Q.2 a. State and prove divergence theorem.

Answer:

Divergence theorem: It states that:- the volume integral of divergence of a vector field D, taken over any volume v is equal to the surface integral of D taken over the closed surface that bounds the volume v. ie

$$\oint D\cdot ds = \int_v (\nabla\cdot D)\,dv$$

Proof. According to Gauss's law

$$\oint D\cdot ds = Q \quad \text{— (1)}$$

$$Q = \int_v \rho_v\,dv \quad \text{— (2)}$$

$$\rho_v = \nabla\cdot D \quad \text{— (3)}$$

$$\therefore Q = \int_v \nabla\cdot D\,dv \quad \text{— (4)}$$

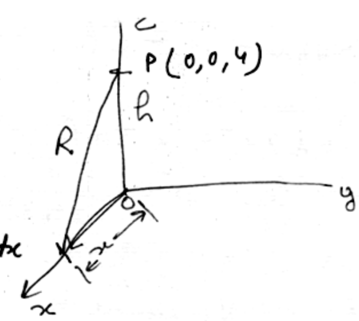
comparing eq. (1) & (4) we get

$$\oint D\cdot dS = \int_v \nabla\cdot D\,dv \quad \text{— (5)}$$

Which is divergence theorem

b. Find the electric field intensity at (0, 0, 4) due to a charge of 2nC distributed uniformly on the line $0 \le x \le 3$.

Answer:



From fig $R = h - x = h a_z - x a_x$

$$a_R = \frac{h a_z - x a_x}{\sqrt{h^2+x^2}}$$

The differential field at P due to line charge of distance dx is

$$dE = \frac{\rho_l\,dl}{4\pi\epsilon_0 R^2}\,a_R$$

field at P, $E = \int \frac{\rho_l}{4\pi\epsilon_0 (x^2+h^2)^{3/2}}\,[h a_z - x a_x]\,dx$

$$= \frac{\rho_l}{4\pi\epsilon_0}\left[\int_0^3 \frac{h}{(h^2+x^2)^{3/2}}\,dx\,a_z - \int_{x=0}^3 \frac{x}{(h^2+x^2)^{3/2}}\,dx\,a_x\right], \quad \frac{\rho_l}{4\pi\epsilon_0} = \frac{2/3\times10^{-9}}{4\pi\,8.854\times10^{-12}} = 6$$

$$= 6\left[\frac{4x}{16\sqrt{16+x^2}}\Big|_0^3 a_z + \frac{1}{\sqrt{16+x^2}}\Big|_0^3 a_x\right]$$

$$= -0.3\,a_x + 0.9\,a_y \text{ V/m} \quad \text{Ans.}$$

Q.3 a. Derive the continuity equation and give its physical interpretation.

Answer:

Continuity Equation: Consider a region bounded by a closed surface. The current through the closed surface is

$$I = \oint_s J \cdot ds \quad \text{—(1)} \qquad \text{Also } I = -\frac{dQ}{dt} \quad \text{—(2)}$$

$$\therefore I = \oint_s J \cdot ds = -\frac{dQ}{dt} \quad \text{—(3)}$$ using the principle of conservation of charge.

Converting surface integral into volume integral

$$\oint_S J \cdot ds = \int_V (\nabla \cdot J)\, dv \quad \text{and}, \quad Q = \int_v \rho_v\, dv$$

Substituting in eq (3)

$$\int_V (\nabla \cdot J)\, dv = -\frac{d}{dt}\int_v \rho_v\, dv = -\int_V \frac{d\rho_v}{dt}\, dv$$

comparing both sides

$$\nabla \cdot J = -\frac{\partial \rho_v}{dt} \quad \text{—(4)}$$ which is continuity equation

It indicates that the current diverging from a small volume per unit volume is equal to the rate of decrease of charge per unit volume at every point.

b. A charge distribution with spherical symmetry has volume charge density

$$\rho_V = \begin{cases} \rho_0 & 0 \le r \le a \\ 0 & r > a \end{cases}$$

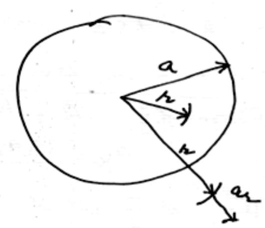
calculate (i) electrical field intensity and (ii) total energy stored.

