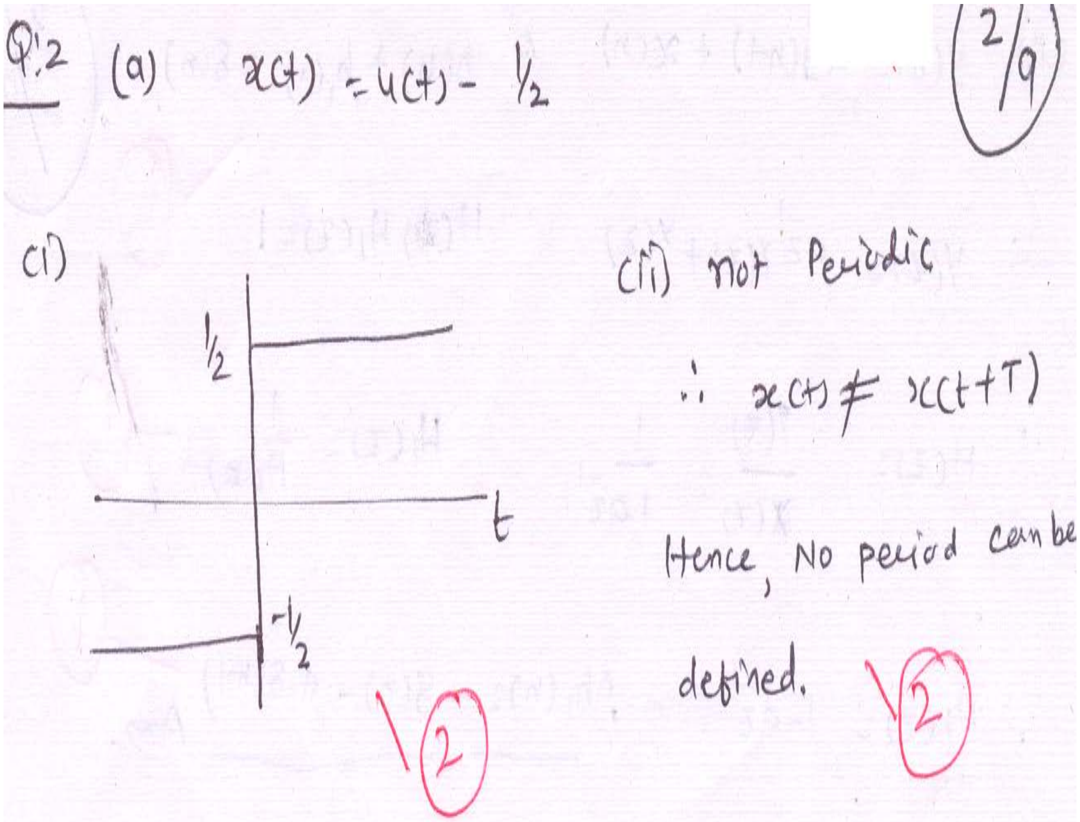
Q.2 a. If $x(t) = u(t) - {}^{1}/_{2}$; (i) Sketch x(t)
(ii) Determine analytically the signal is periodic or not if periodic, state the period



b. Do as directed: (i) Plot x(n) and also plot y[n] = x[n+1] where x[n] is defined as below:

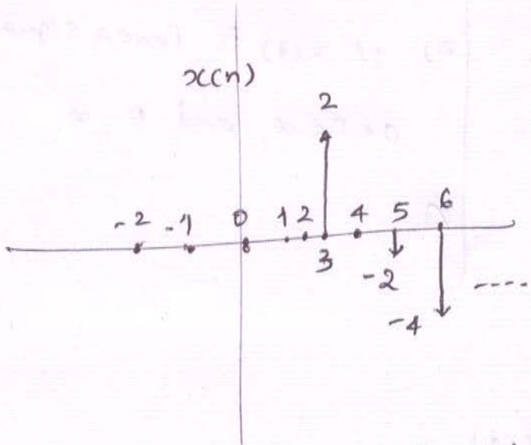
$$x[n] = \begin{cases} 0 & \text{if} \quad n < 2 \\ 2n - 4 & \text{if} \quad 2 \le n \le 4 \\ 4 - n & \text{if} \quad 4 \le n \end{cases}$$

(ii) Determine convolution of $x(t) = e^{2t}u(-t)$ and $h(t) = u(t-3)$ and plot resultant y(t)

(iii) Let h(n) be the impulse response of the LTI causal system described by the difference equation $y(n) = a\,y(n-1) + x(n)$ and let $h(n) * h_1(n) = \delta(n)$. Find $h_1(n)$.

