Q.2a. **If** $x^y = e^{x-y}$**, show that** $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$

Sol Q b

$x^y = e^{x-y}$

Taking log of both sides

$y \log x = x - y$ or $y(1+\log x) = x$ or $y = \dfrac{x}{1+\log x}$ (1) mark (A)

diff. w.r.t. to x,

$y\dfrac{1}{x} + \log x \dfrac{dy}{dx} = 1 - \dfrac{dy}{dx}$

upto this, 3 marks

$\dfrac{dy}{dx} = \dfrac{1-\frac{y}{x}}{1+\log x} = \dfrac{x-y}{x(1+\log x)}$

upto this (5) marks

$= \dfrac{y \log x}{x(1+\log x)}$ using (A)

$= \dfrac{x}{1+\log x} \times \dfrac{\log x}{x(1+\log x)}$ using (A)

$= \dfrac{\log x}{(1+\log x)^2}$

full marks

b. Find the point on the curve y = 7x – 3x^2 where the inclination of the tangent with x – axis is of 45°**, Also find the equation of the normal to the given curve at that point.**

(b) Let $P(x_1, y_1)$ be a point on the curve $y = 7x - 3x^2$, where the tangent is inclined at an angle of 45^0 with the x-axis. Then,

Slope of the tangent at $P = \tan 45^0$

ie $\left(\frac{dy}{dx}\right)_P = \tan 45^0 \Rightarrow \left(\frac{dy}{dx}\right)_P = 1$ —— (i) 1 Mark

Differentiating $y = 7x - 3x^2$ w.r.t. x, we get

$\frac{dy}{dx} = 7 - 6x$

$\therefore \left(\frac{dy}{dx}\right)_P = 7 - 6x_1$ —— (ii) — 2 Marks

From (i) and (ii), we get

$7 - 6x_1 = 1 \Rightarrow x_1 = 1$

Since $P(x_1, y_1)$ lies on the curve $y = 7x - 3x^2$

$\therefore y_1 = 7x_1 - 3x_1^2$ —— 4 Marks

$\Rightarrow y_1 = 7 - 3 = 4$ [Putting $x_1 = 1$]

Hence, the required point on the curve is $(1,4)$ — 6 Marks

$\therefore$ Eqn of normal is $y - y_1 = \text{slope of normal}(x - x_1)$, $\text{slope of normal} = -\frac{1}{\text{slope of tgt}} = -1$

or $y - 4 = -1(x-1)$ or $x + y = 5$ — 8 Marks

Q.3a. **Evaluate** $\int \frac{1+\sin x}{\sin x(1+\cos x)}dx$

Q.3.a. Soln:

Let $I = \int \frac{1+\sin x}{\sin x(1+\cos x)}\,dx$ — 1 Mark

Putting $\sin x = \frac{2\tan x/2}{1+\tan^2 x/2}$ and $\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$, we get

$$I = \int \frac{1+\frac{2\tan x/2}{1+\tan^2 x/2}}{\frac{2\tan x/2}{1+\tan^2 x/2}\left(1+\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)}\,dx$$

$$\Rightarrow I = \int \frac{(1+\tan^2 x/2 + 2\tan x/2)(1+\tan^2 x/2)}{2\tan x/2\,(1+\tan^2 x/2 + 1-\tan^2 x/2)}\,dx$$

$$\Rightarrow I = \int \frac{(1+\tan x/2)^2 \sec^2 x/2}{4\tan x/2}\,dx$$ — 3 Marks

Putting $\tan x/2 = t$ and $(1/2)\sec^2(x/2)\,dx = dt$ — 4 Marks

or $\sec^2(x/2)\,dx = 2\,dt$, we get

$$I = \int \frac{(1+t)^2}{4t}\,2\,dt$$

$$\Rightarrow I = \frac{1}{2}\int \frac{1+t^2+2t}{t}\,dt = \frac{1}{2}\int\left(\frac{1}{t}+t+2\right)dt$$ — 6 Marks

$$\Rightarrow I = \frac{1}{2}\left\{\log|t| + \frac{t^2}{2} + 2t\right\} + c$$

$$\Rightarrow I = \frac{1}{2}\left\{\log|\tan x/2| + \frac{\tan^2 x/2}{2} + 2\tan x/2\right\} + c$$

$$\Rightarrow I = \frac{1}{2}\left\{\log|\tan x/2|\right.$$ — 8 Marks

b.Evaluate $\int_0^{\pi/2} \log \tan x dx$

Q.3.b. Let $I = \int_0 \log$

then, $I = \int_0^{\pi/2} \log \tan\left(\frac{\pi}{2} - x\right) dx$ — 2 Marks

$\Rightarrow I = \int_0^{\pi/2} \log . \cot x \, dx$ ← (ii) $\left[\text{using: } \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \right]$

