Q.2a. Find the value of $v(t)$ using Laplace transform if

$$V(s) = \frac{s+2}{s(s+1)+(s+3)}$$

What is its value at $t = 0^+$ and $t = \infty$?

Q 2(a) Given that

$$V(s) = \frac{s+2}{s(s+1)(s+3)} = \frac{a_1}{s} + \frac{a_2}{s+1} + \frac{a_3}{s+3}$$

$$a_1 = \left| \frac{s+2}{(s+1)(s+3)} \right|_{s=0} = \frac{2}{3}$$

$$a_2 = \left| \frac{s+2}{s(s+3)} \right|_{s=-1} = \left| \frac{-1+2}{-1(-1+3)} \right| = -\frac{1}{2}$$

$$a_3 = \left| \frac{s+2}{s(s+1)} \right|_{s=-3} = \left| \frac{-3+2}{-3(-3+1)} \right| = -\frac{1}{6}$$

Therefore, $V(s) = \frac{2/3}{s} - \frac{1/2}{s+1} - \frac{\frac{1}{6}}{s+3}$

Taking inverse Laplace Transform

$$\therefore v(t) = \mathcal{L}^{-1}\left[\frac{2/3}{s} - \frac{1/2}{s+1} - \frac{1/6}{s+3}\right]$$

$$v(t) = \frac{2}{3}u(t) - \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t}$$

At $t=0$, $V(0^+) = 0$ and at $t=\infty$ $V(\infty) = \frac{2}{3}$

Making use of initial value theorem

$$\lim_{t \to 0} v(t) = \lim_{s \to \infty} sV(s) = \lim_{s \to \infty} \frac{s(s+2)}{s(s+1)(s+3)} = 0$$

and making use of final value theorem

$$\underset{t\to\infty}{v(t)} = \underset{s\to 0}{sV(s)} = \frac{s(s+2)}{s(s+2)(s+3)} = \frac{2}{3}$$

b.State and prove the shifting theorem of Laplace Transform.

Q 2(b) Shifting Theorem

This theorem states that Laplace of any function shifted or ~~dayed~~ delayed by a time interval of $\underline{a}$ is e^{-as} times the transform of the function

Proof :- Let any function $f(t)$ is shifted by time $\underline{a}$. The shifted function may be represented by $f(t-a) \cdot u(t-a)$

Choosing a new variable x, by definition, we have

$$F(s) = \int_0^\infty f(x)\, e^{-sx}\, dx.$$

Let $x = t-a$, the above expression will become

$$F(s) = \int_0^\infty f(t-a)\, e^{-s(t-a)}\, d(t-a)$$

$$= \int_0^\infty f(t-a)\, e^{-s(t-a)}\, dt$$

$$= e^{as}\int_0^\infty f(t-a)\, e^{-as}\, dt$$

The lower limit of integration may be changed from a to 0 if $f(t-a)$ is multiplied by $u(t-a)$

Therefore, $F(s) = e^{as}\int_0^{\infty} f(t-a)\,u(t-a)\,e^{-st}\,dt$

$= e^{as}\mathcal{L}[f(t-a)\,u(t-a)]\,dt.$

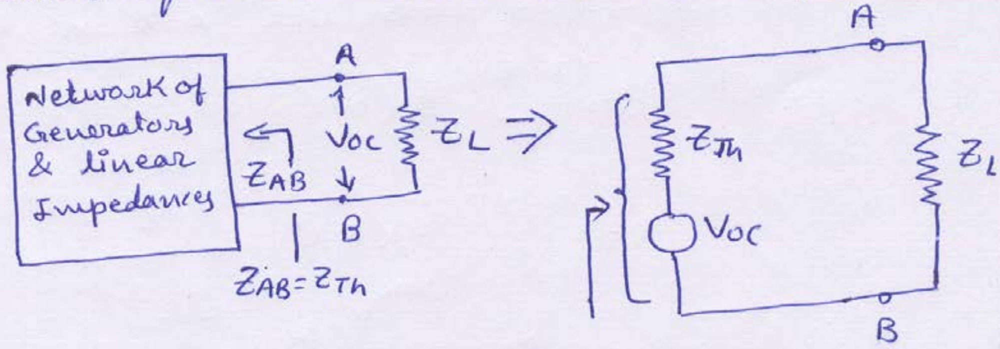
or $= \mathcal{L}[f(t-a)\cdot u(t-a)] = e^{-as}f(s)$

Thus the shifting theorem is proved.

Q.3 a. State and explain Thevenin's theorem. What are its limitations?

Thevenin's Theorem

Thevenin's Theorem state that "Any two terminal consisting of linear impedances and generators may be replaced by an e.m.f. acting in series with an impedance. The e.m.f. is the open circuit voltage at the terminals and the impedance is the impedance viewed at the terminals when all the generators in the network have been replaced by impedances equal to their internal impedances."



Thevenin Source.

Proof :- Consider a general T-network,

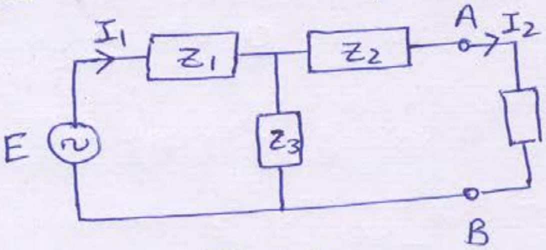


Fig (1)

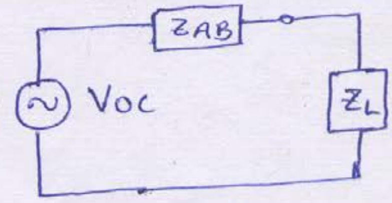


Fig (2).