(b)

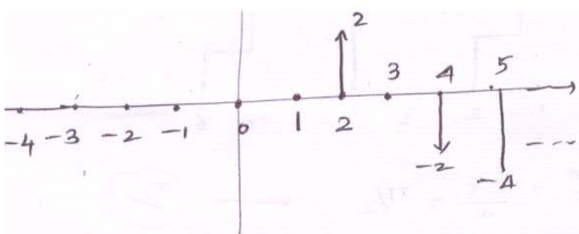
(i) $x(n) = \begin{cases} 0 & n < 2 \\ 2n-4 & 2 \le n < 4 \\ 4-n & 4 \le n \end{cases}$ — (1)



$y(n) = x(n+1)$ i.e. shift left by 1

— (1)

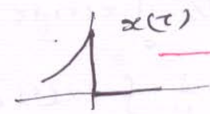
$x(n+1) = y(n)$.



— (1)

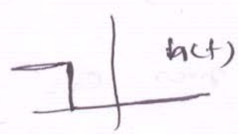
(ii) convolution $x(t) = e^{2t}u(-t)$ & $h(t) = u(t-3)$

$\therefore y(t) = \int_{-\infty}^{t-3} e^{2\tau} d\tau = \frac{1}{2} e^{2(\tau-3)}$

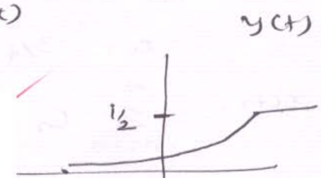


— (1)

$\therefore y(t) = \int_{-\infty}^{0} e^{2\tau} d\tau = \frac{1}{2}.$



($\because$ For $t-3 \geq 0$, product $x(\tau).h(t-\tau)$ is non zero for $-\infty < \tau < 0$)



(iii) $y(n) = a y(n-1) + x(n)$ & $h(n) * h_1(n) = \delta(n)$

3/9

(1)

$\therefore Y(z) = a z^{-1} Y(z) + X(z)$ $\quad H(z) H_1(z) = 1$

$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - a z^{-1}}$ $\quad H_1(z) = \frac{1}{H(z)}$ (1)

$\therefore H_1(z) = 1 - a z^{-1}$ $\quad ; h_1(n) = \delta(n) - a.\delta(n-1)$ Ans. (1)

(C)

Energy Signal	Power Signal
1) Which has finite energy & zero avg. power.	1) which has finite avg. power & infinite energy. ($1\frac{1}{2}$)
2) if $x(t)$ is energy signal $0 < E < \infty$, and $P = 0$.	2) if $x(t)$ is Power signal $0 < P < \infty$, and $E = \infty$. ($1\frac{1}{2}$)
(3)	(3)

Q.3 a. Compute the Fourier series for the following signal shown in Fig.2.

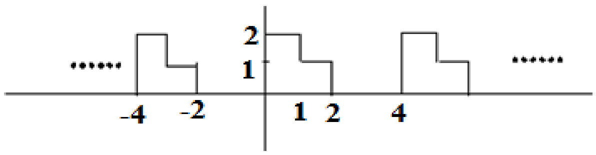
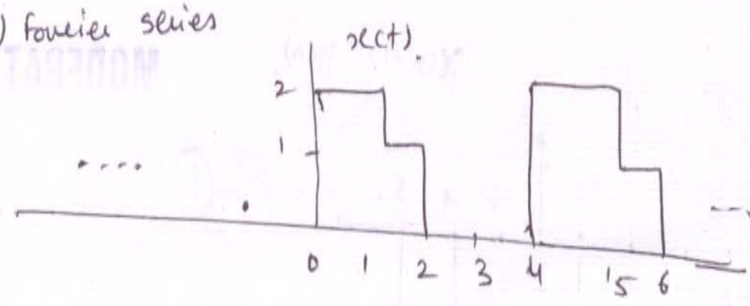


