

Q.2a. Calculate the voltage V_{AB} across terminals A and B in the network, shown in Fig 3.

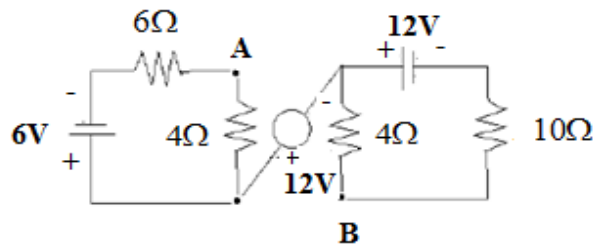


Fig.3

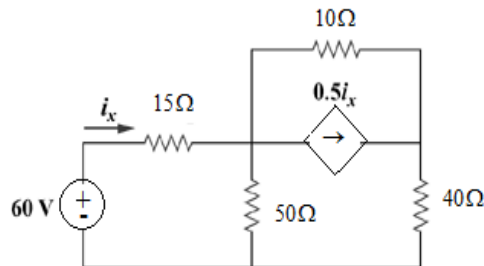


Fig.4

Fig. 2 B

Ans:
The circuit of Fig.2 can be redrawn as shown in Fig.2.1(a)

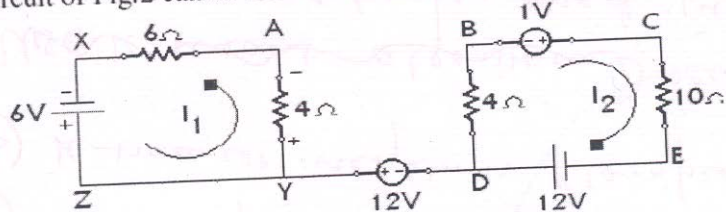


Fig.2.1(a)

In loop XAYZ, loop current I_1 as shown in Fig.2.1(a) is

$$I_1 = \frac{6}{6+4} = \frac{6}{10} = 0.6A$$

In loop BCED, loop current I_2 as shown in Fig.2.1(a) is

$$I_2 = \frac{12}{4+10} = \frac{12}{14} = 0.86A$$

V_A = voltage drop across 4Ω resistor is

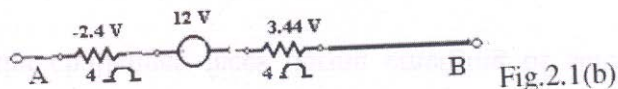
$$V_A = I_1 \times 4\Omega = 0.6 \times 4 = 2.4V$$

V_B = voltage drop across 4Ω resistor is

$$V_B = I_2 \times 4\Omega = 0.86 \times 4 = 3.44V$$

Therefore,

The voltage between points A and B is the sum of voltages as shown in Fig.2.1(b)



Hence, $V_{AB} = -2.4 + 12 + 3.44 = 13.04V$

- b. Using source transformation, calculate the current i_x flowing in the circuit shown in Fig 4.

Fig. 3

As shown in Fig. (a), we transform the dependent current source to a voltage source,

(a) Original circuit: A 60V DC voltage source is in series with a 15Ω resistor. This combination is in parallel with a 50Ω resistor. The circuit then continues in series with a 10Ω resistor, a dependent current source of $5i_x$ (pointing downwards), and a 40Ω resistor.

(b) Circuit after source transformation: The 60V source and 15Ω resistor are in series. This is followed by a parallel combination of three branches: a 50Ω resistor, another 50Ω resistor, and a dependent current source of $0.1i_x$ pointing downwards.

(c) Circuit for KVL: A single loop containing a 60V source, a 15Ω resistor, a 25Ω resistor (the equivalent of the two 50Ω resistors in parallel), and a dependent current source of $2.5i_x$ pointing downwards. A clockwise current i_x is indicated.

In Fig. (b), $50\parallel 50 = 25$ ohms. Applying KVL in Fig. (c),

$$-60 + 40i_x - 2.5i_x = 0, \text{ or } i_x = \underline{1.6 \text{ A}}$$

- Q.3 a. After steady-state current is established in the R-L circuit shown in Fig.5 with switch S in position 'a', the switch is moved to position 'b' at $t = 0$. Find $i_L(0^+)$ and $i(t)$ for $t > 0$. What will be the value of $i(t)$ when $t = 4$ seconds?(8)

