Q.2 a. Evaluate $\underset{x\to 0}{\text{Lt}} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$

Q.2 a Soln: we have,

$$\underset{x\to 0}{\text{Lt}} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{\left(\sqrt{1+\sin x}-\sqrt{1-\sin x}\right)\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)} \quad \text{— 2 Marks}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1-\sin x}\right)^2}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{1+\sin x-1+\sin x}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{2\sin x}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{2}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}, \quad \left[\underset{x\to 0}{\text{Lt}} \frac{\sin x}{x} = 1\right]$$

$$= \frac{2}{2} = 1 \qquad \underline{\text{Ans: } 1} \quad \text{— 8 Marks}$$

b. Show that the function f(x) = $|x|$ **is not differentiable at x = 0**

Q.2.b. Soln:

We have, $Lf'(0) = \underset{h\to 0}{Lt} \dfrac{f(0-h)-f(0)}{-h} = \underset{h\to 0}{Lt} \dfrac{f(-h)-f(0)}{-h}$

$= \underset{h\to 0}{Lt} \dfrac{|-h|-0}{-h} = \underset{h\to 0}{Lt} \dfrac{h}{-h}$

$= \underset{h\to 0}{Lt} (-1) = -1$ — 4M mks

and $Rf'(0) = \underset{h\to 0}{Lt} \dfrac{f(0+h)-f(0)}{h} = \underset{h\to 0}{Lt} \dfrac{f(h)-f(0)}{h}$

$= \underset{h\to 0}{Lt} \dfrac{|h|-0}{h} = \underset{h\to 0}{Lt} \dfrac{h}{h} = \underset{h\to 0}{Lt} 1 = 1$

Thus, $Lf'(0) \neq Rf'(0)$ — 4M mks

Hence, $f(x)$ is not differentiable at $x=0$

Q.3a. Compute the arc-length of the curve $ay^2 = x^3$ from $x = 0$ to a point having $x = 5a$.

Q.3.a. Soln:

$ay^2 = x^3$

$2ay\dfrac{dy}{dx} = 3x^2$ or $\dfrac{dy}{dx} = \dfrac{3x^2}{2ay} = \dfrac{3x^2}{2a\left(\frac{x^3}{a}\right)^{1/2}}$

$\dfrac{dy}{dx} = \dfrac{3x^2}{2a^{1/2}x^{3/2}} = \dfrac{3x^{1/2}}{3a^{1/2}}$

Required length $= \int_0^{5a} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$ — 2M mks

$$= \int_0^{5a} \sqrt{1 + \frac{9}{4}\frac{x}{a}}\, dx$$

$$= \frac{1}{2\sqrt{a}} \int_0^{5a} \sqrt{4a+9x}\, dx$$

$$= \frac{1}{2\sqrt{a}} \frac{2}{3\times 9}\left[(4a+9x)^{3/2}\right]_0^{5a}$$

$$= \frac{1}{27a^{1/2}}\left[(4a+9\times 5a)^{3/2} - (4a)^{3/2}\right]$$

8 Marks

$$= \frac{1}{27a^{1/2}}\left[343a^{3/2} - 8^{3/2}\right] = \frac{335a}{27} \text{ Ans.}$$

b. **Find the length of an arch of the cycloid whose equations are x = a** $(\theta + \sin\theta)$ **and y = a** $(1 + \cos\theta)$

Q.3.b. Soln.

$$x = a(\theta + \sin\theta), \quad \frac{dx}{d\theta} = a(1+\cos\theta)$$

$$y = a(1+\cos\theta), \quad \frac{dy}{d\theta} = -a\sin\theta$$

The limit for half of the curve $\theta = 0$ and $\theta = \pi$

∴ The required length of the arch

$$= 2\int_0^{\pi} \sqrt{\left[\frac{dx}{d\theta}\right]^2 + \left[\frac{dy}{d\theta}\right]^2}\, d\theta \quad \text{— 2 Marks}$$

