Q.2a. Give a simple analysis of an electric water heater system.

Ans- 2.2 of Text Book I

b. Find the equation describing the motion of the mechanical system shown in Fig.1 below K stands for compliance of the spring.

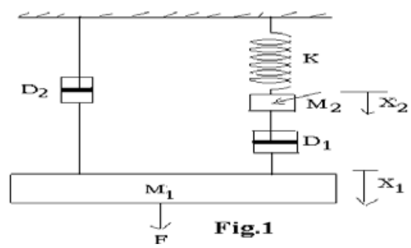

Fig.1

(b) It is a two coordinate system i.e. two variables, x_1 and x_2 are needed to describe the system completely. Consider the forces on mass M_1

External force: f

Resisting forces

(1) Inertia force $f_{m_1} = -M_1 \frac{d^2x_1}{dt^2} = -M_1 \frac{du_1}{dt}$ — (1)

(2) damping force, $f_{D_1} = -D_1 \frac{d}{dt}(x_1 - x_2) = -D_1 (u_1 - u_2)$

$$f_{D_2} = -D_2 \frac{dx_1}{dt} = -D_2 u_1$$ — (1)

Application of D'Alembert's principle gives

$$M_1 \frac{du_1}{dt} + (D_1 + D_2) u_1 - D_1 u_2 = f$$ — (1) (1)

Consider the forces on Mass M_2:

External force: none

Resisting forces

(1) Inertia force, $f_{m_2} = -M_2 \frac{d^2x_2}{dt^2} = -M_2 \frac{du_2}{dt}$ — (1)

(2) damping force, $f_{D_1} = -D_1 \frac{d}{dt}(x_2 - x_1) = -D_1 (u_2 - u_1)$ — (1)

(3) spring force, $f_K = -\frac{1}{K} x_2 = -\frac{1}{K}\left[\int_0^t u_2\, dt + x_2(0)\right]$ — (1)

Hence

$$D_1 u_1 + m_2 \frac{du_2}{dt} + D_1 u_2 + \frac{1}{k}\left[\int_0^t u_2\,dt + x_2(0)\right] = 0 \qquad -(2)$$

Equations (1) and (2) completely describe the motion of the system.

Q.3a. Give a systematic procedure for reduction of complicated block diagrams. Illustrate the procedure with the help of an example.

Ans. 2.4 of Textbook I

b.Determine the overall transfer function C(s) / R(s) of the system represented by the signal flow graph given below.

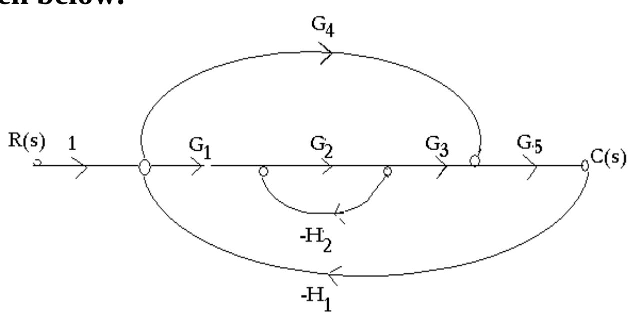

Fig.2

Q 3 b There are two forward paths from the input R(s) to the output C(s). The gains of the paths are

$T_1 = G_1 G_2 G_3 G_5$ $\qquad T_2 = G_4 G_5$

There are three loops in the system with gains as

$L_1 = -G_1 G_2 G_3 G_5 H_1$ $\qquad L_2 = -G_2 H_2$ $\qquad L_3 = -G_4 G_5 H_1$

Because L_2 and L_3 form the only set of non-touching loops, the determinant can be written as

$$\Delta = 1 - (L_1 + L_2 + L_3) + L_2 L_3$$
$$= 1 + (G_1 G_2 G_3 G_5 H_1 + G_2 H_2 + G_4 G_5 H_1) + G_2 G_4 G_5 H_1 H_2$$

Because all three loops touch path T_1, setting $L_1 = L_2 = L_3 = 0$ in the preceeding expression of Δ yields $\Delta_1 = 1$

Because loops L_1 and L_3 touch path T_2, setting $L_1 = L_3 = 0$ in the expression for Δ yields

$\Delta_2 = 1 - L_2 = 1 + G_2 H_2$

Next, Mason's gain formula yields

$$\frac{C(s)}{R(s)} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} =$$

$$= \frac{G_1G_2G_3G_5 + (G_4G_5)(1+G_2H_2)}{1+(G_1G_2G_3+G_4+G_2G_4H_2)G_5H_1 + G_2H_2}$$ Ans. 1

Q.4a. Write a note on stepper motors.