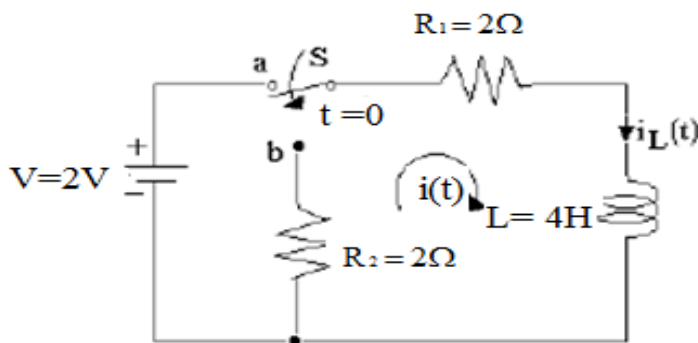


Fig.5

Ans:

When the switch 'S' is in position 'b', Kirchhoff's Voltage Law (KVL) gives

$$L \frac{di}{dt} + R_1 i + R_2 i = 0 \quad \text{----- (1)}$$

But from the Fig.7, $L = 4H$, $R_1 = 2\Omega$ & $R_2 = 2\Omega$

By substituting these values in equation (1), we get

$$4 \frac{di}{dt} + 2i + 2i = 0$$

$$4 \frac{di}{dt} + 4i = 0 \Rightarrow \frac{di}{dt} + i = 0 \quad \text{----- (2)}$$

By applying Laplace Transform to the equation (2), we get

$$sI(s) - i(o^+) + I(s) = 0$$

$$I(s)(s+1) = i(o^+) \quad \text{----- (3)}$$

Now, the current just before switching to position 'b' is given by (shown in Fig.4.8)

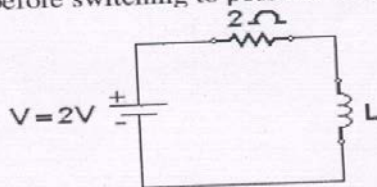


Fig.4.8

$$i(o^-) = \frac{V}{R} = \frac{2}{2} = 1Amp$$

The current $i(o^+)$ after switching to position 'b' must also be 1amp, because of the presence of inductance L in the circuit. Therefore, the equation (3) will become

$$I(s)(s+1) = i(o^+)$$

$$I(s)(s+1) = 1 \Rightarrow I(s) = \frac{1}{s+1} \quad \text{----- (4)}$$

On taking Laplace Transform to equation (4), we get

$$i(t) = e^{-t}$$

When $t = 4$ seconds, finding of $i(t)$: the value of $i(t)$ will be

$$i(t) = e^{-4}$$

b. Determine the amplitude and phase for $F(j2)$ from the pole-zero plot in s -plane for the network function $F(s) = \frac{4s}{(s^2 + 2s + 2)}$

Ans:

The given network function is $F(s) = \frac{4s}{s^2 + 2s + 2}$

In factored form, $F(s)$ will become as

$$F(s) = \frac{4s}{(s+1+j1)(s+1-j1)}$$

$F(s)$ has (i) Zero at $s = 0$

(ii) Poles are located at $(-1+j1)$ & $(-1-j1)$

From the poles and zeros of $F(s)$, draw vectors to the point $j\omega = 2$, as shown in Fig.5.1.

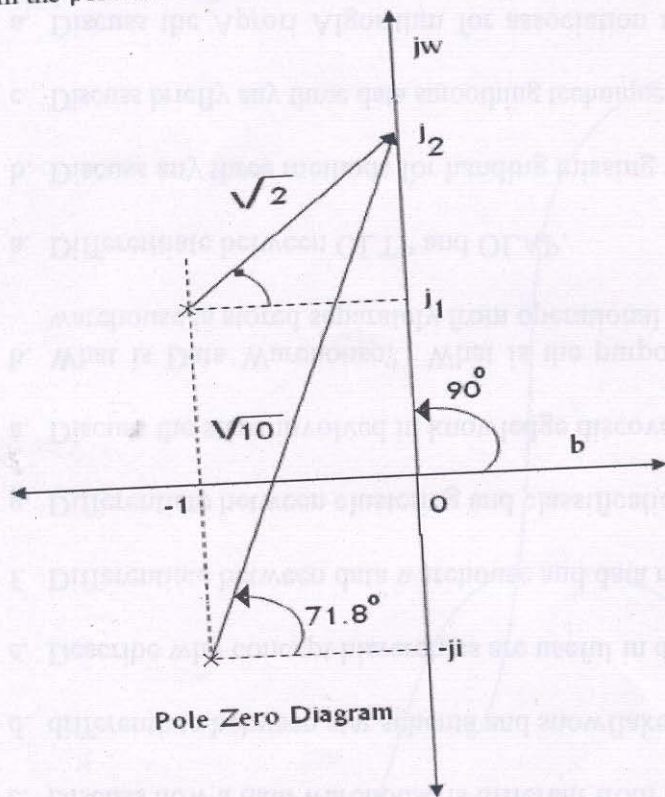


Fig.5.1. Pole-Zero Diagram

From the pole-zero diagram, it is clear that Magnitude at $F(j2)$ is calculated as

$$M(j2) = 4 \left(\frac{2}{\sqrt{2} \times \sqrt{10}} \right) = 1.78 \quad \text{and}$$

Phase at $F(j2)$ is calculated as

$$\phi(j2) = 90^\circ - 45^\circ - 71.8^\circ = -26.8^\circ$$

Q.4 a. Switch K in the circuit shown in Fig.6 is opened at $t = 0^+$ Draw the Laplace transformed network for $t > 0^+$ and find the voltages $V_1(t)$ and $V_2(t)$, $t > 0^+$.