Applying KVL in two loops of fig (1), we have

$$E = I_1 Z_1 + (I_1 - I_2) Z_3 \quad \text{------ (1)}$$

$$0 = -Z_3 (I_1 - I_2) + I_2 Z_2 + I_2 Z_L \quad \text{--- (2)}$$

From second equation

$$0 = -Z_3 I_1 + I_2 (Z_3 + Z_2 + Z_L)$$

or $$I_1 = \frac{I_2 (Z_3 + Z_2 + Z_L)}{Z_3}$$

The first equation can be re-written as,

$$E = I_1 (Z_1 + Z_3) - I_2 Z_3$$

Put value of I_1

$$E = \frac{I_2 (Z_3 + Z_2 + Z_L)}{Z_3}(Z_1 + Z_3) - I_2 Z_3$$

$$= I_2 \left\{ \frac{Z_1 Z_2 + Z_L Z_3 + Z_3 Z_1 + Z_L (Z_1 + Z_3)}{Z_3} \right]$$

or $$I_2 = \frac{E Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1 + Z_L (Z_1 + Z_3)}$$

Dividing the numerator and denominator by $Z_1 + Z_3$

$$I_2 = \frac{E\,\frac{Z_3}{Z_1+Z_3}}{\frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_1+Z_3} + Z_L} \quad \text{Or } I_2 = \frac{V_{OC}}{Z_{AB}+Z_L}$$

Where $Z_{AB} = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_1+Z_3}$ & $V_{OC} = \frac{EZ_3}{Z_1+Z_3}$

Now open circuit Voc which appear across AB of Figur (a).

$$V_{OC} = \frac{EZ_3}{Z_1+Z_3}$$

and if E short circuited, then impedance Z_{AB} Viewed at the terminal AB will be.

$$Z_{AB} = \frac{Z_2Z_1+Z_2Z_3+Z_3Z_1}{Z_1+Z_3}$$

Hence proved.

Limitation of Thevenin Theorem

(1) Thevenin's theorem cannot be applied to a network which contains non linear impedances.

(2) Thevenin's theorem cannot be applied for the power consumed internally, although it gives power in the load correctly.

(3) This theorem cannot be used for determining the efficiency of the circuit.

b.Apply the superposition theorem to the network shown in Fig.1 and obtain the current in the (3 + j4) Ohm impedance.

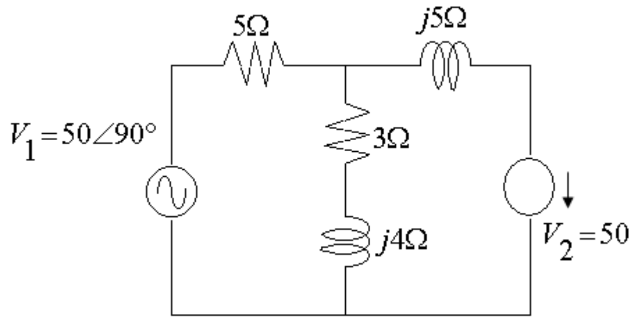


Fig.1

Let V_1 acts alone, then total impedance Z_1 of the circuit is given by

$$Z_1 = 5 + \frac{(3+j4)\,j5}{3+j4+j5} = 5.83 + j2.5 = 6.35\angle 23.3^\circ$$

Current I_1 in the circuit $= \frac{V_1}{Z_1} = \frac{50\angle 90}{6.35\angle 23.3^\circ} = 7.87\angle 66.8^\circ$ Amp.

Suppose I' is the current through $(3+4j)$ when only V_1 acts

$$I' = I_1 \frac{j5}{3+j9} = 7.87\angle 66.8^\circ \times \frac{j5}{3+j9} = \cancel{7.87}\ 4.14\angle 85.3^\circ \text{ Amp.}$$

Now let V_2 acts alone, the total impedance of the circuit Z_2

$$Z_2 = j5 + \frac{5(3+4j)}{8+j4} = 2.5 + j6.25 = 6.74\angle 68.2^\circ$$

Thus the current I_2 in the circuit

$$I_2 = \frac{V_2}{Z_2} = \frac{50\angle 0}{6.74\angle 68.2^\circ} = 7.42\angle -68.2^\circ$$

Suppose I'' is the current through $(3+j4)$ when only V_2 acts

$$I'' = -(7.42\angle -68.2^\circ)\cdot\frac{5}{8+j4} = 4.15\angle 85.3^\circ$$

The minus sign gives the I'' the same direction as the branch current I shown in the diagram.

The total current in the $(3+j4)\Omega$ branch is

$$I = I' + I'' = 4.14\angle 85.3^\circ + 4.15\angle 85.3^\circ = 8.30\angle 85.3^\circ \text{ Amp.}$$

Hence $8.30\angle 85.3^\circ$ Amp (Approximate) current is flowing in the $(3+j4)\Omega$ impedance.