Fig.2

Q.3 (a) Fourier series



Hence $T = 4$, so, $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \pi/2$ — (2)

$$C_0 = 1/4 \int_0^2 x(t)\,d\tau = 3/4 \quad \text{— (1)}$$

$$C_n = \frac{1}{4} \int_0^2 x(t)\, e^{-jn\pi/2}\, dt = \frac{1}{4} \int_0^1 2 e^{-jn\pi/2 t}\, dt + \int_1^2 e^{-jn\frac{\pi}{2}t}\, dt. \quad \text{— (2)}$$

This gives $C_n = \frac{1}{jn\pi}\left[1 - (1/2)(-1)^n - 1/2\, e^{-jn\pi/2}\right] \left(\because e^{-jn\pi} = (-1)^n\right)$ — (2)

$C_0 = 3/4$.

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} C_n e^{-jn\frac{\pi}{2}t} \quad \text{— (1)}$$

Ans.

— 4

b. Consider an LTI system with impulse response h(n) = α^n u(n) , -1 < α < 1; and with the input x(n) = cos (2πn/N). Determine y(n).

Q.3 (b) $x(n) = \cos(2\pi n/N)$ & $h(n) = \alpha^n u(n)$.

$\therefore x(n) = \frac{1}{2} e^{j(2\pi/N)n} + \frac{1}{2} e^{-j(2\pi/N)n}$ (1)

Also,

$$H(e^{j\omega}) = \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (\alpha e^{-j\omega})^n$$

$$\therefore H(e^{j\omega}) = \frac{1}{1-\alpha e^{-j\omega}}$$ (2)

Hence $y(n) = x(n) * h(n)$

If $\frac{1}{1-\alpha e^{-j2\pi/N}} = re^{j\theta}$

$\therefore y(n) = r\cos\left(\frac{2\pi}{N}n + \theta\right)$

For N = 4 (2)

$$\frac{1}{1-\alpha e^{-j\frac{2\pi}{4}}} = \frac{1}{1+\alpha j}$$

$$= \frac{1}{\sqrt{1+\alpha^2}} e^{j(-\tan^{-1}\alpha)}$$

$$= \frac{1}{2} H(e^{j\frac{2\pi}{N}}) \cdot e^{\left(\frac{j2\pi n}{N}\right)} + \frac{1}{2} H(e^{-j\frac{2\pi}{N}}) e^{-j\left(\frac{2\pi n}{N}\right)}$$

$$= \frac{1}{2}\left(\frac{1}{1-\alpha e^{-j2\pi/N}}\right) e^{j\left(\frac{2\pi}{N}\right)n} + \frac{1}{2}\left(\frac{1}{1-\alpha e^{+j2\pi/N}}\right) e^{-j\left(\frac{2\pi}{N}\right)n}$$ — (2)

using this,

$$y(n) = \frac{1}{\sqrt{1+\alpha^2}} \cdot \cos\left(\frac{\pi n}{2} - \tan^{-1}(\alpha)\right)$$ — (1) -- Ans.

Q.4a. State and explain convergence conditions for continuous–time Fourier transform.

Q:4 (a). Topic 4.1.2. Text Book (I)

b. Consider a stable LTI system characterized by the differential equation:
dy(t) /dt + 5 y(t) = x(t). Determine the
(i) frequency response and

(ii) impulse response for the system.

(b) $\frac{dy(t)}{dt} + 5y(t) = x(t)$. $2\frac{1}{2}$

$\therefore H(e^{j\omega}) = \frac{1}{5+j\omega}$ & $h(t) = e^{-5t}u(t)$ — $2\frac{1}{2}$

c. State and prove following properties for continuous time Fourier transforms:
(i) Conjugation
(ii) Time shifting
(iii) Differentiation
(iv) Duality

(c) Topic. 4.3 Text Book (I)

4.3.3; 4.3.2, 4.3.4. 4.3.6.

