}{E_c(s)}=\frac{k_m}{s} \qquad (5)$$

It means that ac servometer works as an integrator. Following figure gives the simplified block diagram of an ac servometer.



Q.3a. The transfer functions for a single-loop non-unity-feedback control system are given as (9)

(i) $G(s)H(s)=\dfrac{4}{(s+1)(s+2)}$

(ii) $G(s)H(s)=\dfrac{2}{s(s+4)(s+6)}$

(iii) $G(s)H(s) = \dfrac{5}{s^2(s+3)(s+10)}$

Find the steady-state errors due to a unit-step input, a unit-ramp input and a parabolic input.

Q.3 (a)

(i) $G(s)H(s) = \dfrac{4}{(s+1)(s+2)}$

Positional error for unit step input

$$e_{ss} = \lim_{s \to 0} \frac{1}{1 + G(s)} = \frac{1}{1 + G(0)} = \frac{1}{1 + k_p}$$

k = G(0) is defined as positional error constant

$k\,p$ = 4/2 = 2 e_{ss} = 1/3

k_v = lim s G(s) is defined as velocity error constant = $\lim s \to 0 \dfrac{s * 4}{(s+1)(s+2)}$

velocity error for ramp input

$$e_{ss} = \lim_{s \to 0} \frac{1}{s(1 + G(0))} = \lim_{s \to 0} \frac{1}{sG(s)} = \frac{1}{k\,v} = □$$

Acceleration error constant $k\,a$

$$e\,ss = \lim_{s \to 0} \frac{1}{s^2(1+G(s))} = \lim_{s \to 0} \frac{1}{s^2G(s)} = \frac{1}{k a} = □$$

lim s^2 G(s) is defined as acceleration error constant= $\lim s \to 0 \dfrac{s^2 * 4}{(s+1)(s+2)}$ = 0

(ii) $G(s)H(s) = \dfrac{2}{s(s+4)(s+6)}$

$k\,p$

$$= \lim \frac{2}{\quad} = \infty$$

$s \to 0 \quad s(s+4)(s+6)$

$$k_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{2s}{s(s+4)(s+6)} = \frac{2}{24} = \frac{1}{12}$$

$$k_a = \lim_{s\to 0} s^2G(s)H(s) = \lim_{s\to 0} \frac{2s^2}{s(s+4)(s+6)} = 0$$

(iii) $G(s)H(s) = \dfrac{5}{s^2(s+3)(s+10)}$

$$k_p = \lim_{s\to 0} G(s)H(s)$$

$$= \lim_{s\to 0} \frac{5}{s^2(s+3)(s+10)} = \infty$$

$$k_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{5s}{s^2(s+3)(s+10)} = \infty$$

$$k_a = \lim_{s\to 0} s^2G(s)H(s) = \lim_{s\to 0} \frac{5s^2}{s^2(s+3)(s+10)} = \frac{1}{6}$$

b. Consider the closed-loop system given by

$$\frac{C(s)}{R(s)} = \frac{wn^2}{s^2 + 2\varsigma wns + wn^2}$$

Determine the values of □□ and wn so that the system responds to a unit step input with approximately 5% overshoot and with a settling time of 2 seconds (use the 2% error criterion)

$$\frac{C(s)}{R(s)} = \frac{wn^2}{s^2 + 2\ \square\square\ wns + wn^2}$$

%Mp=5% = e(-□□□ / √(1 − □²□)) Given;

□□ = 0.698

for 2% $t_s = 4/\square wn$ = 2 sec

wn = 2.89 rad/sec

Q.4 a. Determine the transfer function of a control system shown in Fig.2:

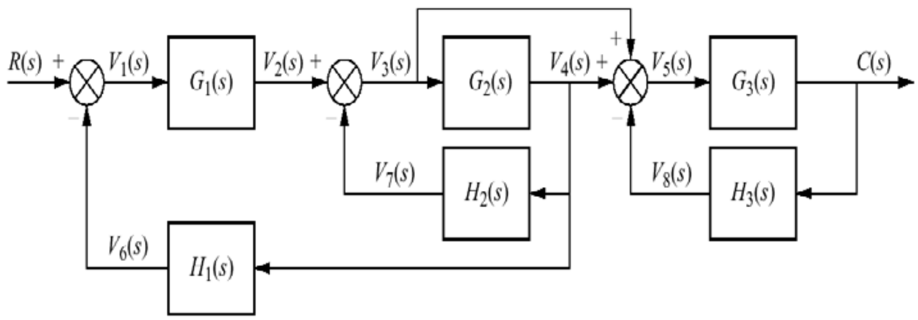
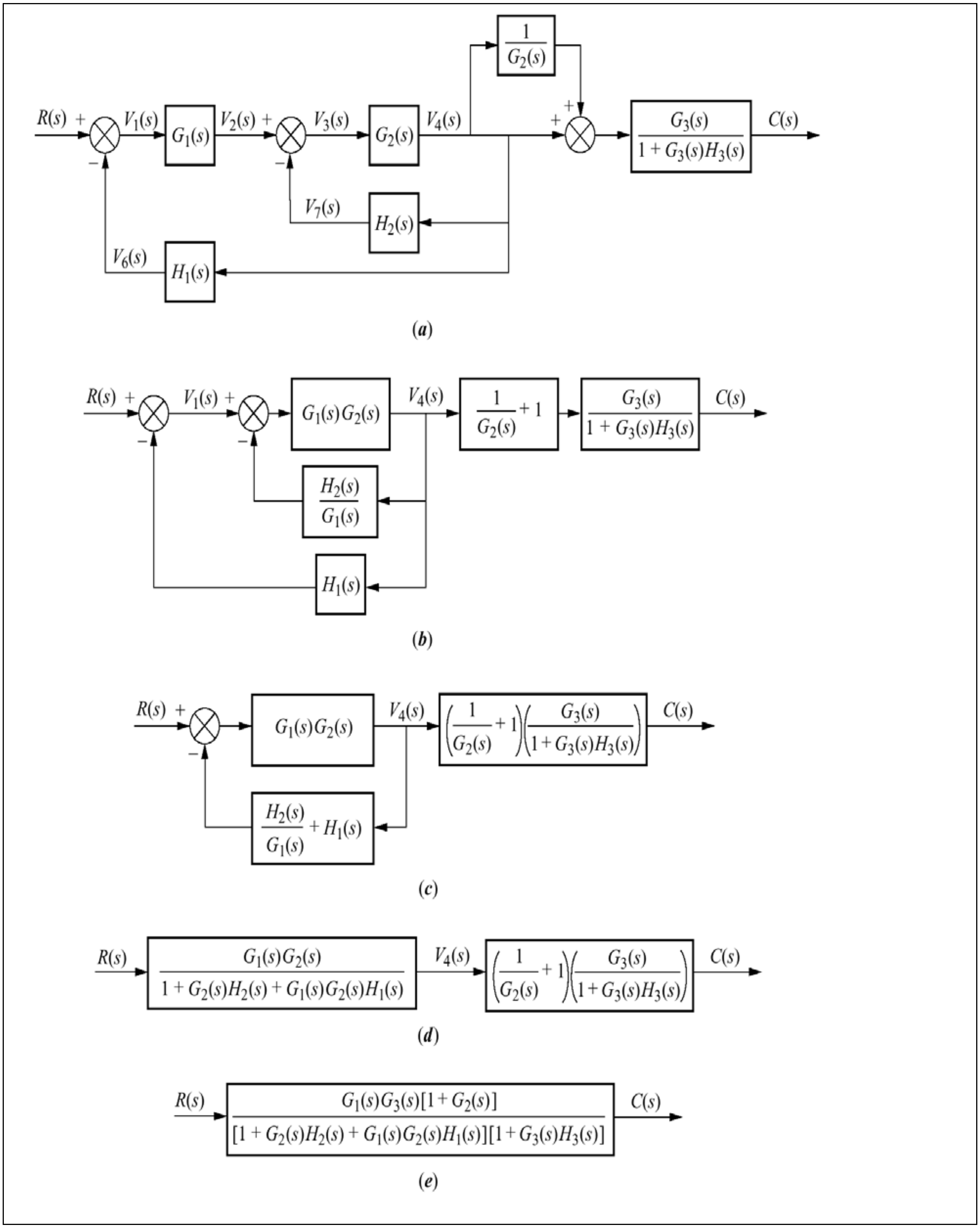