Q.4 a. Find the z-parameters for the resistive network shown in Fig.2

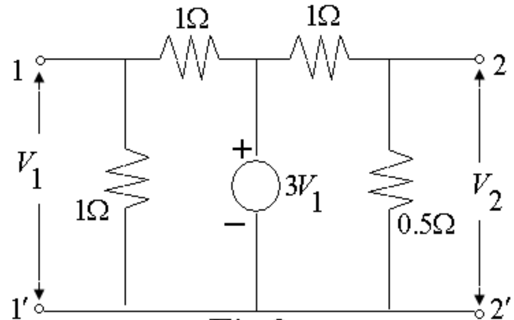
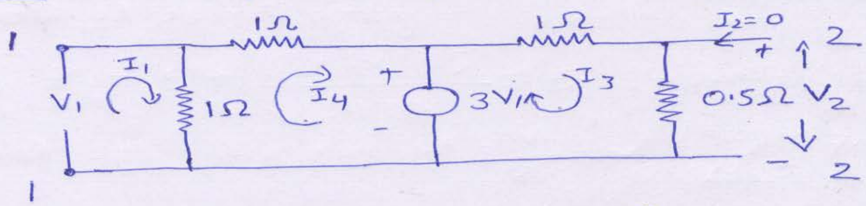


Fig.2

Q 4(a) Solution :- With port 2 open-circuited, we have



$V_2 = I_3 \times 0.5$ ------- (1)

$I_3(1+0.5) - 3V_1 = 0$ ---- (2)

$-I_1 + I_4(1+1) + 3V_1 = 0$ --- (3)

$V_1 = I_1 - I_4$ ---- (4)

From equation (3) & (4)

$$V_1 = I_1 - I_4 = I_1 - \frac{(I_1 - 3V_1)}{2} = \frac{2I_1 - I_1 + 3V_1}{2}$$

or $2V_1 = 3V_1 + I_1$ or $V_1 = -I_1$ --- (5)

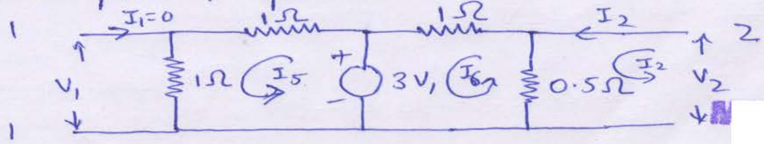
Hence $\boxed{Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = -1\Omega}$

Also from equation (1)

$V_2 = 0.5\, I_3 = 0.5 \cdot \frac{3V_1}{1.5} = V_1 = -I_1$ From equation (5)

$\therefore$ $\boxed{Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{-I_1}{I_1} = -1}$

with port 1 open circuited, we have



$$I_5 \times 1 = V_1 \quad \text{-----} (6)$$

$$2I_5 = 3V_1 \quad \text{-----} (7)$$

$$I_6(0.5+1) + 3V_1 = 0.5\, I_2 \quad \text{---} (8)$$

$$0.5(I_2 - I_6) = V_2 \quad \text{---} (9)$$

From Eqs (6) & (7), we have

$$I_5 = V_1 = 0$$

Hence from equation (8) & (9) we get

$$0.5(I_2 - I_6) = V_2 \quad \text{or} \quad 2V_2 = I_2 - I_6 \quad \text{--} (10)$$

From eqs (8) $I_6(1.5) + 0 = 0.5 I_2$

$$\text{or } I_6 = \frac{0.5}{1.5} I_2 = \frac{1}{3} I_2$$

Put value of I_6 in equation (10)

$$2V_2 = I_2 - I_6 = I_2 - \frac{1}{3} I_2 = \frac{2I_2}{3}$$

$$\therefore V_2 = \frac{1}{3} I_2$$

Therefore $Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1}{3}\,\Omega$

$$\& \quad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 0$$

$$\boxed{\begin{array}{ll} Z_{11} = -1\,\Omega, & Z_{12} = 0 \\ Z_{21} = -1\,\Omega, & Z_{22} = \frac{1}{3}\,\Omega \end{array}}$$

b. **The z-parameters of a two port network are Z_{11} = 10 Ω, Z_{22} = 20Ω, Z_{12} = Z_{21} = 20Ω. Find the ABCD parameters of this two port network.**

Q 4(b) Given $Z_{11} = 10\Omega$, $Z_{22} = 20\Omega$, $Z_{12} = Z_{21} = 5\Omega$s.

We know that ABCD parameters related to Z parameter as follows

$$A = \frac{Z_{11}}{Z_{12}} = \frac{10}{5} = 2$$

$$B = \frac{\Delta Z}{Z_{21}} = \frac{Z_{11}Z_{22} - Z_{12}\cdot Z_{21}}{Z_{21}} = \frac{10\times20 - 5\times5}{5}$$

MODERATION-I

$$= \frac{175}{5} = 35\,\Omega$$

$$C = \frac{1}{Z_{21}}, = \frac{1}{5} = 0.2 \text{ mho}$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{20}{5} = 4$$

[Note 2 marks for each parameter]

Q.5 **a.** **The circuit shown in Fig.3 represents a parallel resonant circuit where R_L is the ohmic resistance of the inductor L. Find the resonant frequency of the circuit.** **(8)** **(8)**