Q.5 a. Consider a stable Causal LTI system whose input x(n) and output y(n) are related through second order difference equation
$y(n) - (3/4)\, y(n-1) + (1/8)\, y(n-2) = 2\, x(n)$; determine the response for the given input $x(n) = (1/4)^n u(n)$

Q:5 (a) given $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 2x(n)$ & (5/9)

$x(n) = (1/4)^n u(n)$

$$Y(e^{j\omega}) = H(e^{j\omega}).X(e^{j\omega}) = \left[\frac{2}{(1-\frac{1}{2}e^{j\omega})(1-\frac{1}{4}e^{j\omega})}\right]\left[\frac{1}{1-\frac{1}{4}e^{-j\omega}}\right]$$

$$= \frac{2}{(1-\frac{1}{2}e^{j\omega})(1-\frac{1}{4}e^{-j\omega})^2} \quad — (2)$$

$$\therefore Y(e^{j\omega}) = \frac{B_1}{(1-\frac{1}{4}e^{j\omega})} + \frac{B_2}{(1-\frac{1}{4}e^{j\omega})^2} + \frac{B_3}{(1-\frac{1}{2}e^{-j\omega})} \quad — (2)$$

on solving

$B_1 = -4, \quad B_2 = -2, \quad B_3 = 8$ — (2)

Substituting & solving.

$$y(n) = \left\{-4(1/4)^n - 2(n+1)(1/4)^n + 8(1/2)^n\right\} u(n) \quad — (2)$$

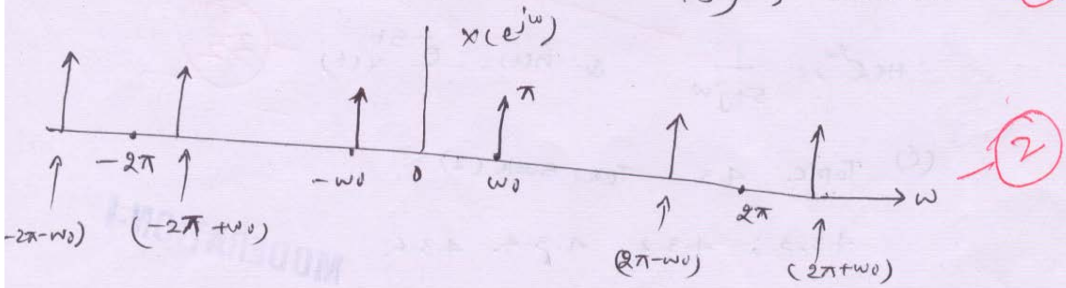
Ans.

b.For signal x(n) = cosw$_o$ n with w$_o$ = 2π /5, obtain and plot X (e^{jw}).

(b) $\cos\omega_0 n = x(n) = \frac{1}{2}e^{j\omega_0 n} + \frac{1}{2}e^{-j\omega_0 n}$ with $\omega_0 = 2\pi/5$

$$\therefore X(e^{j\omega}) = \sum_{l=-\infty}^{\infty} \pi\delta\left(\omega - \frac{2\pi}{5} - 2\pi l\right) + \sum_{l=-\infty}^{\infty} \pi\delta\left(\omega + \frac{2\pi}{5} - 2\pi l\right) \quad — (1)$$

$$X(e^{j\omega}) = \pi\delta\left(\omega - \frac{2\pi}{5}\right) + \pi\delta\left(\omega + 2\pi/5\right); \quad -\pi < \omega < \pi \quad — (1)$$



— (2)

c. State and prove following properties for discrete time Fourier transforms:
(i) Time shifting
(ii) Frequency shifting

(c) Topic. 5.3.3 Text Book (I)

– 6 –

Q.6 a. Explain the concept of :
(i) Non-linear phase
(ii) Group delay
(iii) Continuous-time ideal low pass filter
(iv) First order continuous time system

Q:6 (a) Topic 6.2. Text Book (I)

b. Define sampling, aliasing and Nyquist interval. For the following signal x (t), calculate Nyquist rate.

$x(t) = 6\cos 50\pi t + 20 \sin 300\pi t - 10\cos 100\pi t$.