Fig.2



R(s)
V1(s)
G1(s)
V2(s)
V3(s)
G2(s)
V4(s)
1/G2(s)
G3(s)/(1 + G3(s)H3(s))
C(s)
V7(s)
H2(s)
V6(s)
H1(s)
(a)
G1(s)G2(s)
(1/G2(s) + 1)
H2(s)/G1(s)
(b)
H2(s)/G1(s) + H1(s)
(c)
G1(s)G2(s)/(1 + G2(s)H2(s) + G1(s)G2(s)H1(s))
(d)
G1(s)G3(s)[1 + G2(s)] / [1 + G2(s)H2(s) + G1(s)G2(s)H1(s)][1 + G3(s)H3(s)]
(e)

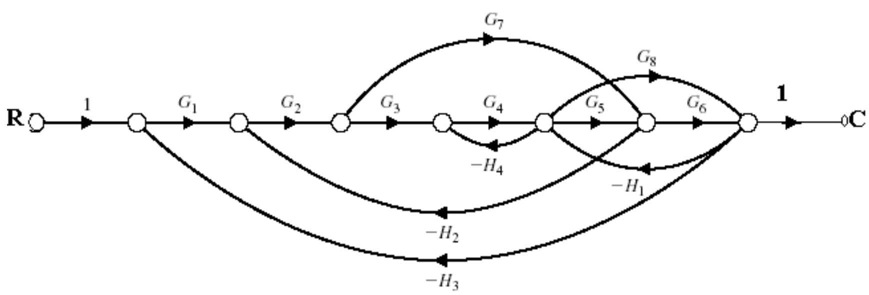
b.Comment on the role of positive feedback and negative feedback in closed –loop control configurations.

Positive feedback is a process in which the effects of a small disturbance on a system include an increase in the magnitude of the perturbation. That is, A produces more of B which in turn produces more of A. In contrast, a system in which the results of a change act to reduce or counteract it has negative feedback.

Mathematically, positive feedback is defined as a positive loop gain around a feedback loop. That is, positive feedback is in phase with the input, in the sense that it adds to make the input larger. Positive feedback tends to cause system instability. When the loop gain is positive and above 1, there will typically be exponential growth, increasing oscillations or divergences from equilibrium. System parameters will typically accelerate towards extreme values, which may damage or destroy the system, or may end with the system latched into a new stable state. Positive feedback may be controlled by signals in the system being filtered, damped, or limited, or it can be cancelled or reduced by adding negative feedback.

Positive feedback is used in digital electronics to force voltages away from intermediate voltages into '0' and '1' states. On the other hand, thermal runaway is a positive feedback that can destroy semiconductor junctions. Positive feedback in chemical reactions can increase the rate of reactions, and in some cases can lead to explosions. Positive feedback in mechanical design causes tipping-point, or 'over-centre', mechanisms to snap into position, for example in switches and locking pliers. Out of control, it can cause bridges to collapse. Positive feedback in economic systems can cause boom-then-bust cycles. A familiar example is the loud squealing or screeching sounds produced by PA systems due to audio feedback, when the microphone picks up sound from its own loudspeakers, and these sounds are re-amplified and sent through the speakers again.

Q.5 a. Obtain the transfer function C(s)/R(s) of the following signal flow graph as shown in fig.3



Ans

$$\frac{Y(s)}{R(s)} = \frac{P_1 + P_2 \cdot \Delta_2 + P_3}{\Delta}$$

$P_1 = G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot G_5 \cdot G_6$ $\quad P_2 = G_1 \cdot G_2 \cdot G_7 \cdot G_6$ $\quad P_3 = G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot G_8$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5 \cdot L_7 + L_5 \cdot L_4 + L_3 \cdot L_4)$$

$\Delta_1 = \Delta_3 = 1$ $\quad \Delta_2 = 1 - L_5 = 1 + G_4 \cdot H_4$

5a

$L_1 = -G_4H_4$

$L_2 = -G_5G_6H_1$

$L_3 = -G_8H_1$

$L_4 = -G_2G_3G_4G_5H_2$

$L_5 = -G_2G_7H_2$

$L_6 = -G_1G_2G_3G_4G_5G_6H_3$

$L_7 = -G_1G_2G_7G_6H_3$

$L_8 = -G_1G_2G_4G_8H_3$

5

Two Non touching loops:

① L_1L_5

② L_1L_7

b. Explain a signal flow graph, a node and a branch with suitable diagrams. State and explain the rules of algebra of signal flow graph.