Adding (i) and (ii), we get

$2I = \int_0^{\pi/2} (\log \tan x + \log \cot x)\, dx$ — 4 Marks

$\Rightarrow 2I = \int_0^{\pi/2} \log(\tan x . \cot x)\, dx$ — 6 Marks

$\Rightarrow 2I = \int_0^{\pi/2} \log 1 . dx = \int_0^{\pi/2} 0 . dx = 0$ — 8 Marks

$\Rightarrow I = 0$ Ans

Q.4 a. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

Q.4.a.

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$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

2 Marks

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a-b)(a+b) & (b-c)(b+c) & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ (a+b) & (b+c) & c^2 \end{vmatrix}$$

(by taking $(a-b)$ and $(b-c)$ common out from C_1 and C_2 respectively) 4 Marks

$$= (a-b)(b-c)\begin{vmatrix} 1 & 1 \\ (a+b) & (b+c) \end{vmatrix}$$

, by expanding the determinant along R_1.

8 Marks

$$= (a-b)(b-c)[(b+c)-(a+b)]$$

$$= (a-b)(b-c)(c-a) \text{ proved.}$$

b.Apply Cramer's rule to solve the following system of linear equations:

$$3x - 2y + 4z = 5$$
$$x + y + 3z = 2$$
$$-x + 2y - z = 1$$

b. $$\Delta = \begin{vmatrix} 3 & -2 & 4 \\ 1 & 1 & 3 \\ -1 & 2 & -1 \end{vmatrix} = -5$$ — 1 Mark

$$\Delta x = \begin{vmatrix} 5 & -2 & 4 \\ 2 & 1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = -33$$ 1 Mark

$$\Delta y = \begin{vmatrix} 3 & 5 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & -1 \end{vmatrix} = -13$$ 1 Mark

$$\Delta z = \begin{vmatrix} 3 & -2 & 5 \\ 1 & 1 & 2 \\ -1 & 2 & 1 \end{vmatrix} = 12$$ 1 Mark

$$x = \frac{\Delta x}{\Delta} = \frac{-33}{-5} = \frac{33}{5}$$ — 1 Mark

$$y = \frac{\Delta y}{\Delta} = \frac{-13}{-5} = \frac{13}{5}$$ — 1 Mark

$$z = \frac{\Delta z}{\Delta} = \frac{12}{-5} = \frac{-12}{5}$$ — 1 Mark

Ans $x = \frac{33}{5}, y = \frac{13}{5}, z = \frac{-12}{5}$ — 8 Marks

Q.5 a. Solve cos(x + y)dy = dx

Q5(a) Soln:

$\cos(x+y)\,dy = dx$

or $\frac{dy}{dx} = \sec(x+y)$ — 1 Mark

On putting $x+y=z$

So that, $1+\frac{dy}{dx} = \frac{dz}{dx}$ or $\frac{dy}{dx} = \frac{dz}{dx}-1$

or $\frac{dz}{dx}-1 = \sec z$ — 1 Mark

or $\frac{dz}{dx} = 1+\sec z$

or $\frac{dz}{dx} = 1+\sec z$

or $\frac{dz}{1+\sec z} = dx$ — 4 Marks

$\int \frac{\cos z}{\cos z+1}\,dz = \int dx + c$

or $\int \left[1-\frac{1}{\cos z+1}\right]dz = x+c$

or $\int \left[1-\frac{1}{2\cos^2\frac{z}{2}-1+1}\right]dz = x+c$ — 6 Marks

or $\int \left(1-\frac{1}{2}\sec^2\frac{z}{2}\right)dz = x+c$

or $z - \tan\frac{z}{2} = x+c$

or $x+y-\tan\frac{x+y}{2} = x+c$

or $y-\tan\frac{x+y}{2} = c$ Ans. — 8 Marks

b. Solve $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}+y=\cos 2x$

Q5.(b) $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}+y=\cos 2x$ Let $D=\left(\dfrac{d}{dx}\right)$