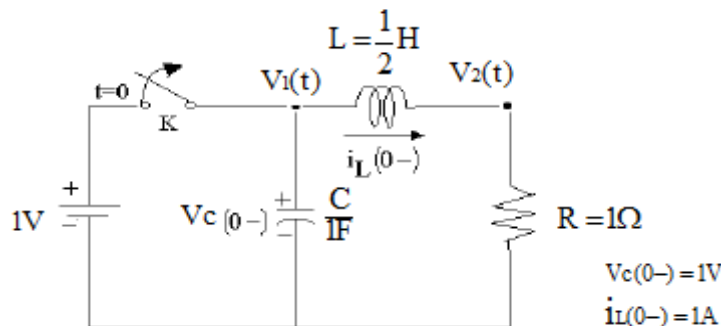
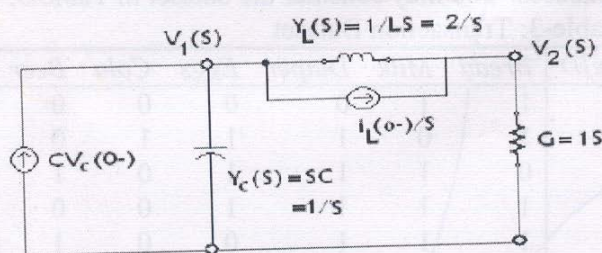


Fig.6

Ans:
The Laplace transformed network for $t > 0^+$ is shown in Fig 5.



Node V_1 :

$$\frac{-i_L(0^-)}{s} + CV_C(0^-) = \left(sC + \frac{1}{sL} \right) V_1(s) - \frac{1}{sL} V_2(s) \quad \text{----- (1)}$$

and Node V_2 :

$$\frac{i_L(0^-)}{s} = -\frac{1}{sL} V_1(s) + \left(\frac{1}{sL} + G \right) V_2(s) \quad \text{----- (2)}$$

Since prior to opening of switch the network has been in steady-state, then we have $V_C(0^-) = 1\text{V}$ and $I_L(0^-) = 1\text{A}$. By substituting the numerical values in eqns (1) & (2) we have

$$1 + \frac{1}{s} = \left(s + \frac{1}{s} \right) V_1(s) - \frac{2}{s} V_2(s) \quad \text{----- (3)}$$

$$\frac{1}{s} = \frac{2}{s} V_1(s) + \left(\frac{2}{s} + 1 \right) V_2(s) \quad \text{----- (4)}$$

Solving the equations (3) & (4) for $V_1(s)$ & $V_2(s)$ we have

$$V_1(s) = \frac{s+1}{(s^2+2s+2)} = \frac{s+1}{(s+1)^2+1} \quad \text{----- (5)}$$

$$V_2(s) = \frac{s+2}{(s^2+2s+2)} = \frac{s+2}{(s+1)^2+1}$$

- b. In the network shown in Fig. 7, the switch 'K' is moved from position 'a' to position 'b' at $t=0$, a steady state having previously been established at position 'a'. Solve the current $i(t)$ using the Laplace transformation method (8)

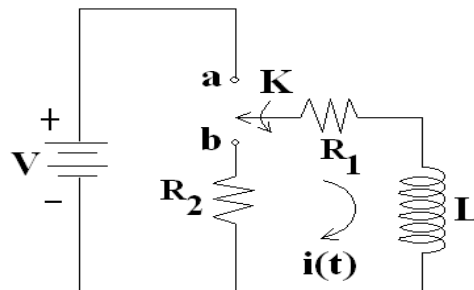


Fig.7

Ans:

KVL to loop of v and series R-L-C with $i(t) \Rightarrow V = L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt$

Differentiation $\Rightarrow 0 = L \frac{d^2i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} \Rightarrow \left(s^2 L + Rs + \frac{1}{C} \right) I(s) = 0$

\therefore Roots of $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \Rightarrow s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\alpha \pm \beta$ (say).

