Q.2a. Show that the function $f(z)=\sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied thereof.

Q.2a. Soln: If $f(z) = \sqrt{|xy|} = u(x,y) + i\,v(x,y)$, then

$u(x,y) = \sqrt{|xy|}$, $v(x,y) = 0$

At the origin, we have,

$$\frac{\partial u}{\partial x} = \underset{x\to 0}{Lt}\ \frac{u(x,0)-u(0,0)}{x} = \underset{x\to 0}{Lt}\ \frac{0-0}{x} = 0 \quad (1)$$

$$\frac{\partial u}{\partial y} = \underset{y\to 0}{Lt}\ \frac{u(0,y)-u(0,0)}{y} = \underset{x\to 0}{Lt}\ \frac{0-0}{y} = 0 \quad (1)$$

$$\frac{\partial v}{\partial x} = \underset{x\to 0}{Lt}\ \frac{v(x,0)-v(0,0)}{x} = \underset{x\to 0}{Lt}\ \frac{0-0}{x} = 0 \quad (1)$$

$$\frac{\partial v}{\partial y} = \underset{y\to 0}{Lt}\ \frac{v(0,y)-v(0,0)}{y} = \underset{y\to 0}{Lt}\ \frac{0-0}{y} = 0 \quad (1)$$

$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ (1)

ie. C.R. equations are satisfied at the origin.

However $f'(0) = \underset{z\to 0}{Lt}\ \frac{f(z)-f(0)}{z-0}$ (1) $= \underset{z\to 0}{Lt}\ \frac{\sqrt{|xy|}-0}{x+iy}$

$$= \underset{x\to 0}{Lt}\ \frac{\sqrt{|mx^2|}-0}{x(1+im)}, \text{ when } z\to 0 \text{ along the line } y=mx$$

$= \frac{\sqrt{|m|}}{1+im}$ (1) which is not unique.

$\therefore f'(0)$ does not exist. Hence $f(z)$ is not analytic at the origin. (1)

b.Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i$ and $z_3 = -2$ into the points $w_1 = 1, w_2 = i$ and $w_3 = -1$.

Q.2. b. soln: We have the transformation is $w = \dfrac{az+b}{cz+d}$ — (i)

$2a + b - 2c - d = 0$ (1) — (ii)

$ai + b + c - di = 0$ (1) — (iii)

$2a - b + 2c - d = 0$ (1) — (iv)

z	w
2	1
i	i
-2	-1

(1)

Adding (ii) and (iv) $2a = d$ — (v)

Now (iv) − (ii), $b = 2c$

from (iii), $ai = 3c$,

So, $2a = d$, $b = 2c$, $ai = 3c$

$\Rightarrow a = \dfrac{d}{2} = \dfrac{3c}{i} = -\dfrac{3b}{2i}$ (3)

$\Rightarrow \dfrac{a}{1} = \dfrac{d}{2} = \dfrac{c}{i/3} = \dfrac{b}{2i/3}$

$\therefore w = \dfrac{3z + 2i}{iz + 6}$ (1) Ans.

Q.3a. Find the Taylor's series expansion of a function of the complex variable $f(z) = \dfrac{1}{(z-1)(z-3)}$ about the point, z = 4. Find its region of convergence.

Q.3.a. Soln.

we have, $f(z) = \dfrac{1}{(z-1)(z-3)}$

$\therefore \quad f(4) = \dfrac{1}{3}$

$f(z) = \dfrac{1}{2}\left[\dfrac{1}{z-3} - \dfrac{1}{z-1}\right]$ 2

$\therefore \quad f'(z) = \dfrac{1}{2}\left[-\dfrac{1}{(z-3)^2} + \dfrac{1}{(z-1)^2}\right]$ 1

Putting $z = 4$, $\quad f'(4) = -\dfrac{4}{9}$

$f''(z) = \dfrac{1}{2}\left[\dfrac{-2}{(z-3)^2} - \dfrac{1}{(z-1)^2}\right]$ 1

Putting $z=4$, $\quad f''(4) = \dfrac{26}{27}$

Similarly, $\quad f'''(4) = -\dfrac{80}{27}$

we have Taylor's series is,

$$f(z) = f(a) + (z-a)f'(a) + \frac{(z-a)^2}{|\underline{2}}f''(a) + \cdots \quad 1$$

$$\frac{1}{(z-1)(z-3)} = \frac{1}{3} + (z-4)\left(-\frac{4}{9}\right) + \frac{(z-4)^2}{|\underline{2}}\left(\frac{26}{27}\right) + \frac{(z-4)^3}{|\underline{3}}\left(-\frac{80}{27}\right)\cdots$$

$$= \frac{1}{3} - \frac{4}{9}(z-4) + (z-4)^2\frac{13}{27} - (z-4)^3\frac{40}{81} + \cdots \quad 2$$