Answer:

i) Using fig, Apply Gauss's law for $r \le a$

$$\int_S D \cdot ds = \int_v \rho_v\, dv$$

$$D\, 4\pi r^2 = \frac{4}{3}\pi r^2 \rho_0 \Rightarrow D = \frac{r\rho_0}{3} \qquad \therefore E = \frac{r\rho_0}{3\epsilon_0}\, a_r \quad 0 \le r \le a$$



For $r > a$, Gauss's law

$$D \cdot 4\pi r^2 = \frac{4}{3}\pi a^3 \rho_0$$

$$D = \frac{a^3\rho_0}{3r^2} \Rightarrow E = \frac{\rho_0 a^3}{3\epsilon_0 r^2}\, a_r \quad r > a$$

At $r = a$, $E = \frac{\rho_0 a}{3\epsilon_0}\, a_r \quad r = a$

(ii) The energy stored is

$$W = \frac{1}{2}\int_v \epsilon_0 E^2\, dv = \frac{1}{2}\int_{v,\, r \le a} \epsilon_0 E^2\, dv + \frac{1}{2}\int_{v,\, r > a} \epsilon_0 E^2\, dv$$

$$= \frac{\rho_0^2}{18\epsilon_0}\int_{r=0}^{a}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^2 r^2 \sin\theta\, dr\, d\theta\, d\phi + \frac{\rho_0^2 a^6}{18\epsilon_0}\int_a^\infty\int_0^\pi\int_0^{2\pi} \frac{1}{r^4} r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= \frac{\rho_0^2 a^5}{18\epsilon_0\, 5}\, 4\pi + \frac{\rho_0 a^6}{18\epsilon_0}\left(\frac{-1}{r}\right)\Bigg|_a^\infty 4\pi = \frac{2\pi\rho_0 a^5}{45\epsilon_0} + \frac{2\pi\rho_0^2 a^5}{9\epsilon_0}$$

$$= \frac{4\pi\rho_0^2 a^5}{15\epsilon_0}\ \text{J} \quad \text{Ans.}$$

Q.4 a. Draw the profiles for (i) the charge density (ii) the electric field intensity (iii) the potential of pn-junction as function of distanced from the centre of the junction.

Answer:

Q.4 a. Uniqueness Theorem :- If a solution can be found to the Poisson's (4) equation or Laplace's equation by any means which satisfies the boundary conditions, then solution is unique.

Proof: Consider ϕ_1 and ϕ_2 be two solutions of Poissions equation, then

$$\nabla^2 \phi_1 = -\frac{\rho_v}{\epsilon}$$

$$\nabla^2 \phi_2 = -\frac{\rho_v}{\epsilon}$$

Let $\phi_0 = \phi_1 - \phi_2$, then $\nabla^2 \phi_0 = \nabla^2(\phi_1 - \phi_2) = 0$

using identity, $\nabla \cdot (U \nabla V) = \nabla U \cdot \nabla V - U \nabla^2 V$

$$\nabla \cdot (\phi_0 \nabla \phi_0) = (\nabla \phi_0)^2$$

Integrating over volume $\int_V (\nabla \phi)^2 dv = \int \nabla \cdot (\phi_0 \nabla \phi_0)\, dv$

Apply divergence theorem

$$\int_V (\nabla \phi_0)^2 dv = \int (\phi \nabla \phi_0)\, dS = \int_S \phi_0 \frac{\partial \phi_0}{\partial n} ds$$

on the surface bounding the volume V, ϕ_0 or $\frac{d\phi_0}{dn} = 0$

Then $\int_V (\nabla \phi_0)^2 dv = 0$ or $\nabla \phi_0 = 0$ ie $\phi_0 = \text{Constant}$

$\phi_0 = 0$ over S and ϕ_0 is continuous through out the entire volume

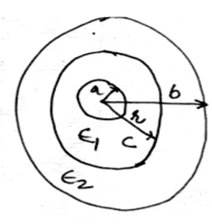
$\therefore \phi_1 - \phi_2 = 0 \Rightarrow \phi_1 = \phi_2 \Rightarrow$ There cannot be two solutions of Poissions eq

b. A coaxial cable consists of an inner conductor of radius 'a' and outer conductor of radius 'b'. The space between the conductor is filled with the dielectric of permittivity '$\in$'. Determine the capacitance of cable per unit length.