Where $\alpha = \frac{R}{2L}$, $\omega_0^2 = \frac{1}{LC}$, and $\beta = \sqrt{\alpha^2 - \omega_0^2}$.

\therefore Transient current

$i(t) = Ae^{s_1 t} + Be^{s_2 t}$, A ----- (1) (A, B \rightarrow constants, derived using initial conditions)

- Q.5 a. Determine the equivalent Norton network at the terminals a and b of the circuit shown in Fig.8 below.

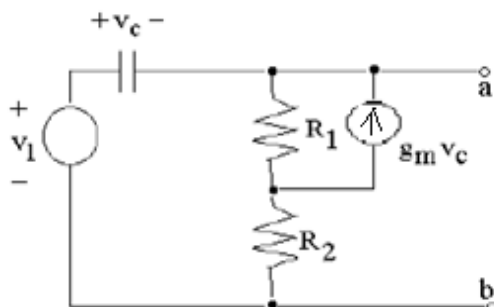


Fig.8

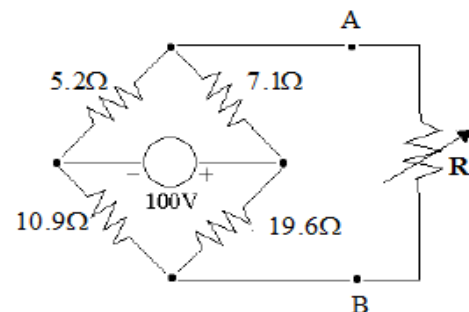
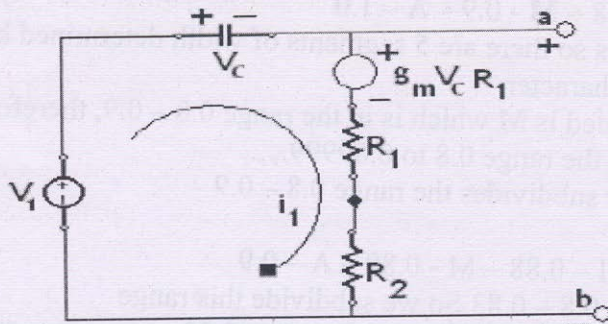
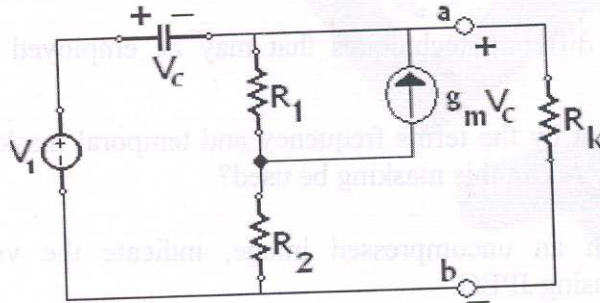


Fig.9

Ans:



By writing loop equations for the circuit shown

$$-V_1 + V_c + g_m V_c R_1 + i_1 (R_1 + R_2) = 0$$

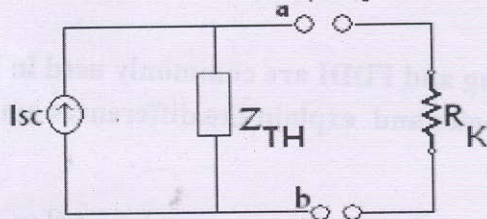
$$\text{OR } i_1 (R_1 + R_2) = V_1 - V_c (1 + g_m R_1)$$

$$\text{OR } i_1 = \frac{V_1 - V_c (1 + g_m R_1)}{(R_1 + R_2)}$$

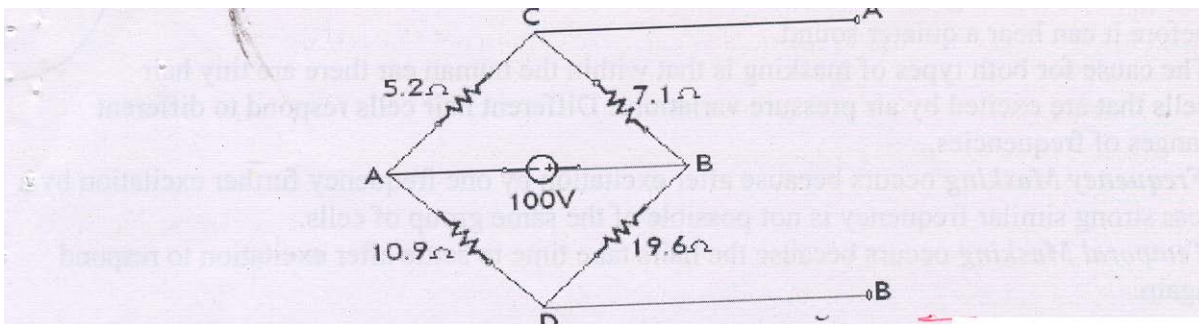
Thevenin's Impedance for the circuit is given by

$$Z_{Th} = \frac{R_1 + R_2}{1 + sC(R_1 + R_2) + g_m R_1}$$

$$\text{OR } I_{sc} = V_1(s) \cdot \frac{[sC(R_1 + R_2) + g_m R_1]}{R_1 + R_2}$$



b. Use the Thevenin equivalent of the network shown in Fig.9 to find the value of R which will receive maximum power. Find also this power.