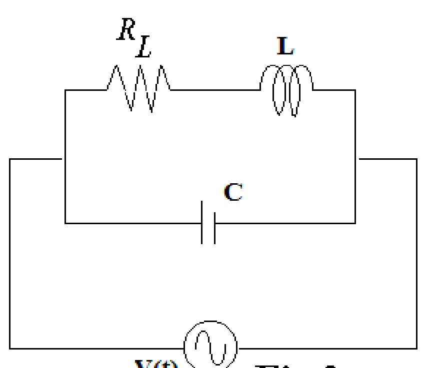
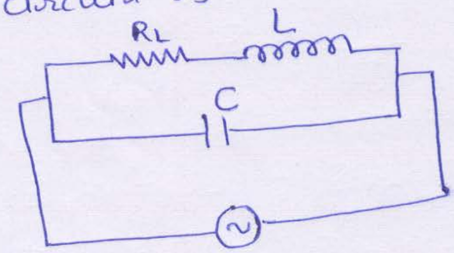


Fig.3

Q 5(a) Given circuit is



$$Y_L = \frac{1}{R_L + j\omega L} = \frac{1}{R_L + j\omega L} \times \frac{R_L - j\omega L}{R_L - j\omega L} = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2}$$

$$Y_c = \frac{1}{1/j\omega C} = j\omega C$$

$$\therefore\ Y_T = Y_L + Y_C = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + j\omega C$$

$$= \frac{R_L}{R_L^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R_L^2\ \omega^2 L^2}\right)$$

For resonance, susceptance will be zero, that is

$$\omega_0\left[C - \frac{L}{R_L^2 + \omega_0^2 L^2}\right] = 0$$

$$\text{Or}\quad C = \frac{L}{R_L^2 + \omega_0^2 L^2} \quad \text{or} \quad R_L^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0^2 = \frac{L}{L^2 C} - \frac{R_L^2}{L^2} \quad \text{or} \quad \omega_0 = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

$$\text{Or}\quad \boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}}$$ required resonant frequency

If R is negligible

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

b. A 220 V, 100 Hz AC source supplies a series LCR circuit with a capacitor and a coil. If the coil has 50 mΩ resistance and 5 mH inductance and resonant frequency of the circuit is 100 Hz, than find the value of capacitor. Also calculate Q factor and half power frequencies of the circuit.

Q 5b. It is given that $V = 200$ Volt, $f_0 = 100$ Hz.

$L = 5 \times 10^{-3}$ Henry

$R_L = 50 \times 10^{-3}\ \Omega$

We know that $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$100 = \frac{1}{2\pi\sqrt{5 \times 10^{-3} \times C}}$ or $C = \frac{1}{4\pi^2 \times 100^2 \times 5 \times 10^{-3}}$

or $\boxed{C = 506.6\ \mu F}$

$Q = \frac{\omega_0 L}{R} = \frac{2\pi \times 100 \times 5 \times 10^{-3}}{50 \times 10^{-3}} = 62.83$

$\boxed{Q = 62.83}$

(BW) Bandwidth $= \frac{f_0}{Q} = \frac{100}{62.83} = 1.59$

∴ Half power frequency $f_2 = f_0 + \frac{BW}{2}$

or $f_2 = 100 + \frac{1.59}{2} = 100.8$ Hz.

& $f_1 = f_n - \frac{BW}{2} = 100 - \frac{1.59}{2}$

or $\boxed{f_1 = 99.2 \text{ Hz. } \quad \& \quad f_2 = 100.8 \text{ Hz.}}$

Q.6 a. Define and explain phase velocity and group velocity of a uniform transmission line.

Q 6 (a) The velocity of propagation along the line based on the observation of change in phase along the line.

Since the change of 2π in phase angle represents One cycle in time 't' and occurs in a distance of one wave length λ, then

$$\lambda = v_p \times t = v_p \times \frac{1}{f}$$

$$\text{or} \quad v_p = \lambda f$$

we Know that $\lambda = \frac{2\pi}{\beta}$ where β is phase shift.

$$\therefore \quad v_p = \frac{2\pi}{\beta} f = \frac{\omega}{\beta}$$

Group Velocity :-

In case of distortion less or loss less line β phase shift is not a constant multiple of ω. As a result of this the components in a complex waveform normally shift in phase relation during propagation. This phenomenon is Known as dispersion which results in distortion.

In Small dispersion, a significant velocity of propagation is group velocity.

Thus group velocity is defined as the velocity of the envelope of a complex signal. It is denoted by the letter v_g.

The group velocity is of special importance in the transmission of modulated wave, pulses and transmission through wave guides.

Let ω_1 and ω_2 be the two close angular frequencies being transmitted and β_1 and β_2 be the corresponding phase constant, then the group velocity v_g will be given as

$$v_g = \frac{\omega_2 - \omega_1}{\beta_2 - \beta_1} = \frac{d\omega}{d\beta}$$

$\frac{d\omega}{d\beta}$ is evaluated at the carrier of centre frequency

Also we know that

$$v_p = \frac{\omega}{\beta} \quad \text{---} \quad (1)$$

differentiating w.r.t. ω, we get

$$\frac{dv_p}{d\omega} = \frac{1.\beta - \omega \frac{d\beta}{d\omega}}{\beta^2} = \frac{1 - \frac{\omega}{\beta}\left(\frac{d\beta}{d\omega}\right)}{\beta}$$

or $$\beta . \frac{dv_p}{d\omega} = 1 - v_p . \frac{1}{v_g}$$

or $$\frac{v_p}{v_g} = 1 - \beta \frac{dv_p}{d\omega}$$

or $$v_g = \frac{v_p}{1 - \beta . \frac{dv_p}{d\omega}}$$

Put value of β from equation (1), we get

$$v_g = \frac{v_p}{1 - \frac{\omega}{v_p}\left(\frac{dv_p}{d\omega}\right)}$$

MODERATION

$v_g = v_p$, when $\frac{dv_p}{d\omega} = 0$

This is consistant with the definition of group velocity.