Region of convergence is $|z-4| < 1$ 1

Aliter
Soln to Q.3(a)

$$f(z) = \frac{1}{(z-1)(z-3)}$$

$$= \frac{1}{2}\left[\frac{(z-1)-(z-3)}{(z-1)(z-3)}\right]$$

$$f(z) = \frac{1}{2}\left[\frac{1}{z-3} - \frac{1}{z-1}\right] \quad (2)$$

For expansion about $z=4$, put $z-4=u$

$\Rightarrow z = 4+u$

$$\Rightarrow f(z) = \frac{1}{2}\left[\frac{1}{u+1} - \frac{1}{u+3}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(1+u)} - \frac{1}{3\left(1+\frac{u}{3}\right)}\right]$$

$$= \frac{1}{2}\left[(1+u)^{-1} - \frac{1}{3}\left(1+\frac{u}{3}\right)^{-1}\right] \quad (1)$$

$$= \frac{1}{2}\left[\sum_{n=0}^{\infty}(-1)^n u^n - \frac{1}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{u}{3}\right)^n\right]$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}(-1)^n(z-4)^n \; (2) \; - \frac{1}{6}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z-4}{3}\right)^n \quad (2)$$

Region of convergence:

Largest circle having centre at $z=4$ and enclosing no singularity of $f(z)$ is $|z-4|=1$

$\therefore$ Region of convergence is $|z-4|<1$ (1)

b. Determine the poles of the function $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ **and find residue of** $f(z)$ **of at each pole.**

Ans

Q.3.b. Soln We have, $f(z) = \dfrac{z^2}{(z-1)^2 (z+2)}$

For the pole putting the denominator $= 0$

$\therefore (z-1)^2 (z+2) = 0 \Rightarrow z = 1, 1, -2$

Here $z = 1$ is a pole of order 2 and $z = -2$ is a simple pole

Now residue at $z = -2$,

$$\lim_{z \to -2} \left[(z+2) f(z)\right] = \lim_{z \to 2} \frac{z^2}{(z-1)^2} = \frac{4}{9}$$

and residue at $z = 1$,

P.T.O.

Q3.b. Soln: Remaining part.

$$\lim_{z \to 1} \frac{d}{dz}\left[\frac{z^2}{z+2}\right] = \lim_{z \to 1}\left[\frac{(z+2)\cdot 2z - z^2}{(z+2)^2}\right]$$

Q.4a. Evaluate grad e^{r^2} where $r^2 = x^2 + y^2 + z^2$.

Not required

Q.4.a. Soln: We have $\text{grad } e^{r^2} = \nabla e^{r^2}$ 1

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) e^{r^2} \quad 1$$

$$= \hat{i}\frac{\partial}{\partial x}(e^{r^2}) + \hat{j}\frac{\partial}{\partial y}(e^{r^2}) + \hat{k}\frac{\partial}{\partial z}(e^{r^2})$$

$$= \hat{i}(e^{r^2})\, 2r\frac{\partial r}{\partial x} + \hat{j}(e^{r^2})\, 2r\frac{\partial r}{\partial y} + \hat{k}(e^{r^2})\, 2r\frac{\partial r}{\partial z} \quad 2$$

$$= 2re^{r^2}\left(\hat{i}\frac{\partial r}{\partial x} + \hat{j}\frac{\partial r}{\partial y} + \hat{k}\frac{\partial r}{\partial z}\right) \longleftarrow (1)$$

But we have $r^2 = x^2 + y^2 + z^2$

$$\Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r} \text{ and } \frac{\partial r}{\partial z} = \frac{z}{r}. \quad 2$$