Ans. 4.4 of Text book I

b.Given a closed loop control system with the forward path transfer function = 32 and the feedback path transfer function = 0.01. Calculate the closed loop transfer function if the system is

(i) Negative feedback

(ii) Positive feedback

Q4b. The closed loop transfer function with negative feedback is

$$M(s) = \frac{G(s)}{1+G(s)H(s)}$$ 2

$G(s) = 32 \quad H(s) = 0.01$

$$M(s) = \frac{32}{1+32\times 0.01} = \frac{32}{1.32} = \underline{24.24}$$ 2 Ans

Similarly, the closed loop transfer function with positive feedback is

$$M(s) = \frac{G(s)}{1-H(s)G(s)} = \frac{32}{1-32\times 0.01} = \frac{32}{0.68}$$ 2

$$= \underline{47.06}$$ Ans. 2

Q.5 a. In reference to control system engineering define the term performance index. What are various qualities which a suitable performance index should possess?

Ans. 5.5 of textbook I

b. What possible difficulties may be faced while implementing the Routh – Hurwitz

criterion for determination of stability of linear control systems? Explain through examples how these difficulties can be faced?

Ans 5.5 of textbook I

Q.6 Give a stepwise procedure to draw the root locus of a given control system. Illustrate the procedure with the help of an example.

Ans 7.3 of textbook I

Q.7 a. Define the terms gain crossover, phase crossover, gain margin and phase margin. Show these quantities on a typical Nyquist plot.

Ans 7.3 of textbook I

b.Explain how the initial slope of the log-magnitude versus frequency plot of a transfer function is related to the type of the system represented by the given transfer function.

Ans 8.4 of textbook I

Q.8 a. Explain the reaction curve method for the experimental determination of controller setting of a given control system as given by loop. (8)

Ans 10.4 of textbook I

b.The open – loop transfer function of a unity feedback control system is given by G(s)

$$= \frac{10}{s(s+4)}$$

Design a suitable compensator so that the static velocity error constant of the compensated system be 50 sec^{-1} without appreciably changing the original closed – loop poles located at -2 $\pm j\sqrt{5}$

Ans

Q 8 b. : Since the design specification relates to the steady-state performance and requires [illegible] of the transient response of the original system, the compensator to be designed should be a lag-compensator. Let the transfer function of the compensator be

$$D(s) = \frac{K_c(s+z)}{s+p}$$

The static velocity error constant of the original system can be determined as

$$K_v = \lim_{s \to 0} sG(s) = \lim_{s \to 0} \frac{s \times 10}{s(s+4)}$$

$$= \frac{10}{4} = 2.5 \text{ sec}^{-1}.$$ (1)

The desired value of the static velocity error constant of the compensated system is 50 sec^{-1}!

Therefore,

$$\frac{z}{p} = \frac{K_v'}{K_v} = \frac{50}{2.5} = 20$$ (1)

Let us consider the compensating zero at $s = -0.1$ and the compensating pole at $s = -0.005$ so that the ratio z/p is maintained at 20.

The transfer function of the lag compensator would be

$$D(s) = \frac{K_c(s+0.1)}{s+0.005} \quad [\text{Assuming } K_c = 1]$$

$$= \frac{s+0.1}{s+0.005} \quad \underline{\text{Ans}}$$ (2)

and the open-loop transfer function of the compensated system becomes

$$D(s)\,G(s) = \frac{s+0.1}{(s+0.005)} \cdot \frac{10}{s(s+4)}$$ 1

Before accepting the design let us determine the angle contribution at $-2+j\sqrt{5}$ by the compensating pole-zero pair

$$\angle D(s) = \angle\left[\frac{s+0.1}{s+0.005}\right]_{s=-2+j\sqrt{5}}$$

$$= \left[\angle(s+0.1) - \angle(s+0.005)\right]\Big|_{s=-2+j\sqrt{5}}$$

$$= \angle(-2+j\sqrt{5}+0.1) - \angle(-2+j\sqrt{5}+0.005)$$

$$= \tan^{-1}\frac{\sqrt{5}}{-1.9} - \tan^{-1}\frac{\sqrt{5}}{-1.995}$$ 3

$$= 130.36^\circ - 131.74^\circ = 1.38^\circ$$

This is very small and justifies design of the

lag compensator

Q.9 a. Determine stability of the system described by equation: (8)

$$\dot{X} = \mathbf{AX}$$

$$\mathbf{A} = \begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix}$$

by using liapunov's direct method.

Ans. Page No. 652 of Text book –I, 13.3

b.Develop a state space model for a system whose dynamics is represented by the following equation.

$$\frac{d^3y(t)}{dt^3} + 3\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 7y = 11u(t)$$

Q9b. As the first step, let us choose

$x_1 = y$

Then we can choose other state variables as

$x_2 = \dot{y}$

$x_3 = \ddot{y}$ 1

The given equation governing the system dynamics can be written in a simplified manner as follows

$\dddot{y} + 3\ddot{y} + 5\dot{y} + 7y = 11\,u(t)$ 1

The state equations can be written as

3

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -7x_1 - 5x_2 - 3x_3 + 11u(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 11 \end{bmatrix} u(t)$$

3

Ans.

Textbook

I. Control Systems Engineering,), I.J. Nagrath and M. Gopal, Fifth Edition (2007 New Age International Pvt. Ltd