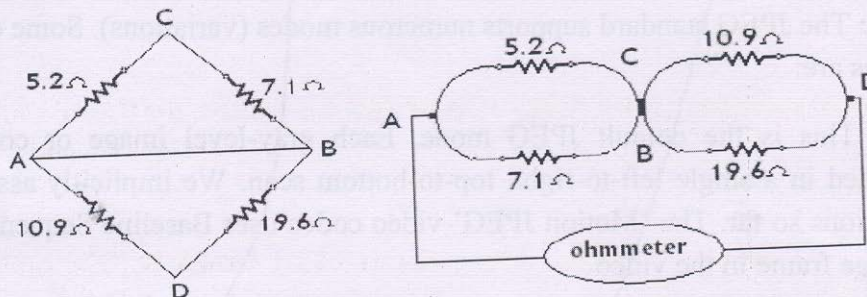
Consider the point A (the reference point) at ground potential,

$$V_A = \frac{5.2}{5.2+7.1} \times (100v) = \frac{5.2}{12.3} \times (100v) = 42.27 \text{ volts}$$

$$\text{Similarly, } V_B = \frac{10.9}{10.9+19.6} \times (100v) = \frac{10.9}{30.5} \times (100v) = 35.73 \text{ volts}$$

$$\text{Therefore, } V_{Th} = V_{CD} = 42.27 - 35.73 = 6.53 \text{ volts}$$

Now, apply the second step of Thevenin's Theorem, to find R_{Th} . For finding of R_{Th} , short the points C & D together, that is replace the voltage generator by its internal resistance (considered here as a short) and measure the resistance between points A and B. This is illustrated in Fig.



$$R_{Th} = (5.2 \parallel 7.1) + (10.9 \parallel 19.6)$$

$$= \frac{5.2 \times 7.1}{5.2 + 7.1} + \frac{10.9 \times 19.6}{10.9 + 19.6} = \frac{36.92}{12.3} + \frac{213.64}{30.5} = 10\Omega$$

The Maximum Power is

$$P_{R(Max)} = I_R^2 \cdot R = \frac{V_s^2}{4R^2} (R) = \frac{V_s^2}{4R} = \frac{(6.53)^2}{4 \times 10} = 1.066 \text{ W}$$

Q.6a. Test the following polynomial for the Hurwitz property.

$$P(s) = s^4 + s^3 + 2s^2 + 3s + 2$$

(i) The even part $e(s)$ and odd part $o(s)$ of the given function is

$$e(s) = s^4 + 2s^2 + 2$$

$$\text{And } o(s) = s^3 + 3s$$

Continued fraction expansion. $F_1(s) = \frac{e(s)}{o(s)}$ can be obtained by dividing $e(s)$ by $o(s)$ and

then investing and dividing again as follows:-

$$\begin{array}{l} s^3 + 3s \overline{) s^4 + 2s^2 + 2} \left\{ S \right. \\ \underline{s^4 + 3s^2} \\ -s^2 + 2 \left\{ -S \right. \\ \underline{-s^2 + 3s} \\ -s^2 + 2 \left\{ -\frac{S}{5} \right. \\ \underline{-s^2 + 2} \\ \left\{ \frac{5S}{2} \right. \\ \left\{ \frac{5S}{2} \right. \\ \left\{ \frac{5S}{2} \right. \end{array}$$

Hence continued expansion $F_1(s)$ is

$$F_1(s) = \frac{e(s)}{o(s)} = S + \frac{1}{-S + \frac{1}{-\frac{S}{5} + \frac{1}{\frac{5S}{2}}}}$$

Since two quotient terms -1 and $-\frac{1}{5}$ out of the total quotient terms $1, -1, -\frac{1}{5}$ and $-\frac{5}{2}$ are negative. Therefore, $F_1(s)$ is not Hurwitz.