— X — X — X —

b.The primary constants of a line per loop km are R = 196 Ω; C = 0.09 μF; L = 7.1mH and leakage conductance is negligible. Calculate the characteristic impedance and the propagation constant at angular frequency of 5000 radians/sec.

Q6b Given that

$w = 2\pi f = 5000 Hz,$

$R = 196\,\Omega$

$L = 7.1\ mH$

$C = 0.09\ \mu F$

Series impedance $Z = R + j\omega L$

$= 196 + j\,5000 \times 7.1 \times 10^{-3}$

$= 196 + j\,35.5 = 199.2 \angle 10.5^\circ$

Similarly, shunt admittance

$Y = G + j\omega C = 0 + j \times 5000 \times 0.09 \times 10^{-6}$

$= 0.45 \times 10^{-3} \angle 90^\circ$

$\therefore Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{199.2 \angle 10.5^\circ}{0.45 \times 10^{-3} \angle 90^\circ}}$

~~666~~ $= 665.33 \angle -39.75\ \Omega$ = Characteristic Impedance.

and propagation constant

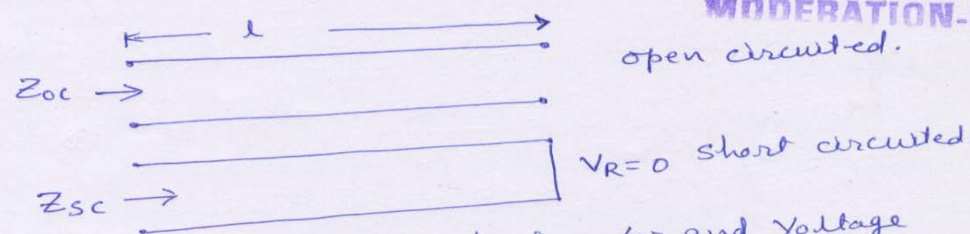
$P = \sqrt{Z \cdot Y} = \sqrt{199.2 \angle 10.5^\circ \times 0.45 \times 10^{-3} \angle 90^\circ}$

$= 2994 \times 10^{-4} \angle 50.25^\circ = 0.299 \angle 50.25^\circ$

MODERATION-I

Q.7 a. Derive expression for input impedance of open and short circuited line and show that characteristic impedance $Z_0 = \sqrt{Z_{OC} \times Z_{SC}}$

Q 7 (a) Input impedance of open and short circuited line



Consider a length of line l, having far end Voltage and current V_R and I_R respectively.

Therefore when $x = l$, $V = V_R$ & $I = I_R$, we get

$$V_R = V_s \cosh Pl - I_s \sinh Pl$$

$$I_R = I_s \cosh Pl - \frac{V_s}{Z_0} \sinh Pl$$

where I_s = Sending current & V_s = sending Voltage

P = propagation constant

In an open circuit line, $I_R = 0$,

$$\therefore \quad 0 = I_s \cosh Pl - \frac{V_s}{Z_0} \sinh Pl$$

$$\frac{V_s}{I_s} = Z_0 \frac{\cosh Pl}{\sinh Pl} = Z_0 \coth Pl$$

Since $\frac{V_s}{I_s}$ is the input impedance Z_{oc} of open circuited line, we have

$$Z_{oc} = Z_0 \coth Pl \quad ----- (1)$$

Similarly, in a short circuited line, $V_R = 0$,

$$\therefore \quad 0 = V_s \cosh Pl - I_s \sinh Pl$$

Or $\quad \frac{V_s}{I_s} = Z_0 \frac{\sinh Pl}{\cosh Pl} = Z_0 \tanh Pl$ ---

But $\frac{V_s}{I_s}$ = short circuited line input impedance Z_{sc}

or $Z_{sc} = Z_0 \tanh Pl$ — (2)

b.A 100 km long transmission line is terminated by a resistance of 200 ohm. It has characteristics impedance $Z_0 = 600\angle 0^o$ ohms, attenuation constant α = 0.01 neper / km, phase shift constant β = 0.03 rad / km. Find the reflection coefficient and the impedance.

Multiplying Eq. ① & ② we get

$$Z_{oc} \cdot Z_{sc} = Z_0 \coth Pl \times Z_0 \tanh Pl$$

$$Z_{oc} \cdot Z_{sc} = Z_0^2$$

or $$Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}$$

Hence Proved.