Answer:

b. Gauss' Law for $a \leq r \leq b$

$\oint D \cdot ds = Q_{enclosed}$ or $D \cdot 2\pi r \cdot 1 = Q \Rightarrow D = \frac{Q}{2\pi r}$ — (1)



$E_1 = \frac{Q}{2\pi \epsilon_1 r}$ $\quad a \leq r \leq c$ — (2)

$E_2 = \frac{Q}{2\pi \epsilon_2 r}$ $\quad c \leq r \leq b$ — (3)

Potential difference across ϵ_1

$V_1 = \int_c^a E \cdot dl = \int_c^a \frac{Q}{2\pi \epsilon_1 r} dr = \frac{Q}{2\pi \epsilon_1} \ln \frac{c}{a}$ — (4)

Similarly $V_2 = \frac{Q}{2\pi \epsilon_2} \ln b/c$ — (5)

Potential difference $V = V_1 + V_2 = \frac{Q}{2\pi} \left[\frac{1}{\epsilon_1} \ln \frac{c}{a} + \frac{1}{\epsilon_2} \ln \frac{b}{c} \right] = \frac{Q}{2\pi \epsilon_0} \left[\frac{1}{\epsilon_{r1}} \ln \frac{c}{a} + \frac{1}{\epsilon_{r2}} \ln \frac{b}{c} \right]$

$\therefore \epsilon = \epsilon_0 \epsilon_r$

capacitance per unit length

$$C = \frac{2\pi \epsilon_0}{\frac{1}{\epsilon_{r1}} \ln \frac{c}{a} + \frac{1}{\epsilon_{r2}} \ln \frac{b}{c}}$$ Ans.

Q.5 a. What is vector magnetic potential? Derive its expression.

Answer:

Q.5a Magnetic vector Potential: Vector magnetic potential A is defined as $B = \nabla \times A$

$\nabla \times B = \mu J$

$\nabla \times B = \nabla \times \nabla \times A = \mu J$

$\nabla \times \nabla \times A = \mu J$

$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A$

$\nabla^2 A = a_x \nabla^2 A_x + a_y \nabla^2 A_y + a_z \nabla^2 A_z$

$\because \nabla \cdot A = 0$

$\therefore -\nabla^2 A = \mu J$

$\nabla^2 A = -\mu(a_x J_x + a_y J_y + a_z J_z)$

$\nabla^2 A_x = -\mu J_x$

$\nabla^2 A_y = -\mu J_y$

$\nabla^2 A_z = -\mu J_z$

Also using Poisson equation

$\nabla^2 V = -\rho_v/\epsilon_0$

$V = \frac{1}{4\pi\epsilon_0}\int_v \frac{\rho_v \, dv}{r}$

$\therefore A_x = \mu/4\pi \int_v \left(\frac{J_x}{r}\right) dv$

$A_y = \frac{\mu}{4\pi}\int_v J_y/r \, dv$

$A_z = \frac{\mu}{4\pi}\int_v J_z/r \, dv$

$\therefore \boxed{A = \frac{\mu}{4\pi}\int \frac{J}{r} dv}$ with $B = \nabla \times A$ which give magnetic vector potential.

b. What is curl operator? Explain stokes theorem.

Answer:

b. Curl of a vector $B = B_x a_x + B_y a_y + B_z a_z$ is $\nabla \times B$

$$\nabla \times B = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)a_x + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)a_y + \left(\frac{\partial B_y}{\partial z} - \frac{\partial B_x}{\partial y}\right)$$

in cartesian coordinates.

$$= \begin{vmatrix} a_r & a_\phi & a_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ B_r & rB_\phi & B_z \end{vmatrix}, \quad \nabla \times B = \begin{vmatrix} \frac{a_r}{r^2 \sin\theta} & \frac{a_\theta}{r\sin\theta} & \frac{a_\phi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ B_r & rB_\theta & r\sin\theta B_\phi \end{vmatrix}$$

in cylindrical coordinates, in spherical coordinates

Stokes Theorem: The surface integral of curl of a vector field over an open surface equals the line integral of the vector field over the closed curve bounding the surface area.

$$\int_s (\nabla \times A) \cdot ds = \oint_c A \cdot dl$$

Q.6 a. What are magnetic circuits? Compare it with electrical circuits.

Answer:

a. Magnetic circuits: The fundamental techniques involved in solving a class of magnetic problems are known as magnetic circuits. (5

Page: 284: Text book-I: Engineering Electromagnetics – by Hayt

b. Calculate the values for χ_m, M and H for ferrite material operating in a linear mode with B = 0.05 T and $\mu_r = 50$.