(b)

Sampling :- The process of converting an analog signal into a discrete signal or making an analog (continuous) signal to occur at a particular interval of time is known as sampling. — 1

Aliasing :- When a continuous band limited signal is sampled at a rate lower than Nyquist rate $f_s < 2f_m$, then successive cycles of the spectrum of sampled signal g(t) overlap with each other is called aliasing. — 1

Nyquist Interval:- Max. sampling interval is called Ny. int.

$T_s = \frac{1}{2f_m}$ sec. —— (1)

$x(t) = 6\cos 50\pi t + 20 \sin 300\pi t - 10\cos 100\pi t.$

$f_1 = \frac{\omega_1}{2\pi} = \frac{50\pi}{2\pi} = 25$ Hz —— (1)

$f_2 = \frac{\omega_2}{2\pi} = \frac{300}{2\pi} = 150$ Hz —— (1)

$f_3 = \frac{\omega_3}{2\pi} = \frac{100}{2\pi} = 50$ Hz. —— (1)

∴ Highest freq. component of the given message signal will be $f_{max} = 150$ Hz. —— (1)

∴ Nyquist rate $= 2f_{max} = 300$ Hz Ans. —— (2)

Q.7a. Discuss the all-pass system using Laplace transform with necessary diagrams.

(a) All Pass-system: Topic. 9.4.3 Text Book (I).

b. Obtain the Laplace transform of:

(i) $x(t) = e^{-at}u(t)$ (ii) $x(t) = -e^{-at}u(-t)$

(b) (i) $x(t) = e^{-at} u(t)$.

$$X(s) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{st} dt$$

$$= \int_0^{\infty} e^{-at} e^{st} dt$$

$$= \int_0^{\infty} e^{(-s+a)t} dt$$ — (1)

with $s = \sigma + j\omega$,

$$X(\sigma + j\omega) = \int_0^{\infty} e^{-(\sigma+a)t} e^{-j\omega t} dt.$$

$$= \frac{1}{(\sigma+a)+j\omega} \quad \sigma + a > 0$$ — (1)

$$\therefore X(s) = \frac{1}{s+a} \quad Re\{s\} > -a.$$

$$\therefore e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a} \quad Re\{s\} > -a.$$ — (1)

Ans

(ii) $x(t) = -e^{-at} u(-t)$.

$$\therefore X(s) = \int_{-\infty}^{\infty} e^{-at} e^{-st} u(-t) dt$$ — (1)

$$= -\int_0^{\infty} e^{-(s+a)t} dt.$$

$$\therefore X(s) = \frac{1}{s+a}$$ for convergence $Re\{s+a\} < 0$. — (1)

$\therefore Re\{s\} < -a$.

$$\therefore e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a} \quad Re\{s\} < -a.$$ — (1)

A.

c. State initial value and final value theorems for Laplace Transform. Also state its usefulness.

( )

Initial Value Th. Final Value Th.

$x(0^+) = \lim_{s\to\infty} sX(s)$ ($1\frac{1}{2}$)

$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$. ($1\frac{1}{2}$)

Used:- To check correctness of the Laplace transform calculation for a signal. (1)

Q.8 a. Let the z- transform of x(n) be X(z). Show that the z-transform of x (-n) is X(1/z)

Q:8 (a) $x(n) \overset{z}{\longleftrightarrow} X(z)$

Let

$y(n) = x(-n)$.

$\therefore Y(n) = \sum_{n=-\infty}^{\infty} x(-n) z^{-n} = \sum_{r=-\infty}^{\infty} x(r) z^{+r}$ — (2)

$= \sum_{r=-\infty}^{\infty} x(r) \cdot (z^{-1})^{-1} = X(z^{-1}) = X(1/z)$. — (2)

b. Determine the region of convergence of the z-transform of the signal $x(n) = 2^n u(n) - 3^n u(-n-1)$

(b) $x(n) = 2^n u(n) - 3^n u(-n-1)$

$2^n u(n) \longleftrightarrow \frac{1}{1-2z^{-1}}$; $|z| > 2$ ($1\frac{1}{2}$)

$3^n u(-n-1) \longleftrightarrow \frac{1}{1-3z^{-1}}$, $|z| < 3$ ($1\frac{1}{2}$)

$\therefore$ ROC is $2 < |z| < 3$ (1)

c. Find Inverse Z-Transform of following:

(i) $X(z) = 1/(1 - az^{-1})$, $|z| > |a|$

(ii) $X(z) = \log(1 + az^{-1})$, $|z| > |a|$

(c)

(i)

$X(z) = \frac{1}{1-az^{-1}}$ $|z| > |a|$.