Putting in (1), we get

$$\text{grad } (e^{r^2}) = 2re^{r^2}\left(\hat{i}\frac{x}{r} + \hat{j}\frac{y}{r} + \hat{k}\frac{z}{r}\right)$$

$$= 2e^{r^2}(x\hat{i} + y\hat{j} + z\hat{k}) \quad 2 \quad = 2e^{r^2}\vec{r} \quad \underline{\text{Ans}}$$

b. **Prove that** $div\left\{\dfrac{f(r)\cdot \vec{r}}{r}\right\} = \dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2 f\right)$

Q4.b. Soln: We have,

$$div\left\{\frac{f(r)\vec{r}}{r}\right\} = div\left\{\frac{f(r)}{r}(x\hat{i} + y\hat{j} + z\hat{k})\right\}$$

$$= \frac{\partial}{\partial x}\left\{\frac{f(r)}{r}x\right\} + \frac{\partial}{\partial y}\left\{\frac{f(r)}{r}y\right\} + \frac{\partial}{\partial z}\left\{\frac{f(r)}{r}z\right\} \quad 1 \quad \cdots (1)$$

Now, $$\frac{\partial}{\partial x}\left\{\frac{f(r)}{r}x\right\} = \frac{f(r)}{r} + x\frac{d}{dr}\left\{\frac{f(r)}{r}\right\}\frac{\partial r}{\partial x} \quad 1$$

$$= \frac{f(r)}{r} + x\left\{\frac{f'(r)}{r} - \frac{1}{r^2}f(r)\right\}\frac{x}{r} \quad 1$$

$$= \frac{f(r)}{r} + \frac{x^2}{r^2}f'(r) - \frac{x^2}{r^3}f(r). \quad 1$$

Similarly $$\frac{\partial}{\partial y}\left\{\frac{f(r)}{r}y\right\} = \frac{f(r)}{r} + \frac{y^2}{r^2}f'(r) - \frac{y^2}{r^3}f(r)$$

and $$\frac{\partial}{\partial z}\left\{\frac{f(r)}{r}z\right\} = \frac{f(r)}{r} + \frac{z^2}{r^2}f'(r) - \frac{z^2}{r^3}f(r) \quad 1$$

Putting these values in (1), we get

$$div\left\{\frac{f(r)\vec{r}}{r}\right\} = \frac{3}{r}f(r) + \frac{r^2}{r^2}f'(r) - \frac{r^2}{r^3}f(r) \quad 1, \quad \text{P.T.O.}$$

$$= \frac{2}{r} f(r) + f'(r) \; (1) = \frac{1}{r^2}\left[2r f(r) + r^2 f'(r)\right]$$

$$= \frac{1}{r^2}\frac{d}{dr}\left[r^2 f(r)\right] \; (1) \; \text{Ans}$$

Q.5 a. Apply Green's theorem to evaluate $\oint_C 2y^2dx + 3xdy$

Where C is the boundary of closed region bounded between y = x and $y = x^2$

Q.5.a. Soln: Statement of Green's theorem (1)

By Green's Theorem.

$$\oint_C 2y^2 dx + 3x\,dy = \iint_S (3-4y)\,dx\,dy \quad (1)$$

$$= \int_0^1 \int_{y=x^2}^{y=x} (3-4y)\,dy\,dx \quad \text{(1) For limits of y} \quad \text{(1) For limits of x}$$

$$= \int_0^1 (3y-2y^2)_{x^2}^{x}\,dx \quad (1)$$

$$= -\int_0^1 \left[3x^2 - 2x^4 + 2x^2 - 3x\right] dx \quad (1)$$

$$= -\int_0^1 \left[5x^2 - 2x^4 - 3x\right] dx$$

$$= -\frac{5}{3} + \frac{2}{5} + \frac{3}{2} = -\frac{50-12-45}{30} = +\frac{7}{30} \quad \text{Ans} \quad (2)$$

b. Apply Stoke's theorem to calculate $\oint_C 4ydx + 2zdy + 6ydz$

where C is the curve of intersection of $x^2 + y^2 + z^2 = 6z$ **and** $z = x + 3$

b. Soln:

$$\oint_C \bar{F}\cdot d\bar{r} = \oint_C (4y\hat{i} + 2z\hat{j} + 6y\hat{k})\cdot(\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz) \quad ①$$

using Stoke's Theorem,

$$\oint_C \bar{F}\cdot d\bar{r} = \iint_S \nabla\times(4y\hat{i} + 2z\hat{j} + 6y\hat{k})\cdot\bar{n}\,ds \quad ①$$