$$= 2a\int_0^{\pi} \sqrt{(1+\cos\theta)^2 + \sin^2\theta}\, d\theta$$

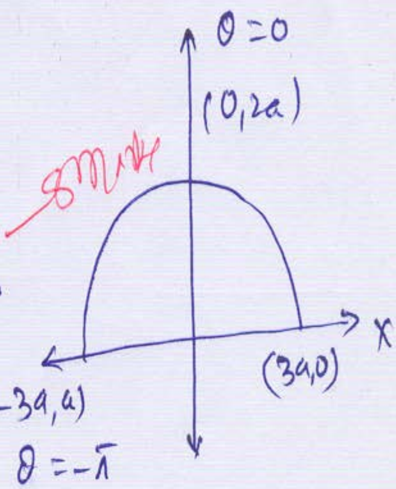
$$= 2a\int_0^{\pi} \sqrt{(1+\cos^2\theta + 2\cos\theta + \sin^2\theta)}\, d\theta$$

$$= 2\sqrt{2}\,a\int_0^{\pi} \sqrt{(1+\cos\theta)}\, d\theta$$

$$= 2\sqrt{2}\cdot\sqrt{2}\,a\int_0^{\pi} \cos\frac{\theta}{2}\, d\theta$$

$$= 8a\left[\sin\frac{\theta}{2}\right]_0^{\pi} = 8a \text{ Ans.}$$

8 Marks



Q.4a. Use De Moivre's Theorem to solve the equation $x^7 + x^4 + x^3 + 1 = 0$

Q.4.a. Soln.

We have $x^7 + x^4 + x^3 + 1 = 0$

or $x^4(x^3+1) + (x^3+1) = 0$

or $(x^4+1)(x^3+1) = 0$

$\therefore x = (-1)^{1/3}$ or $(-1)^{1/4}$

Now, $x = (-1)^{1/3} \Rightarrow x = (\cos\pi + i\sin\pi)^{1/3}$

$= [\cos(2m\pi+\pi) + i\sin(2m\pi+\pi)]^{1/3}$

2Marks $= \cos(2m+1)\frac{\pi}{3} + i\sin(2m+1)\pi/3$

$[m = 0, 1, 2]$

$\frac{266}{1}$

Putting $m=0,1,2$ we get the roots as

$$\cos \pi/3 + i \sin \pi/3 = \frac{1+i\sqrt{3}}{2},$$

$$\cos \pi + i \sin \pi = -1$$

and $\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} = \frac{1}{2} - \frac{i\sqrt{3}}{2}$

2 Marks

$$x = (-1)^{1/4} \Rightarrow x = (\cos \pi + i \sin \pi)^{1/4}$$

$$= [\cos(2m\pi + \pi) + i \sin(2m\pi + \pi)]^{1/4}$$

$$= \cos(2m+1)\frac{\pi}{4} + i \sin(2m+1)\pi/4$$ — 2 Marks

Putting $m=0,1,2,3$, we get the roots as

$$\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}(1+i)$$

$$\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}(1-i),$$

$$\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} = -\frac{1}{2}(1-i)$$

$$\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} = \frac{1}{\sqrt{2}}(1-i)$$

2 Marks

Hence all the roots of the given eqn are,

$-1, \frac{1}{2}(1 \pm i\sqrt{3}), \frac{1}{\sqrt{2}}(1 \pm i), \frac{1}{\sqrt{2}}(-1 \pm i)$ Ans.

b.Two circuits of impedance 2 + 4j ohms and 3 + 4j ohms are connected in parallel and A.C. voltage of 100 volts is applied across the parallel combination. Calculate the magnitude of the currents as well power factor for each circuit and magnitude of the total current for the parallel combination and its power factor.

Q.4.b. Soln.

$z_1 = 2+4j$, $z_2 = 3+4j$

$i_1 = \frac{V}{z_1} = \frac{100}{2+4j} = \frac{50}{1+2j} = \frac{50(1-2j)}{(1+2j)(1-2j)}$

$= \frac{50(1-2j)}{1+4} = 10(1-2j) = 10-20j$ — 2 Marks

$|i_1| = \sqrt{100+400} = 10\sqrt{5}$ Amp. — 2 Marks

Power factor $= \frac{R_1}{|z_1|} = \frac{2}{\sqrt{20}} = \frac{1}{\sqrt{5}} = 0.447$

and $i_2 = \frac{V}{z_2} = \frac{100}{3+4j} = \frac{100(3-4j)}{(3+4j)(3-4j)} = \frac{100(3-4j)}{9+16}$ — 2 Mn