Q.5 (b)

SFG provides the relation between system variables without requiring any reduction procedure.

Basic elements of SFG

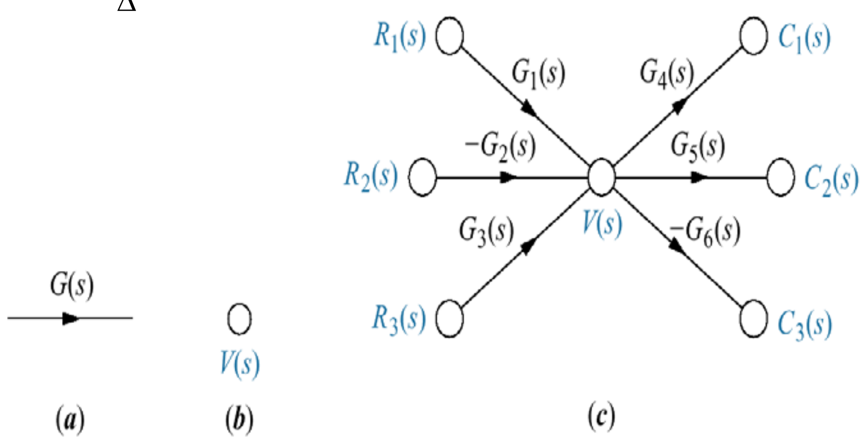
- Branch: A unidirectional path segment
- Nodes: The input and output points or junctions
- Path: A branch or a continuous sequence of branches that can be traversed from one

node to anther node.

- Loop: A closed path that originates and terminates on the same node, and along the path no node is met twice.
- Non-touching loops: If two loops do not have a common node.
- Touching loops: Two touching loops share one or more common nodes

Mason's gain formula
The linear dependence (Gain) T_{ij} between input variable x_i and output variable x_j is given by the following formula:

$$T_{ij} = \frac{\sum_k P_{ijk}\Delta_{ijk}}{\Delta}$$



Signal-flow graph components:
a. system;
b. signal;
c. interconnection of systems and signals

Q.6 a. A unity negative feedback system has open loop transfer function of

$$G(s) = \frac{K}{s+4}$$

Consider a cascade compensator $G_C(s) = \frac{s+\alpha}{s}$

Select the value of K & α to achieve
(i) Peak over shoot of 20%
(ii) Setting time (2% basis) $\cong$ 1 sec **(10**

6(a). $G(s)G_c(s) =$

$1 + G(s)H(s) = 0$

$s^2 + (4+K)s + K\alpha = 0$

$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.2$ —— (3)

so $\xi = 0.456$

$t_s = \frac{4}{\xi\omega_n} = 1$ —— (1)

$\omega_n = 8.77$ —— (1)

$K\alpha = \omega_n^2 = 76.9$ —— (2)

$K + 4 = 2\xi\omega_n = 8$ —— (2)

$K = 4$ & $\alpha = 19.23$

so $G_P(s) = \frac{4}{s+19.23}$ —— (1)

b. Find sensitivity of overall transfer function w.r.t. forward path transfer function.

6 (b)

Sensitivity: Measure of the effectiveness of feedback in reducing the influence of the variations (changing environment) on system performance It gives an assessment of the system performance as affected due to parameter variation. sensitivity of overall transfer function w.r.t. forward path transfer function

$$M(s) = \frac{C(s)}{R(S)} = \frac{G(s)}{1+G(S)H(S)} \rightarrow (1)$$

$$C(S) + \Delta\mathrm{C(S)} = \frac{[\mathrm{G(S)} + \Delta\mathrm{G(S)}]}{1+[G(S)H(S) + \Delta\mathrm{G(S)H(S)}]}\mathrm{R(S)}$$

$$= \frac{[\mathrm{G(S)R(S)}\ ]}{1+[G(S)H(S) + \Delta\mathrm{G(S)H(S)}]} + \frac{\Delta\mathrm{G(S)R(S)}}{1+[G(S)H(S) + \Delta\mathrm{G(S)H(S)}}$$