$s^3 + 5s^2 + 9s + 3$ is Positive real function. (8)

b. Determine if the function $F(s) = \frac{s^3 + 5s^2 + 9s + 3}{s^3 + 4s^2 + 7s + 9}$ is Positive real function.

b. Determine if the function $F(s) = \frac{s^3 + 5s^2 + 9s + 3}{s^3 + 4s^2 + 7s + 9}$ is Positive real function. (8)

Ans:

The given function is $F(s) = \frac{s^3 + 5s^2 + 9s + 3}{s^3 + 4s^2 + 7s + 9}$

Now let us proceed with the testing of the function $F(s)$ for positive realness:-

- (i) Since all the coefficients in the numerator and denominator are having positive values, hence, for real value of S , $Z(s)$ is real.
- (ii) To find whether the poles are on the left half of the S -plane, let us apply the Hurwitz criterion to the denominator using continued fraction method.

$$\text{Let } P(s) = s^3 + 4s^2 + 7s + 9 = M_2(s) + N_2(s)$$

$$\text{Where } M_2(s) = 4s^2 + 9 \text{ and } N_2(s) = s^3 + 7s.$$

$$\varphi(s) = \frac{N_2(s)}{M_2(s)} = \frac{s^3 + 7s}{4s^2 + 9}$$

$$4s^2 + 9 \overline{) \begin{array}{r} s^3 + 7s \\ \underline{s^3 + 9\frac{s}{4}} \\ \hline \frac{19s}{4} \end{array} \left(\frac{s}{4} \right)}$$

$$\frac{19s}{4} \overline{) \begin{array}{r} 4s^2 + 9 \\ \underline{4s^2} \\ \hline 9 \end{array} \left(\frac{16}{19} s \right)}$$

$$\frac{19s}{4} \overline{) \begin{array}{r} 9 \\ \underline{9} \\ \hline 0 \end{array} \left(\frac{19s}{4 \times 9} \right)}$$

Since all the quotients are positive in the continued fraction expansion, hence, the polynomial of $Z(s)$ in the denominator is Hurwitz.

- (iii) In order to find whether $\operatorname{Re} Z(j\omega) \geq 0$ for all ω , let us adopt slightly more mathematical manipulation.

Let $F(s) = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$ where

$$M_1(s) = 5s^2 + 3; \quad N_1(s) = s^3 + 9s;$$

$$M_2(s) = 4s^2 + 9 \text{ and } N_2(s) = s^3 + 7s$$

$$F(s) = \frac{M_1 + N_1}{M_2 + N_2} \cdot \frac{M_2 - N_2}{M_2 - N_2}$$

Rationalising,

$$= \frac{M_1 M_2 - N_1 N_2}{M_2^2 - N_2^2} + \frac{N_1 M_2 - M_1 N_2}{M_2^2 - N_2^2}$$

Here, even part of $F(s)$ is $\frac{M_1 M_2 - N_1 N_2}{M_2^2 - N_2^2}$

$$\therefore \operatorname{Real} F(j\omega) = \frac{M_1 M_2 - N_1 N_2}{M_2^2 - N_2^2} \Big|_{s=j\omega} = \frac{D(s)}{M_2^2 - N_2^2}$$

Since, $(M_2^2 - N_2^2)$ is always positive for $s = j\omega$, $F(j\omega) \geq 0$ provided $D(j\omega) \geq 0$ for any ω .

In this problem,

$$D(s) = M_1 M_2 - N_1 N_2 = (5s^2 + 3)(4s^2 + 9) - (s^3 + 9s)(s^3 + 7s)$$

$$\text{Or } D(s) = 20s^4 + 45s^2 + 12s^2 + 27 - s^6 - 7s^4 - 9s^4 - 63s^2$$

$$= -s^6 + 4s^4 - 6s^2 + 27$$

$$\text{Therefore, } D(j\omega) = -(j\omega)^6 + 4(j\omega)^4 - 6(j\omega)^2 + 27$$

$$\text{Or } D(j\omega) = \omega^6 + 4\omega^4 + 6\omega^2 + 27$$

i.e., $D(j\omega) > 0$ for any value of ω . Thus, $Z(j\omega) \geq 0$ for any value of ω .

The above three (i), (ii), & (iii) tests certify that the given function $F(s)$ is a PR function.

Q.7 a. Following short circuit currents and voltages are obtained experimentally for a two port network. Determine Y parameters.