$= 4(3-4j) = 12-16j$

$|i_2| = \sqrt{144+256} = 20$ Amp

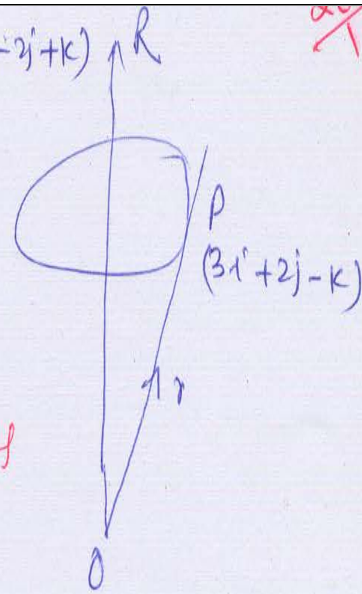
Power factor $= \frac{R_2}{|z_2|} = \frac{3}{5} = 0.6$ — 2 Marks

$i = i_1 + i_2 = (10-20j)+(12-16j) = 22-36j$

Magnitude $|i| = \sqrt{484+1296} = 2\sqrt{445}$ Ans.

Q.5a. A rigid body is rotating with angular velocity 2 radian/sec about an axis OR where R is $2i - 2j + k$ and O is the origin. Find the velocity of the point 3i + 2j – k on the body.

Q.5.a. Soln:



Angular velocity = 2 rad/sec.

$\vec{OR} = 2i - 2j + k$

$\vec{\omega} = \dfrac{2(2i-2j+k)}{\sqrt{4+4+1}}$ 2 Mark

$= \dfrac{2}{3}(2i - 2j + k)$

$\vec{r} = \vec{OP} = 3i + 2j - k$

$\vec{V} = \vec{\omega} \times \vec{r}$ 2 Mark

$= \dfrac{2}{3}(2i - 2j + k) \times (3i + 2j - k)$

$= \dfrac{2}{3}\begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = \dfrac{2}{3}\left[(2-2)i - (-2-3)j + (4+6)k\right]$

$= \dfrac{2}{3}[5i + 10k]$

$= \dfrac{10}{3}[j + 2k]$ Ans: 6 Mark

b. **Forces 2i + 7j, 2i – 5j + 6k, –i + 2j – k act at a point P whose position vector is 4i – 3j – 2k. Find the vector moment of the resultant of three forces acting at P about the point Q, whose position vector is 6i + j – 3k.**

Q5. b. Soln.

Let $\vec{F_1} = 2i + 7j$, $\vec{F} = 2i - 5j + 6k$, $\vec{F_3} = -i + 2j - k$, then the resultant forces $\vec{F}$ is given by,

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

$$\Rightarrow \vec{F} = 3i + 4j + 5k$$ — 2 Marks

Let $\vec{r} = \vec{QP}$

$$\Rightarrow \vec{r} = \text{P.V. of } P - \text{P.V. of } Q$$ — 2 Marks

$$\Rightarrow \vec{r} = (4i - 3j - 2k) - (6i + j - 3k)$$

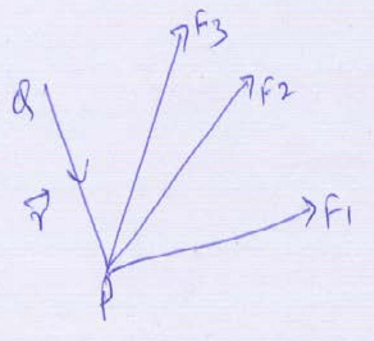
$$\Rightarrow \vec{r} = -2i - 4j + k$$

Let $\vec{M}$ be the moment of the resultant force $\vec{F}$ about

Then, $\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ -2 & -4 & 1 \\ 3 & 4 & 5 \end{vmatrix}$ — 8 Marks

$$= (-24i + 13j + 4k) \text{ Ans.}$$

Fig.



Q.6 a. Solve the differential equation $(D-1)^2 (D^2+1)y = \sin\frac{1}{2}x$, $D = \frac{d}{dx}$