$NEGLECT\ \Delta\mathrm{G(S)\ AS}\ \Delta\mathrm{G(S)} << \mathrm{G(S)}$

$\therefore \Delta\mathrm{G(S)H(S)\ CAN\ BE\ NEGLECTED}$

$$= \frac{[\mathrm{G(S)R(S)}\ ]}{1+[G(S)H(S)]} + \frac{\Delta\mathrm{G(S)R(S)}}{1+[G(S)H(S) + \Delta\mathrm{G(S)H(S)}}$$

$PUT\ EQN.1\ \mathrm{EQN.3}$

$$\Delta\mathrm{C(S)} = \frac{\Delta\mathrm{G(S)}}{1+G(S)H(S)}\mathrm{R(S)}$$

Sensitivity of overall transfer function w.r.t. fwd path transfer function in case of closed loop system is reduced by 1+G(s)H(s) as compared to open loop system

Q.7 a. Draw the complete Nyquist plot for a unity feedback system having the open loop transfer function $G(s)H(s) = \frac{6}{s(1+0.5s)(6+s)}$. From this plot obtain all the information regarding absolute as well as relative stability.

Ans

$$G(s)H(s) = \frac{6}{s(1+0.5s)(6+s)}$$

G(jw) at w = 0 = □□□ 90^0G(jw) at w= 00 0□90

$G(jw)H(jw) = \frac{6}{(s+0.5s^2)(6+s)}$ at s=jw $\quad G(jw)H(jw) = \frac{6}{(jw-0.5w^2)(6+jw)}$

=6(-0.5w2 – jw)

Nyquist contour is given by Semicircle around the origin represented by

S=$\varepsilon e^{j\theta} \varepsilon \rightarrow 0$

θ varying from -90^0 through 0^0 to 90^0

Maps into

$$\lim \varepsilon \to 0 \frac{6}{6\varepsilon e^{j\theta}\left(1+0.5\varepsilon e^{j\theta}\right)\left(1+\frac{1}{6\varepsilon e^{j\theta}}\right)} = \infty$$

θ varying from -90^0 through 0^0 to 90^0

Mapping into imaginary axis (w=0+ to ∞+)

Calculate magnitude and phase value of transfer function

$G(jw)H(jw) = \frac{6}{6jw(1+0.5jw)\left(1+\frac{1}{6jw}\right)}$ at various values of w

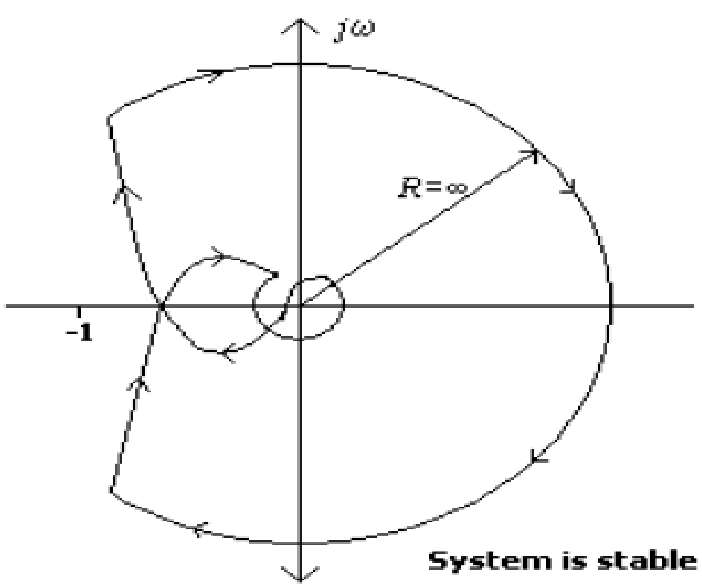
	1	10	50	100	500
Magnitude	-1.085	-39.89	-80.389	-98.39	-140.32
phase	234	132.37	99.16	94.59	90.91

Mapping of infinite semicircular arc of the nyquist contour represented by

S=$Re^{j\theta}$ (varying from +90 through 0 to -90 as R→ ∞)

$$\lim R \to \infty \frac{6}{6\,\mathrm{Re}^{j\theta}\left(1+0.5\,\mathrm{Re}^{j\theta}\right)\left(1+0.166\,\mathrm{Re}^{j\theta}\right)} = 0e^{-3j\theta}$$

$$-270^0 through\ to\ 270^0$$



NYQUIST PLOT

b. Discuss relative stability.