(i) With output short circuited $I_1=5\text{mA}$, $I_2 = -0.3\text{mA}$, $V_1=25\text{V}$

(ii) With input short circuited $I_1 = -5\text{mA}$, $I_2 = -10\text{mA}$, $V_2=30\text{V}$

Ans:

The given short circuit currents and voltages, when the output is short-circuited i.e. when $V_2 = 0$ are

$$I_1 = 5\text{mA}$$

$$; I_2 = -0.3\text{mA} \text{ and } V_1 = 25\text{V} \Big|_{\text{when } V_2=0}$$

And the given short circuit currents and voltages, when the input is short-circuited i.e., when $V_1 = 0$ are

$$I_1 = -5\text{mA}; I_2 = 10\text{mA} \text{ and } V_2 = 30\text{V} \Big|_{\text{when } V_1=0}$$

Therefore, the Y-parameter equations are

$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2 \text{ and}$$

$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2$$

$$\text{Hence } Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} = \frac{5\text{mA}}{25\text{volts}} = 0.2 \times 10^{-3} \text{ mho}$$

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = \frac{-0.3\text{mA}}{25\text{volts}} = -0.012 \times 10^{-3} \text{ mho}$$

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{10\text{mA}}{30\text{volts}} = 0.333 \times 10^{-3} \text{ mho}$$

$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = \frac{-5\text{mA}}{30\text{volts}} = -0.166 \times 10^{-3} \text{ mho}$$

Therefore the Y-parameters are

$$Y_{11} = 0.2 \times 10^{-3} \text{ mho}; Y_{22} = 0.333 \times 10^{-3} \text{ mho} \text{ and}$$

$$Y_{21} = -0.012 \times 10^{-3} \text{ mho}; Y_{12} = -0.166 \times 10^{-3} \text{ mho}$$

b. Derive the Relationship between Z and Y parameter.

Ans:

The Z-parameters of a two-port network are given by

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{----- (1) and}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{----- (2)}$$

Where Z_{11} , Z_{12} , Z_{21} , Z_{22} are called Z-Parameters or impedance (z) parameters.

These parameters can be represented by matrix form as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{----- (3)}$$

From equation (3), the current I_1 and I_2 are

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\Delta_Z} \quad \text{----- (4)}$$

$$I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\Delta_Z} \quad \text{----- (5)}$$

Where Δ_Z is the determinant of Z matrix given by

$$\Delta_Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} \quad \text{----- (6)}$$

From equations (4) and (5), we can write

$$I_1 = \frac{Z_{22}V_1 - Z_{12}V_2}{\Delta_Z} \quad \text{----- (7)}$$

$$I_2 = \frac{-Z_{21}V_1 + Z_{11}V_2}{\Delta_Z} \quad \text{----- (8)}$$

The Y-parameters of a two-port network are given as

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{----- (9)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{----- (10)}$$

Comparing equation (7) with equation (9), we have

$$Y_{11} = \frac{Z_{22}}{\Delta_Z}; \quad Y_{12} = \frac{-Z_{12}}{\Delta_Z} \quad \text{and}$$

Comparing equation (8) with equation (10), we have

$$Y_{21} = \frac{-Z_{21}}{\Delta_Z} \quad \text{and} \quad Y_{22} = \frac{Z_{11}}{\Delta_Z}$$

Q.8 a. Consider the system given by system function $Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)}$. Design a RC network.

The given impedance function given by

$$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)}$$

Design of R-C N/W :-

The partial fraction expansion of $Z(s)$ will yield negative residues at poles $s = -2$ and $s = -6$.

Therefore, $Z(s)$ have to expand as $\frac{Z(s)}{s}$ and latter multiply by s .

Hence,

$$\frac{Z(s)}{s} = \frac{a_1}{s} + \frac{a_2}{(s+2)} + \frac{a_3}{(s+6)}$$

Where,

$$a_1 = \left. \frac{2(s+1)(s+3)}{(s+2)(s+6)} \right|_{s=0} = \frac{1}{2}$$

$$a_2 = \left. \frac{2(s+1)(s+3)}{s(s+6)} \right|_{s=-2} = \frac{1}{4} \quad \text{and}$$

$$a_3 = \left. \frac{2(s+1)(s+3)}{s(s+6)} \right|_{s=-6} = \frac{5}{4}$$

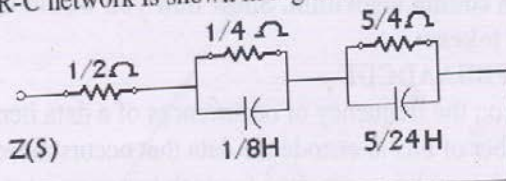
$$\text{Therefore, } \frac{Z(s)}{s} = \frac{1}{2s} + \frac{1}{4(s+2)} + \frac{5}{4(s+6)}$$

Now, it is clear that none of the residues are negative.