Answer:

b. Since $\mu_r = 1 + \chi_m \Rightarrow \chi_m = \mu_r - 1 = 50 - 1 = \underline{49}$

Also $B = \mu H \Rightarrow B = \mu_r \mu_0 H \Rightarrow H = \dfrac{B}{\mu_r \times \mu_0} = \dfrac{0.05}{50 \times 4\pi \times 10^{-7}} = \underline{796}$ A/m

Magnetization $\chi_m H = 39000$ A/m

$B = \mu_0 (H + M) \Rightarrow 0.05 = 4\pi \times 10^{-7} (796 + 39000)$

$\Rightarrow M = \dfrac{B - \mu_0 H}{\mu_0} = \dfrac{0.05 - 4\pi \times 10^{-7} \times 796}{4\pi \times 10^{-7}} =$

Q.7 a. Explain Faradays' law for time varying fields.

Answer:

a. Any current carrying conductor produces magnetic field. Faraday observed that magnetic field can produce current in a closed circuit with a requirement that magnetic flux linking the circuit must be changing. An emf arises from conductor moving in a magnetic field or from changing magnetic field. The Faraday's law is stated as

$emf = -\dfrac{d\phi}{dt}$ —① The minus sign indicates that emf is in the direction of opposing flux,

A non zero value of $\dfrac{d\phi}{dt}$ may result

(i) A time changing flux linking a stationary closed path.

(ii) Relative motion between a steady flux and closed path.

(iii) A combination of two.

we define emf as $emf = \oint E \cdot dl$ —②

Total flux through circuit $\phi = \int_s B \cdot ds$ —③

using eq. ① & eq. ③ $emf = -\dfrac{d}{dt}\int_s B \cdot ds$ —④

∴ using eq. ② & ④

$emf = \oint E \cdot dl = -\dfrac{d}{dt}\int_s B\, ds$ —⑤

$\oint E \cdot dl = -\int_s \dfrac{dB}{dt} \cdot ds$ —— ⑥

using Stokes theorem $\oint E \cdot dl = \int_s (\nabla \ldots$

∴ eq. 6 becomes $\int_s \nabla \times E \cdot ds = -\int_s \dfrac{\partial B}{\partial t} \cdot ds$

∴ $\nabla \times E = -\dfrac{\partial B}{\partial t}$ —⑦ which is Faraday's law in differential form

b. Explain the concept of displacement current.

Answer:

b. Displacement current:

Maxwell's equation in differential form

$\nabla \times E = -\frac{\partial B}{\partial t}$ — (1)

Ampere's law for steady magnetic field.

$\nabla \times H = +J$ — (2)

$\nabla \cdot \nabla \times H = 0 = \nabla \cdot J$

$\therefore \nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$

eq.(2) is true if $\frac{\partial \rho_v}{\partial t} = 0$, which is unrealistic.

$\therefore$ eq.(2) must be amended before it for time varying fields.

$\therefore \nabla \times H = J + G$

Taking divergence

$0 = \nabla \cdot J + \nabla \cdot G$

Thus $\nabla \cdot G = \frac{\partial \rho_v}{\partial t}$

Replace $\rho_v = \nabla \cdot D$

$\nabla \cdot G = \frac{\partial}{\partial t}(\nabla \cdot B) = \nabla \cdot \frac{\partial D}{\partial t}$

$\therefore G = \frac{\partial D}{\partial t}$

Thus Ampere's law in point form

$\nabla \times H = J + \frac{\partial B}{\partial t}$ — (3)

The additional term $\frac{\partial D}{\partial t}$ has the dimensions of current-density and is refered as displacement current density. $\therefore \nabla \times H = J + J_d$

$J_d = \frac{\partial D}{\partial t}$

Q.8 a. Explain the reflection and refraction of electromagnetic waves by a conducting medium.

Answer:

2 Text book - II : Electronic communication system.
George Kennedy and Bernard Davis.
Pages: 229 - 232

b. Explain the following: (4+4)
(i) Ground wave propagation
(ii) Directional Antennas

Answer:

b: (i) Pages. 237 - 238, (ii) pages 261 - 262

Q.9 Write short notes on : (4+4+4+4)
(i) Antenna resistance
(ii) Grounded Antennas
(iii) Antenna couplers
(iv) Lens Antenna

Answer:
(i) Page 264 - 264
(ii) Pages 267 - 268
(iii) Pages 273 - 274
(iv) Pages 293 - 295

TEXT BOOKS

I. Engineering Electromagnetics, W. H. Hayt and J. A. Buck, Seventh Edition, Tata McGraw Hill, Special Indian Edition 2006.
II. Electronic Communication Systems, George Kennedy and Bernard Davis, Fourth Edition (1999), Tata McGraw Hill Publishing Company Ltd.