260/1

Q.6.a. Soln. $(D-1)^2(D^2+1)y = \sin\frac{1}{2}x$

A.E. is $(m-1)^2(m^2+1)=0$

$\Rightarrow$ Eighter $(m-1)^2=0$ ie, $m=1,1$

or $m^2+1=0$, ie, $m^2=-1 \Rightarrow m=\pm i$

C.F. $= (c_1+c_2x)e^x + c_3\cos x + c_4 \sin x$ — 3 Marks

$$P.I. = \frac{1}{(D-1)^2(D^2+1)}\sin\frac{1}{2}x$$

$$= \frac{1}{(D-1)^2(-\frac{1}{4}+1)}\sin\frac{1}{2}x \qquad \left(D^2=-\frac{1}{2^2}\right)$$

$$= \frac{4}{3}\frac{1}{(D-1)^2}\sin\frac{x}{2}$$ — 2 Marks

$$= \frac{4}{3}\frac{1}{D^2-2D+1}\cdot\sin\frac{x}{2}$$

$$= \frac{4}{3}\frac{1}{-\frac{1}{4}-2D+1}\cdot\sin\frac{x}{2}$$

$$= \frac{4}{3}\frac{1}{-2D+\frac{3}{4}}\cdot\sin\frac{x}{2}$$

$$= \frac{4}{3}\frac{2D+\frac{3}{4}}{-4D^2+\frac{9}{16}}\cdot\sin\frac{x}{2}$$ — 2 Marks

$$= \frac{4}{3}\frac{2D+\frac{3}{4}}{-4(-\frac{1}{4})+\frac{9}{16}}\cdot\sin\frac{x}{2}$$

$$= \frac{4}{3}\cdot\frac{16}{25}\left[2D\cdot\sin\frac{x}{2}+\frac{3}{4}\sin\frac{x}{2}\right]$$

$$= \frac{64}{75}\cdot\left(\cos\frac{x}{2}+\frac{3}{4}\sin\frac{x}{2}\right)$$ — 8 Marks

Complete solution is

$$y = (c_1+c_2x)e^x + c_3\cos x + c_4\sin x + \frac{64}{75}\left(\cos\frac{x}{2}+\frac{3}{4}\sin\frac{x}{2}\right)$$

Ans

b. Solve the equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\sin 2x$

Q.6.b. Sol^n: $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\sin 2x$

$(D^2+3D+2)y=\sin 2x$

A.E. is $m^2+3m+2=0$

$m=-1,-2$

C.F. $=c_1e^{-x}+c_2e^{-2x}$ — 2 Marks

P.I. $=\frac{1}{D^2+3D+2}$

$=\frac{1}{-4+3D+2}\sin 2x$ $\quad [D^2=-(2)^2=4]$

— 2 Marks

Q6.b. Soln   Remaining part

$$P.I. = \frac{1}{3D-2}\sin 2x$$

$$= \frac{3D+2}{9D^2-4}\sin 2x$$

$$= \frac{3D+2}{9(-4)-4}\sin 2x$$

$$= -\frac{1}{40}(3D+2)\sin 2x$$

$$= -\frac{1}{40}\left[3D(\sin 2x) + 2\sin 2x\right]$$

$$= -\frac{1}{40}\left[6\cos 2x + 2\sin 2x\right]$$

$$= -\frac{1}{20}\left[3\cos 2x + \sin 2x\right]$$

$$y = c_1 e^{-x} + c_2 e^{-2x} - \frac{1}{20}\left(3\cos 2x + \sin 2x\right)$$ Ans

8 Mark

Q.7 Examine the following series

(i) $\sum \frac{1}{n^p}$

(ii) $\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \ldots$ **(16)**

Ans (i) Arti 8, page 634 of text book Engg Mathematic by N P Bali

(ii) Ex 1, page 652 of text book Engg Mathematic by N P Bali

Q.8 a. Obtain the Inverse Laplace Transform of $\cot^{-1}\left(\frac{s+3}{2}\right)$

Q.8.a. Soln.

We know that, $L^{-1}\left[F(s)\right] = -\frac{1}{t}L^{-1}\left[\frac{d}{ds}F(s)\right]$ – 2M

$$\therefore L^{-1}\left[\cot^{-1}\left(\frac{s+3}{2}\right)\right] = -\frac{1}{t}L^{-1}\left[\frac{d}{ds}\cot^{-1}\left(\frac{s+3}{2}\right)\right]$$

$$= -\frac{1}{t}L^{-1}\left\{\frac{-\frac{1}{2}}{1+\left(\frac{s+3}{2}\right)^2}\right\}$$

$$= \frac{1}{2t}L^{-1}\left\{\frac{4}{4+(s+3)^2}\right\}$$

$$= \frac{1}{t}L^{-1}\left\{\frac{2}{2^2+(s+3)^2}\right\}$$

$$= \frac{1}{t}e^{-3t}L^{-1}\left\{\frac{2}{2^2+s^2}\right\}$$

$$= \frac{e^{-3t}}{t}\sin 2t \quad \text{Ans.}$$

– 8 Mark

b. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$

Q.8.b. Soln:

Here $f(t) = \dfrac{\cos at - \cos bt}{t}$

we know that, $L(\cos at - \cos bt) = L\cos at - L\cos bt$

$$= \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}$$ 2 marks

$$L\left\{\frac{\cos at - \cos bt}{t}\right\} = \int_s^\infty \left(\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right) ds$$ — 2 marks

$$= \left[\frac{1}{2}\log(s^2+a^2) - \frac{1}{2}\log(s^2+b^2)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\frac{s^2+a^2}{s^2+b^2}\right]_s^\infty = \frac{1}{2}\left[\log\frac{1+\frac{a^2}{s^2}}{1+\frac{b^2}{s^2}}\right]_s^\infty$$

$$= \frac{1}{2}\log 1 - \frac{1}{2}\log\frac{1+\frac{a^2}{s^2}}{1+\frac{b^2}{s^2}}$$

$$= 0 - \frac{1}{2}\log\frac{s^2+a^2}{s^2+b^2} \quad (\log 1 = 0)$$

$$= \frac{1}{2}\log\frac{s^2+b^2}{s^2+a^2}$$ Ans.

8 marks

Q.9 a. Obtain the Laplace transform of $t^2e^t.\sin 4t$. (8)

Q.9.a. Soln:

$$L(\sin 4t) = \frac{4}{s^2+16}$$ 2 Mark

$$L(e^t \sin 4t) = \frac{4}{(s-1)^2+16}$$ 2 Mark

$$L(te^t \sin 4t) = -\frac{d}{ds}\left(\frac{4}{s^2-2s+17}\right) = \frac{4(2s-2)}{(s^2-2s+17)^2}$$ 2 Mark

$$L(t^2e^t \sin 4t) = -\frac{d}{ds}\left(\frac{4(2s-2)}{(s^2-2s+17)^2}\right)$$

$$= -4\frac{(s^2-2s+17)^2 2-(2s-2)2(s^2-2s+17)(2s-2)}{(s^2-2s+17)^4}$$

$$= -4\frac{(s^2-2s+17)2-2(2s-2)^2}{(s^2-2s+17)^3}$$

$$= \frac{-4(2s^2-4s+34-8s^2+16s-8)}{(s^2-2s+17)^3}$$

$$= \frac{-4(-6s^2+12s+26)}{(s^2-2s+17)^3} = \frac{8[3s^2-6s-13]}{(s^2-2s+17)^3}$$ Ans. 8 Marks

b. **Using the Laplace transforms, find the solution of the initial value problem $y'' + 9y = 6\cos 3t$, $y(0) = 2$, $y'(0) = 0$**

Q.9.b. Soln

$$y'' + 9y = 6\cos 3t \quad \text{--- (i)}$$

$$y(0) = 2,\ y'(0) = 0$$

Taking Laplace Transform of (i), we get

$$[s^2\bar{y} - sy(0) - y'(0)] + 9\bar{y} = 6\frac{s}{s^2+9} \quad \text{--- (ii)}$$

2M/KK

Putting the values of $y(0)$ and $y'(0)$ in eqn (ii), we have,

$$s^2\bar{y} - 2s + 9\bar{y} = \frac{6s}{s^2+9}$$

$$\Rightarrow (s^2+9)\bar{y} = 2s + \frac{6s}{s^2+9} \Rightarrow \bar{y} = \frac{2s}{s^2+9^2} + \frac{6s}{(s^2+9^2)}$$

2M/KK

$$y = L^{-1}\left\{\frac{2s}{s^2+9}\right\} + L^{-1}\left\{\frac{6s}{(s^2+9)^2}\right\} = 2\cos 3t + L^{-1}\frac{d}{ds}\left[\frac{-3}{(s^2+9)}\right]$$

$$= 2\cos 3t - t\sin 3t$$

Ans

8M/KK

TEXT BOOKS

1. Engineering Mathematics – Babu Ram, Pearson Education Limited, 2012
2. Applied Mathematics for Polytechnic, H.K.Dass, 10th Edition, 2012, CBS Publishers & Distributors, New Delhi