Multiplying both the sides by s , we have

$$\begin{aligned} Z(s) &= \frac{1}{2} + \frac{s}{4(s+2)} + \frac{5s}{4(s+6)} \\ &= \frac{1}{2} + \frac{1}{\frac{1}{4} + \frac{1}{s}} + \frac{1}{\frac{5}{4} + \frac{1}{s/24}} \end{aligned}$$

The resulting R-C network is shown in Fig.9.1.



b. Design a one-port RL network to realize the driving point admittance function $F(s) = \frac{3(s+2)(s+4)}{s(s+3)}$

$$F(s) = \frac{3(s+2)(s+4)}{s(s+3)} = \frac{3(s^2 + 4s + 2s + 8)}{s^2 + 3s}$$

$$\text{Or } F(s) = \frac{3(s^2 + 6s + 8)}{s^2 + 3s} = \frac{3s^2 + 18s + 24}{s^2 + 3s}$$

Therefore, the driving point function $F(s)$ is realized by the continues fraction expansion
 i.e., (CAUER Form - I)

$$\begin{array}{l}
 s^2 + 3s \left\{ \frac{3s^2 + 18s + 24}{3s^2 + 9s} \right\} 3 \leftarrow Y_1(s) = \frac{1}{3} \Omega \\
 \frac{9s + 24}{s^2 + 3s} \left\{ \frac{1}{9} s \leftarrow Z_2(s) = \frac{1}{9} H \right. \\
 \left. \frac{s^2 + \frac{8}{3}s}{\frac{1}{3}s} \right\} \\
 \frac{1}{3}s \left\{ \frac{9s + 24}{9s} \right\} 27 \leftarrow Y_3(s) = \frac{1}{27} \Omega \\
 \frac{24}{\frac{1}{3}s} \left\{ \frac{s}{72} \leftarrow Z_4(s) = \frac{1}{72} H \right. \\
 \left. \frac{1}{3}s \right\} \\
 \times
 \end{array}$$

The synthesised RL network

Q.9 a. Synthesise the network that has a transfer impedance $Z_{21}(s) = \frac{2}{s^3 + 3s^2 + 4s + 2}$
 and 1Ω termination at the output.

Ans:

$$\text{Given that } Z_{21}(s) = \frac{2}{s^3 + 3s^2 + 4s + 2}$$

$$Z_{21}(s) = \frac{P(s)}{Q(s)} = \frac{2}{s^3 + 3s^2 + 4s + 2}$$

Here, all three zeros of transmission at $s = \infty$. Since the numerator $P(s)$ is a constant 2 & $(3s^2 + 2)$ with the odd part of the denominator i.e. $s^3 + 4s$ as

$$Z_{21} = \frac{2}{s^3 + 4s} \quad \text{and}$$

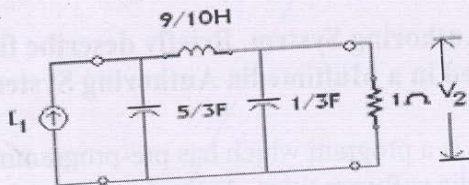
$$Z_{22} = \frac{3s^2 + 2}{s^3 + 4s}$$

b. Show that the filter described by the transfer function $H(s) = \frac{1}{(s^2 + 0.73536s + 1)(s^2 + 1.84776s + 1)}$ is a low pass filter.

Therefore Z_{21} and Z_{22} have the same poles. Synthesize Z_{22} , so that the network has the transmission zeros of Z_{21} . Synthesize Z_{22} to give LC network structure by the following continued fraction expansion of $\frac{1}{Z_{22}}$.

$$\begin{aligned} & \frac{3s^2+2}{s^2+\frac{2}{3}s} \left(\frac{1}{3}s \leftarrow Y \right) \\ & \frac{\frac{10}{3}s}{\frac{3s^2+2}{3s^2}} \left(\frac{9}{10}s \leftarrow Z \right) \\ & \frac{\frac{10}{3}s}{0} \left(\frac{5}{3}s \leftarrow Y \right) \end{aligned}$$

Since Z_{22} is synthesized from the 1-Ω termination toward the input end, the final network takes the form shown in Fig.



b. Show that the filter described by the transfer function $H(s) = \frac{1}{(s^2 + 0.73536s + 1)(s^2 + 1.84776s + 1)}$ is a low pass filter. (6)

Ans:

The given transfer function $H(s)$ is

$$H(s) = \frac{1}{(s^2 + 0.76536s + 1)(s^2 + 1.84776s + 1)}$$

In low-pass filter design, all the zeros of the system function are at infinity. Therefore, in the given transfer function $H(s)$, the zeros in the numerator are at infinity. Hence the given transfer function $H(s)$ is a Low Pass filter.

Text books

1. Network Analysis, M.E.Van Valkenberg, 3rd Edition, Prentice-Hall India, EEE 2006
2. Network Analysis and Synthesis, Franklin F Kuo, 2nd Edition, Wiley India Student Edition 2006