Power series by long division. (1)

$$1-az^{-1} \overline{)\,1\,} = 1 + az^{-1} + a^2z^{-2} + \cdots$$

$1 - az^{-1}$

az^{-1}

$az^{-1} - a^2z^{-2}$

a^2z^{-2} ... (2)

$\therefore \frac{1}{1-az^{-1}} = 1 + az^{-1} + a^2z^{-2}$ (1)

(ii) $X(z) = \log(1 + az^{-1})$ $|z| > |a|$

$$\log(1+v) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} v^n \quad |v| < 1$$ (1)

$$\therefore X(z) = \sum_{n=1}^{\infty} \frac{(-1)^n a^n z^{-n}}{n}$$ (2)

$$\therefore x(n) = \begin{cases} (-1)^{n+1} \frac{a^n}{n} & n \geq 1 \\ 0 & n \leq 0. \end{cases}$$ (1)

Q.9 a. Explain the following random processes:

(i) ergodic

(ii) non-ergodic

(iii) stationary

(iv) non-stationary

Q:-9 (a) Text book (II).

b. A random variable V = b + x ; where x is a Gaussian distributed random variable with mean 0 and variance σ^2 with ‘ b’ a constant. Show that V is a Gaussian distributed random variable with mean b and variance σ^2.

(b) R.V. $V = b + X$ $b =$ const

$X =$ Gaussian dist. R.V.

By writing Gaussian dist^n fu^n for X.

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-m)^2/2\sigma^2} \quad (A)$$

here $m = 0$.

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2} \quad (B).$$

Given $V = b + X \Rightarrow X = V - b$. (1)

$$f_V(v) = \frac{1}{2\pi\sigma^2} e^{-(v-b)^2/2\sigma^2} \quad (C)$$

By comparing (A) & (C),

$V - b = x - m$ ∴ **mean = b**

To find Variance σ^2

$$\sigma_V^2 = \int_{-\infty}^{\infty} (v-b)^2 f_V(v)\,dv \quad (2)$$

$$= \int_{-\infty}^{\infty} (v-b)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(v-b)^2}{2\sigma^2}}\,dv$$

Let $\frac{v-b}{\sigma\sqrt{2}} = P \Rightarrow dv = \sigma\sqrt{2}\,dP$.

$v \to -\infty$ $P \to -\infty$

$v \to +\infty$ $P \to +\infty$ (1)

$(v-b)^2 = 2P^2\sigma^2$

Hence $$\sigma_V^2 = \int_{-\infty}^{\infty} 2\sigma^2 P^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-P^2} \sigma\sqrt{2}\,dP$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} P^2 e^{-P^2}\,dP.$$

$$\sigma_V^2 = \frac{4\sigma^2}{\sqrt{\pi}} \int_0^{\infty} P^2 e^{-P^2}\,dP \quad (2)$$

comparing with standard formula

$$\int_0^{\infty} x^{2n} e^{-ax^2}\,dx = \frac{(2n-1)}{2^{n+1} a^n} \sqrt{\pi/a}.$$

$$\therefore \sigma_V^2 = \frac{4\sigma^2}{\sqrt{\pi}} \times \frac{1}{2^{1+1}} \cdot \frac{\sqrt{\pi}}{1}$$

$$= \frac{4\sigma^2}{\sqrt{\pi}} \times \frac{1}{4} \times \sqrt{\pi}$$

$$\boxed{\sigma_V^2 = \sigma^2}$$ Ans (2)

Textbooks

1.Signals and Systems, A.V. Oppenheim and A.S. Willsky with S. H. Nawab, Second Edition, PHI Private limited, 2006

2. Communication Systems, Simon Haykin, 4th Edition, Wiley Student Edition, 7th Reprint 2007