Ans: article 11.11 of text book I

Q.8 Draw the root locus plot for a unity feedback control system having open loop transfer function as $G(s)=\dfrac{k(s+3)}{s(s^2+2s+2)(s+4)(s+5)}$.Thus find the value of K at a point where the complex poles provide a damping factor of 0.5.

$$G(s) = \frac{K(s + 3)}{s(s2 +2s +2)(s+5)(s+9)}$$

Location of poles and zeros
s2 +2s +2 = 0
poles s=0, -1±j1, -5,-9
zeros s = -3

$$\text{angle of asympototes} = \frac{180(2q + 1)}{(n – m)}$$

n =5, m = 1
± 45 , ± 135

$$\text{centroid} = \frac{-5-9-1-1-(-3)}{4} = -13/4$$

Break away point

$$\frac{C(S)}{R(s)} = \frac{k(s+3)}{s(s^2 + 2s+2)(s+5)(s+9) +k(s+3)}$$

Characterstic equation;
$(s^3 + 2s^2 + 2s)(s^2 + 14s +45) + k(s+3) =0$
$s^5 +14s^4 +45s^3 +2s^4 +28s^3 +90s^2 +2s^3 +28s^2 +90s)+k(s+3) =0$
ds (s+3)^2
$(4s^5+63s^4+342s^3+793s^2+708s+270)=0$

s = -7.407, -3.522±j1.22,-0.64±j0.48
-7.4 is the breaking point

Angle of departure at A

□1=90^0

□□ = 180^0-tan-1(1/1) = 135^0

□3 = tan-1(1/4) = 14.03^0

□4 = tan-1(1/8) = 7.125^0

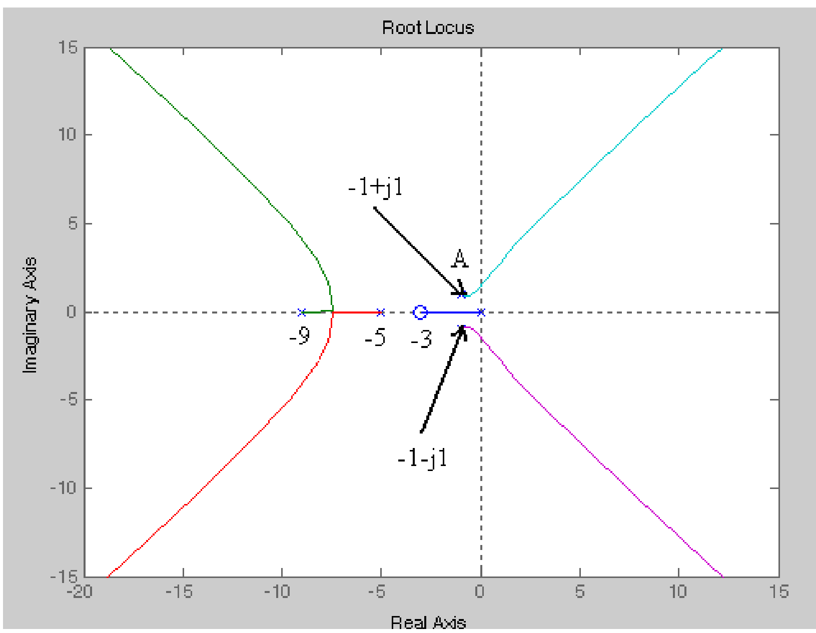
A = tan-1 (1/2)= 26.56^0

$180^0 – (90^0+135^0+14.03^0+7.125^0) +26.56^0= -39.6^0$

To find k at ζ=0.5

$\alpha = \cos^{-1}0.5 = 60$

$$\text{value of k} = \frac{\text{product of length of vector from all poles to point}}{\text{product of length from all zeros to point}}$$



Q.9 a. Obtain Bode Plots for the system: **(8)**

$$G(s) = \frac{K}{s(s+1)(s+10)}$$

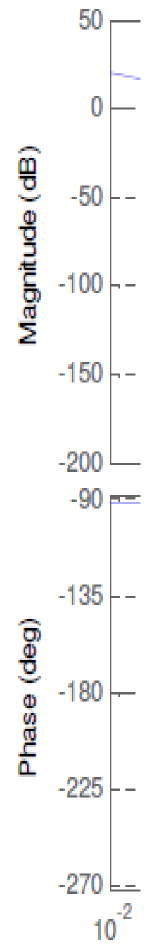
Also obtain GM and PM. Comment on stability.