Q 7(b) Given $Z_R = 200\,\Omega$

$Z_0 = 600\angle 0\,\Omega$

$\alpha = 0.01$ neper/Km

$\beta = 0.03$ rad/Km.

line length = 100 Km

Formula $$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{200-600}{200+600} = -0.5$$

Now propagation constant $P = \alpha + j\beta$

$$P = 0.01 + j0.03$$

or $2Pl = 2(0.01 + j0.03) \times 100 = 2 + j6$

We know that

$$Z_{in} = Z_0 \frac{1+|K|e^{-2Pl}}{1-|K|e^{-2Pl}}$$

$$= 600\left[\frac{1+|(-0.5)|\,e^{-(2+j6)}}{1-|(-0.5)|\,e^{-(2+j6)}}\right]$$

$$= 600\left[\frac{1+0.5\,e^{-2}\cdot e^{-j6}}{1-0.5\,e^{-2}\cdot e^{-j6}}\right] = 600\left[\frac{1+0.5\times 0.135\,e^{-j6}}{1-0.5\times 0.135\,e^{-j6}}\right]$$

$$= 600\left[\frac{1+0.675(\cos 343.8^\circ + j\sin(-343.8^\circ))}{1-0.675(\cos 343.8^\circ + j\sin(-343.8^\circ))}\right]$$

$$= 600\left[\frac{1+0.675(\cos 16.2 + j\sin 16.2)}{1-0.675(\cos 16.2 + j\sin 16.2)}\right] = 600\left[\frac{1+0.675(0.96+j0.2}{1-0.675(0.96+j0.2}\right.$$

$$= 600\left[\frac{1+0.65+j0.19}{1-0.65-j0.19}\right] = 600\left[\frac{1.65+j0.19}{0.35-j0.19}\right]$$

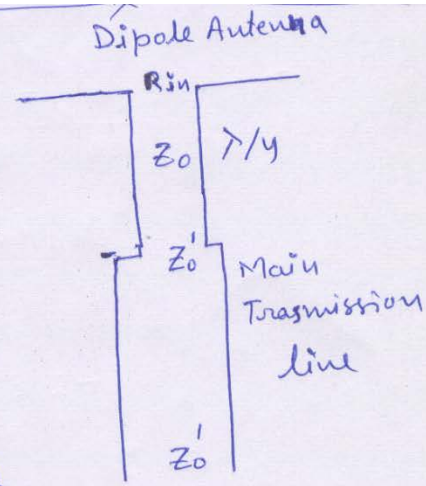
$$= 600\left[\frac{1.65+j0.19}{0.35-j0.19}\right] \times \left[\frac{0.35+j0.19}{0.35+j0.19}\right]$$

$$= 600\left[\frac{0.5414+j0.382}{0.1586}\right] = 3760(0.5414+j0.382)$$

$$\boxed{Z_{in} = (2003+j1416)\,\Omega}$$

Q.8 a. Explain the operation and use of a quarter wave transformer.

Q 8(a) We know that the input impedance Z_{in} of a uniform transmission line in terms of secondary line constants is give by



$$Z_{in} = Z_0 \frac{Z_R \cosh Pl + Z_0 \sinh Pl}{Z_0 \cosh Pl + Z_R \sinh Pl}$$

Since high frequency line can be considered loss free, $\alpha = 0$ resulting $P = j\beta$ only, we have

$$Z_{in} = Z_0 \frac{Z_R \cosh j\beta l + Z_0 \sinh j\beta l}{Z_0 \cosh j\beta l + Z_R \sinh j\beta l} = Z_0 \frac{Z_R \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_R \sin\beta l}$$

For the line which is quarter wave length

$$l = \lambda/4$$

But $\beta = \frac{2\pi}{\lambda}$

$$\therefore \quad \beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

MODERATION-I

Put value of βl in above equation, we get

$$Z_{in} = Z_0 \frac{Z_R \cos\frac{\pi}{2} + jZ_0 \sin\frac{\pi}{2}}{Z_0 \cos\frac{\pi}{2} + jZ_R \sin\frac{\pi}{2}} = Z_0 \frac{Z_0}{Z_R} = \frac{Z_0^2}{Z_R}$$

MODERATION-I

Or $Z_0 = \sqrt{Z_{in} \cdot Z_R}$ - - - - - (1)

This means that the product of the input impedance and load impedance is equal to the square of the characteristic impedance of line.

From the equation (1) it is clear that when load impedance is high, the input impedance low and vice versa. So depending upon the value of Z_R, the quarter wave line acts like a step up or step down transformer.

An application is shown above fig. in which a dipole Antenna with a resistive input impedance Rin is matched to the main transmission line having characteristic impedance Z_0', by means of a $\frac{\lambda}{4}$ wave transformer having characteristic impedance $Z_0 = \sqrt{Z_0' \times R_{in}}$.

The disadvantage of quarter wave transformer is that it is sensitive to change in frequency.

b. A low loss line with Z_0 = 70 Ω terminates in an impedance of $Z_R = 115 - j80$. The wave length of the transmission is 2.5 metres; using the smith chart find:

(i) Standing wave ratio

(ii) Maximum and minimum line impedance

(iii) Distance between load and first voltage maximum

Q8b. Given that

$Z_0 = 70$

$Z_R = 115 - j80$

$\lambda = 2.5$ metres.