$(D^2+D+1)y=\cos x$

Auxiliary equation is $D^2+D+1=0$

$D=\dfrac{-1\pm\sqrt{-3}}{2}$, C.F. $=e^{-x/2}\left[A\cos\dfrac{\sqrt{3}}{2}x+B\sin\dfrac{\sqrt{3}}{2}x\right]$ — 2

P.I. $=\dfrac{1}{D^2+D+1}\cdot\cos 2x$

$=\dfrac{1}{-(2)^2+D+1}\cdot\cos 2x=\dfrac{1}{D-3}\cdot\cos 2x$ Marks

$=\dfrac{D+3}{D^2-9}\cdot\cos 2x=\dfrac{D+3}{-(2)^2-9}\cos 2x$ — 5 Marks

$=-\dfrac{1}{13}(D+3)\cos 2x=-\dfrac{1}{13}(-2\sin 2x+3\cos 2x)$ — 7 Marks

Complete solution is

$y=e^{-x/2}\left[A\cos\dfrac{\sqrt{3}x}{2}+B\sin\dfrac{\sqrt{3}x}{2}\right]+\dfrac{1}{13}\left[2\sin 2x-3\cos 2x\right]$ Ans. — full

Q.6a. **Prove that, cos2A. cos2B + sin^2(A –B) – sin^2(A +B) = cos (2A + 2B)**

Q.6.a. Soln

LHS $= \cos 2A \cdot \cos 2B + \sin^2(A-B) - \sin^2(A+B)$

$= \cos 2A \cdot \cos 2B + \sin(A-B+A+B) \cdot \sin(A-B-A-B)$

$(\because \sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B)$ 2 Marks

$= \cos 2A \cdot \cos 2B + \sin 2A \cdot \sin(-2B)$

$= \cos 2A \cdot \cos 2B - \sin 2A \cdot \sin 2B$ $(\because \sin(-\theta) = -\sin\theta)$

$= \cos(2A+2B)$

$=$ RHS. Hence proved 8 Marks

b. **If A + B + C = $\pi/2$, then prove that** **sin2A +**

sin2B + sin2C = 4 cosA . cosB . cosC

Q6b. LHS $= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A+2B}{2}\right)\cdot\cos\left(\frac{2A-2B}{2}\right) + 2\sin C\cdot\cos C$ 2 Marks

$= 2\sin(A+B)\cdot\cos(A-B) + 2\sin C\cdot\cos C$

$= 2\cos C\cdot\cos(A-B) + 2\cos C\cdot\sin C$ — 4 Marks

$= 2\cos C\left[\cos(A-B) + \sin C\right]$

$= 2\cos C\left[\cos(A-B) + \cos(A+B)\right]$

$= 2\cos C\cdot 2\cos A\cdot\cos B$

$= 4\cos A\cdot\cos B\cdot\cos C$

$=$ RHS. Hence proved 8 Marks

$A+B+C = \pi/2$

$A+B = \pi/2 - C$

$\sin(A+B) = \sin(\pi/2 - C)$

$= \cos C$

$\therefore C = \pi/2 - A+B$

$\sin C = \sin\left(\frac{\pi}{2} - A+B\right)$

$= \cos(A+B)$

Q.7 a. Find the term independent of x in the expansion of $\left(x^2 + \frac{1}{x}\right)^9$.

Q.7.a. Soln. Comparing $\left(x^2 + \frac{1}{x}\right)^9$ with $(x+a)^n$

$x = x^2, \quad a = \frac{1}{x}, \quad n = 9$

$T_{r+1} = {}^nC_r \, x^{n-r} \cdot a^r$ —— 2 Marks

$= {}^9C_r \, (x^2)^{9-r} \left(\frac{1}{x}\right)^r$

$= {}^9C_r \, x^{18-2r} \cdot \frac{1}{x^r}$

$= {}^9C_r \, x^{18-3r}$ —— 4 Marks

$\Rightarrow 18 - 3r = 0$ As this term is independent of x

$\therefore r = 6$ —— 6 Marks

$T_7 = {}^9C_6 = \frac{9.8.7}{3.2.1} = 84$ Ans

b. In a G. P. the first term is 7, the last term is 448 and the sum is 889. Find the common ratio and the series.

Q7.b Soln:

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Here, $a = 7$, $l = T_n = 448$ and $S_n = 889$

Now, $S_n = \dfrac{a(r^n - 1)}{r-1}$ and $l = T_n = ar^{n-1} \Rightarrow ar^n = lr$

2 Marks

$S_n = \dfrac{lr - a}{r-1} \Rightarrow 889 = \dfrac{448r - 7}{r-1}$

2 Marks

$\Rightarrow 889r - 889 = 448r - 7$

$\Rightarrow 441r = 882$ 6 Marks

$\Rightarrow \boxed{r = 2}$ Ans

8 Marks

The Series is $7 + 14 + 28 + 56$ Ans.