Ans.

$$G(s) = \frac{k}{s(s+1)\,(s+10)}$$

GM = 40.8 dB PM = 83.7
Ø gc = 73.7 – 180 = -106.3
Gain at -106.3 is -8.64
20 log k = 8.64 k = 1.5

	0.01	0.1	1	10
l G(jw) l	20	0	-23.2	-63.2
<G(jw)	-91.1	-96.7	-141	-220



b. Discuss M and N circles.

Q.9 (b)

(i) The magnitude of closed loop transfer function with unity feedback can be shown to be in the form of circle for every value of M. These circles are called Mcircles. If the phase of closed loop transfer function with unity feedback is α, then it can be shown that tan α will be on the form of circle for every value of α. These circles are called N circles.The M and N circles are used to find the closed loop frequency response graphically form the open loop frequency response G(jw) without calculating the magnitude and phase of the closed loop transfer for at each frequency .

For M circles:

Consider the closed loop transfer function of unity feedback system.

C(s) / R(s) = G(s) / 1+G(s)

Put s = jw;

C(jw) /R(jw) =G(jw) / 1 + G(jw)

$(X + M^2 / M^2-1)^2 + Y^2 = (M^2 / M^2-1)^2$ ---- (1)

The equation of circle with centre at (X1,Y1) and radius r is given by

$(X - X1)^2 + (Y – Y1)^2 = r^2$ -----(2)

On comparing eqn (1) and (2)

When M = 0;

$X1 = - M^2 / M^2-1 =0$

Y1 = 0

$R = M / M^2-1 = 0$;

When M= ∞

$X1 = - M^2 / M^2-1 = -1$

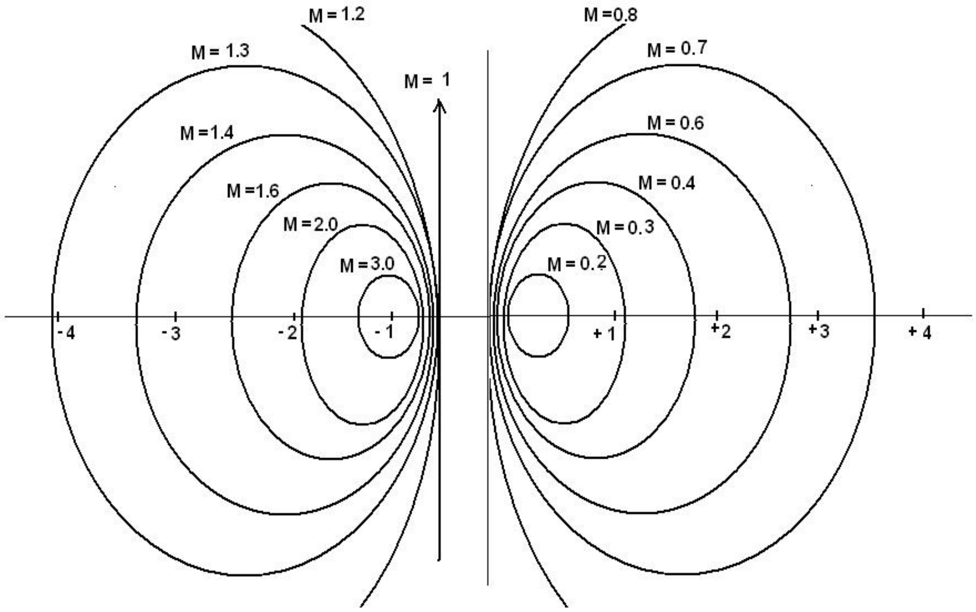
Y1= 0;

$R = M / M^2-1 = 1/M = 0$

When M = 0 the magnitude circle becomes a point at (0, 0)

When M=∞, the magnitude circle becomes a point at (-1, 0)

From above analysis it is clear that magnitude of closed loop transfer function will be in the form of circles when M ¹ 1 and when M=1 , the magnitude is a straight line passing through (-1/2 , 0).