So normalised terminating impedance

$$Z_r = \frac{Z_R}{Z_0} = \frac{115 - j80}{70} = 1.64 - j1.14$$

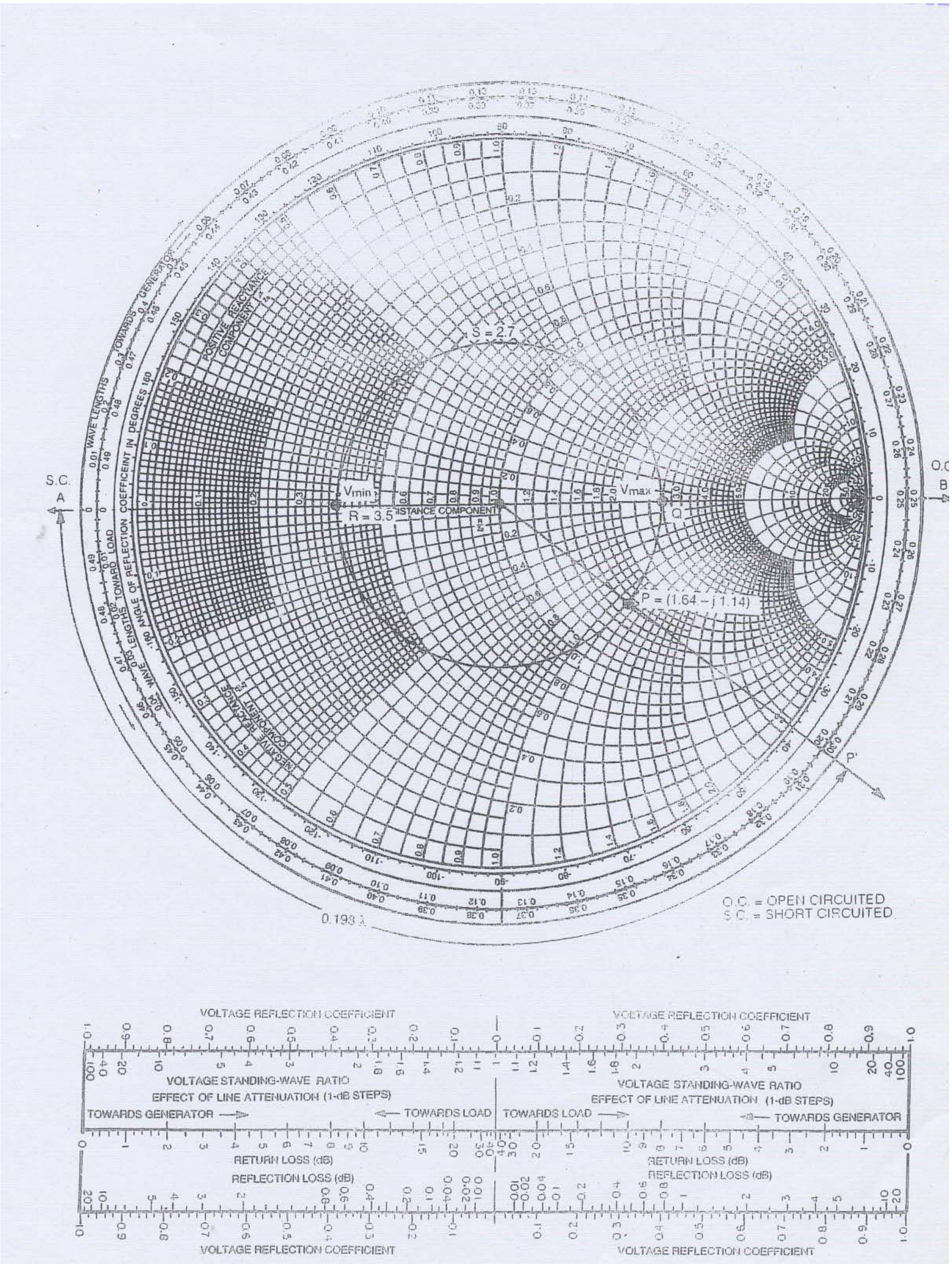
The point $(1.64 - j1.14)$ is plotted as "P" on smith chart

(i) Draw S-circle with centre O and radius OP. Since circle cuts the right hand side of the horizontal axis at Q whose reading is 2.7

$\therefore S = 2.7$

(ii) Vmax occurs at Q and Vmin at R. Therefore, normalized line impedance

$= 2.7 + j0$


S = 2.7
Vmin
R = 3.5
Vmax
P = (1.64 – j 1.14)
P'
S.C.
A
O.C
B
0.193 λ
POSITIVE REACTANCE COMPONENT
NEGATIVE REACTANCE COMPONENT
RESISTANCE COMPONENT
WAVE LENGTHS TOWARD LOAD
WAVE LENGTHS TOWARDS GENERATOR
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
O.C. = OPEN CIRCUITED
S.C. = SHORT CIRCUITED
VOLTAGE REFLECTION COEFFICIENT
VOLTAGE STANDING-WAVE RATIO
EFFECT OF LINE ATTENUATION (1-dB STEPS)
TOWARDS GENERATOR
TOWARDS LOAD
RETURN LOSS (dB)
REFLECTION LOSS (dB)