Q.8 a. Show that the lines x – y = 6, 4x – 3y = 20 and 6x + 5y = -8 are concurrent. Also find their point of intersection.

Q.8.a. Show that the lines $x-y=6$, $4x-3y=20$ and $6x+5y=-8$ are concurrent. Also find their point of intersection.

Soln: The given lines are re-written as

$x-y-6=0$ — (i)

$4x-3y-20=0$ — (ii)

$6x+5y+8=0$ — (iii)

Solving (i) and (ii), we get

$x-y-6=0$ and $4x-3y-20=0$

$$\frac{x}{20-18} = \frac{y}{-24+20} = \frac{1}{-3+4}$$

or $\frac{x}{2} = \frac{y}{-4} = \frac{1}{1} = 1$ — 4 Marks

$\Rightarrow x=2,\ y=-4$

Hence first two lines intersect at the $(2,-4)$

Putting this point in the equation $6x+5y+8=0$, we get, $6\times2+5\times-4+8=0$ — 8 Marks

$\therefore (2,-4)$ lies on $6x+5y+8=0$, ~~respect~~

Hence the given lines are concurrent and their common point of intersection is $(2,-4)$.

b.Find the equation of the straight lines through the point (2, -1) and making an angle of 45° with the line 6x + 5y – 1 = 0

Q.8.b. Soln. Equation of st. line through $(2,-1)$ and having slope m is $y+1 = m(x-2)$ —(i)

(i) makes an angle 45° with line

$$6x + 5y - 1 = 0 \quad \text{—(ii)}$$

2 Marks

Slope of (ii) is $-\frac{6}{5}$

$$\tan 45^\circ = \pm \frac{m + \frac{6}{5}}{1 - \frac{6m}{5}}$$

4 Marks

or $1 = \pm \dfrac{5m+6}{5-6m}$

Taking +ve sign:

$$5 - 6m = +(5m+6) \Rightarrow m = -\frac{1}{11}$$

Taking −ve sign

$$5 - 6m = -(5m+6) \Rightarrow m = 11$$

6 Marks

Putting the values of m in (i) we get

$$y+1 = -\frac{1}{11}(x-2) \text{ and } y+1 = 11(x-2)$$

or $x + 11y + 9 = 0$ and $11x - y - 23 = 0$ Ans.

are the required eqn of st. lines.

2 Marks

Q.9 a. Find the equation of the circle which passes through the points (3, -2), (-2, 0) and having its centre on the line 2x – y = 3.

Q.9.a. Soln: Let the equation of the circle be (i)

$$x^2+y^2+2gx+2fy+c=0 \quad \text{--- (i)}$$ — 1 Mark

As (i) passes through (3, -2)

$\therefore \quad 9+4+6g-4f+c=0$

or $\quad 6g-4f+c=-13$ — (ii) — 2 Mark

Also (i) passes through (-2, 0)

$\therefore \quad 4+0-4g-0+c=0$

or $\quad 4g-c=4$ — (iii) — 3 Mark

The Centre (-g, -f) of (i) lies on $2x-y=3$

$-2g+f=3$ — (iv) — 4 Marks

Adding (ii) and (iii), we get

$10g-4f=-9$ — (v) — 5 Mark

Solving (iv) and (v), we get

$g=\frac{3}{2},\ f=6$ — 6 Mark

Putting g in (iii), we get $c=2$ — 6 Mark

Substituting these values of g, f and c in (i) we get

$$x^2+y^2+3x+12y+2=0$$ — 8 Marks

which is the reqd. eqn. of circle. Ans.

b. **Find the equation of the parabola with focus (3, -4) be the directrix** $6x-7y+5=0$

Q.9.b. Soln:

Let $S(3,-4)$ be the focus and ZM be the directrix and $P(x,y)$ be any point on the parabola

Now, $SP = \sqrt{(x-3)^2+(y+4)^2}$ —(i) — 1 Mark

and $PM = \dfrac{6x-7y+5}{\sqrt{36+49}}$ —(ii) — 1 Mark

TEXTBOOKS

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III. Engineering Mathematics, HK Dass, S Chand and Company Ltd, 13th Edition, New Delhi