Family of M- Circles

Family of N- circles:
For constant N circles

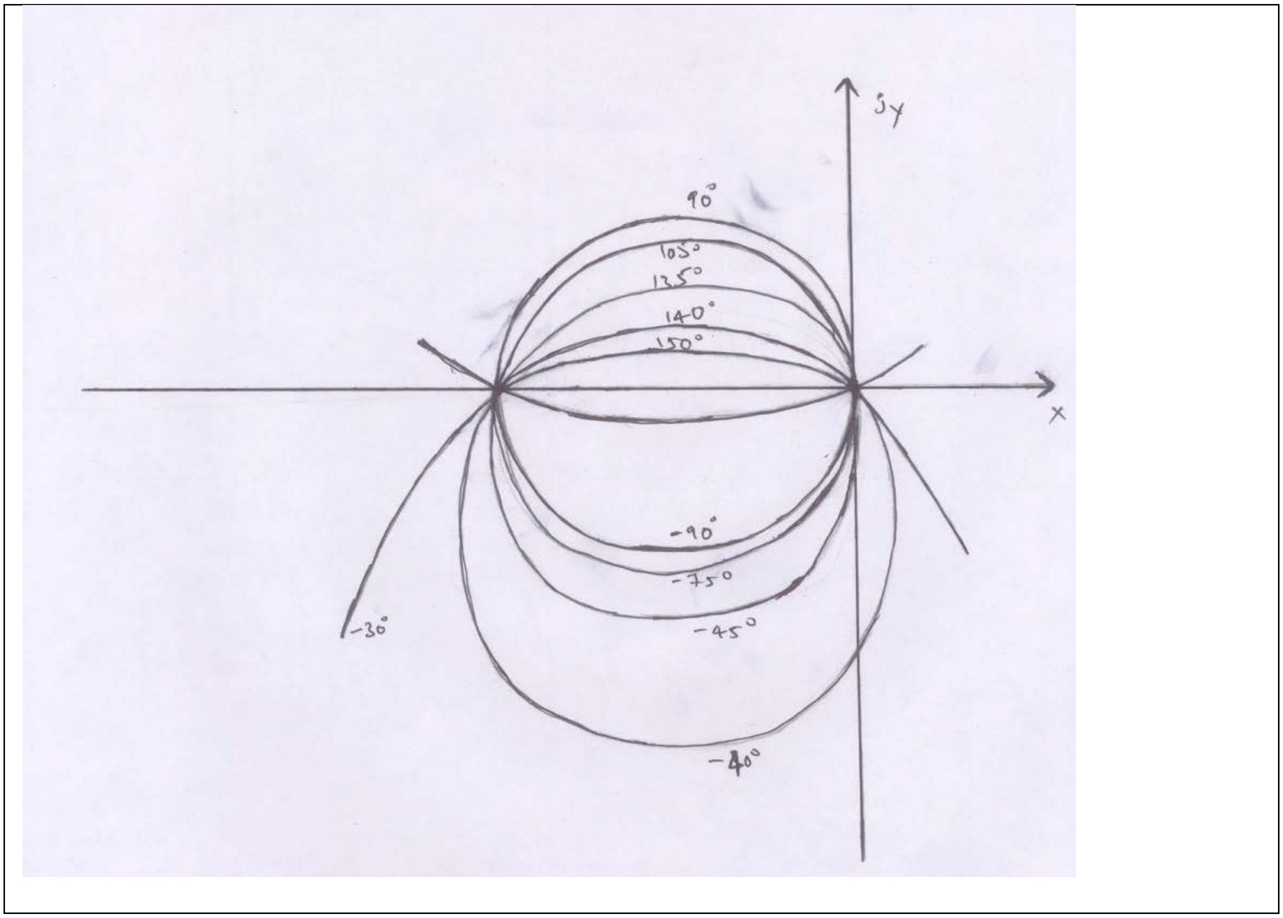
Tan a = N $= \dfrac{y}{x^2 + y^2 + x}$

Constant N-circles have centre as

X_0 =1/2 ;

Y_0 = 1/2N ;

Radius= $= \dfrac{(N^2 + 1)^{1/2}}{2N}$



Text book

1. Feedback and Control Systems (Schaum's Outlines), Joseph J DiStefano III, Allen R. Stubberud and Ivan J. Williams, 2nd Edition, 2007, Tata McGraw-Hill Publishing Company Ltd