Thus, max ~~minimum~~ line impedance $= 70 \times 2.7 = 189$ ohms

minimum normalised line impedance $= 0.36 + j0$

maximum line impedance $= 70 \times 0.36 = 25.2\,\Omega$

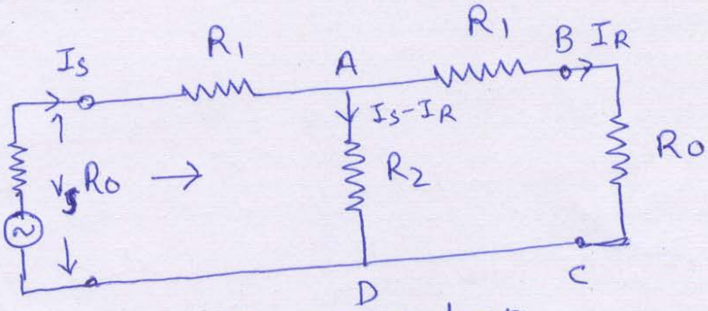
(iii) l_{max} = length of arc RP' measured on wave length scale

$= 0.198\lambda = 0.198 \times 2.5$ metre $= 0.495$ metre

$= 495$ cms.

Q.9 a. Draw the circuit of a symmetrical-T attenuator and derive the design equation.

Q 9(a) Symmetrical T– attenuator



Applying KVL in ABCD loop

$$R_2(I_S - I_R) = I_R(R_1 + R_0)$$

$$I_S R_2 = I_R(R_1 + R_0 + R_2)$$

or $$\frac{I_S}{I_R} = N = \frac{R_1 + R_2 + R_0}{R_2} \quad \text{--- (1)}$$

But impedance looking into the attenuator will be R_0 when it is terminated in R_0

$$R_0 = R_1 + \frac{R_2(R_1 + R_0)}{R_2 + R_1 + R_0} = R_1 + \frac{R_1 + R_0}{N} \quad \text{From (1)}$$

or $$N R_0 = N R_1 + R_1 + R_0$$

$\Rightarrow$ $$(N-1)R_0 = R_1(1+N)$$

or $$\boxed{R_1 = R_0 \frac{N-1}{N+1}}$$

From equation (1), we have

$$\cancel{(N-1)R_2} \cdot N R_2 = R_1 + R_2 + R_0$$

or $$(N-1)R_2 = R_1 + R_0$$

or $$R_2 = \frac{R_1 + R_0}{N-1}$$

Put value of R_1

$$(N-1) = \frac{R_0 + \frac{N-1}{N+1} R_0}{R_2}$$

or $$R_2 = R_0 \left[\frac{N+1+N-1}{(N-1)(N+1)} \right] = \frac{2N}{N^2-1} R_0$$

or $$\boxed{R_2 = \frac{2N}{N^2-1} R_0}$$

The attenuation introduced by a symmetrical T attenuator is

$$\cosh\alpha = \left[1 + \frac{Z_1}{2Z_2}\right] = \left[1 + \frac{R_1}{R_2}\right] \text{neper}$$

$$\therefore \alpha = \cosh^{-1}\left[1 + \frac{R_1}{R_2}\right] \text{ neper}$$

b.Design m-derived low pass filter having a design impedance of 600 Ω and cut-off frequency of 5000 Hz. The frequency of infinite attenuation is 6250 Hz.

Q 9(b) Given that

$f_c = 5000\,Hz$, $R_K = 600\,\Omega$, $f_\infty = 6250\,Hz$.

LPF for constant K

$$L = \frac{R_K}{\pi f_c} = \frac{600}{3.14 \times 5000} = 38.2\,mH$$

$$C = \frac{1}{\pi R_K f_c} = \frac{1}{3.14 \times 600 \times 5000} = 0.106\,\mu F$$

To obtained m-derived LPF, the value of m is

$$m = \sqrt{\left(1 - \left(\frac{f_c}{f_\infty}\right)^2\right)} = \sqrt{1 - \left(\frac{5000}{6250}\right)^2} = \sqrt{1 - (0.8)^2}$$

or $m = \sqrt{0.36} = 0.6$

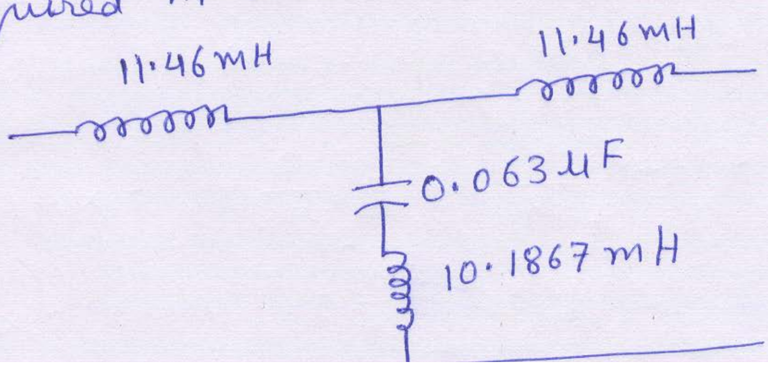
The elements of m-derived LPF, T Section will be

$$\frac{mL}{2} = \frac{0.6 \times 38.2}{2}\,mH = 11.46\,mH$$

$$mC = 0.6 \times 0.106\,\mu F = 0.0636\,\mu F$$

and
$$\frac{1-m^2}{4m} L = \frac{1 - 0.36}{4 \times 0.6} \times 38.2\,mH.$$
$$= \frac{0.64}{2.4} \times 38.2\,mH = 10.1867\,mH$$

The required m-derived LPF will be.



TEXT BOOKS

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