Where S is the region bounded by C. It will be on the plane $x - z = -3$. The unit normal $\hat{n}$ to the surface S is,

$$= \frac{+i-k}{\sqrt{2}} \quad ①$$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4y & 2z & 6y \end{vmatrix} = (6-2)i \cancel{+4k} - 4k \quad ①$$

$$\therefore \iint_S \nabla\times F\cdot\bar{n}\,ds = \iint_S 4(i-k)\cdot\frac{(i-k)}{\sqrt{2}} = \frac{8}{\sqrt{2}}\iint_S ds \quad ①$$

Since $x^2+y^2+(z-3)^2 = 9$ equation of sphere

$z = x+3$ equation of plane.

Intersection will be the circle, with centre $(0,0,3)$ and radius 3

$$\therefore \iint_S ds = 9\pi \quad ②$$

$$\therefore \text{Required integral} = \frac{9\times 8\pi}{\sqrt{2}} = 36\sqrt{2}\,\pi \quad ① \quad \text{Ans}$$

Q.6a. Using Langrange's interpolation formula, find the value of y corresponding to x = 10 from the following table:

x	5	6	9	11
y	12	13	14	16

Q.6.a. Soln: Langrange's interpolation formula

$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} f(x_1) + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} f(x_2)$$

$$+ \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} f(x_3) + \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} f(x_4)$$

Here $x = 10$

$x_1 = 5,\ x_2 = 6,\ x_3 = 9,\ x_4 = 11$

$f(x_1) = 12,\ f(x_2) = 13,\ f(x_3) = 14,\ f(x_4) = 16$

$$f(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} \times 13$$

$$+ \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} \times 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$= \frac{4\times1\times-1}{-1\times-4\times-6} \times 12 + \frac{5\times1\times-1}{1\times-3\times-5} \times 13 + \frac{5\times4\times-1}{4\times3\times-2} \times 14$$

$$+ \frac{5\times4\times1}{6\times5\times2} \times 16$$

$$= 2 - \frac{13}{3} + \frac{35}{3} + \frac{16}{3} = 14\frac{2}{3} \text{ Ans.}$$

MODERATION-I

b. Approximate the integral, $I = \int_0^1 e^x dx$ using Simpson's $^1/_3$rd Rule with n = 8 interval. Round off the result at 4 digits.

Q.6.b. Soln: In the case of Simpson's Rule, for n intervals, there are n+1 points which must be evaluated starting at $x = 0$ and increasing by h until $x = 1$.

$$h = \frac{b-a}{n} = \frac{1}{8} = 0.125 \quad (1) \qquad a = 0,\; b = 1$$

The data table is formed:

j	0	1	2	3	4	5	6	7	8
x	0	.125	.25	.375	.5	.625	.75	.875	1
f(x)	1	1.133	1.284	1.455	1.649	1.868	2.117	2.399	2.718

(1) (1) (1) (1)

Applying Simpson's Rule now gives, Formula for Simpson's $\frac{1}{3}$rd rule (1)

$$\int_a^b f(x)\,dx \approx \frac{.125}{3}\left[1 + 4(1.133 + 1.455 + 1.868 + 2.399) + 2(1.284 + 1.649 + 2.117) + 2.718\right]$$

$$= 1.718 \quad (2)$$

Q.7a. Apply Charpit method to solve the equation $px + qy = pq$

Q.7.a. Soln: $f(x,y,z,p,q)=0$ is $px+qy-pq=0$ — (I)

$$\frac{\partial f}{\partial x}=p,\ \frac{\partial f}{\partial y}=q,\ \frac{\partial f}{\partial z}=0,\ \frac{\partial f}{\partial p}=x-q,\ \frac{\partial f}{\partial q}=y-p \quad 1 \quad 1 \quad 1$$

Charpits equations are,

$$\frac{dx}{-\frac{\partial f}{\partial p}}=\frac{dy}{-\frac{\partial f}{\partial q}}=\frac{dz}{-p\frac{\partial f}{\partial p}-q\frac{\partial f}{\partial q}}=\frac{dp}{\frac{\partial f}{\partial x}+p\frac{\partial f}{\partial z}}=\frac{dq}{\frac{\partial f}{\partial y}+q\frac{\partial f}{\partial z}}=\frac{d\phi}{0} \quad 1$$

$$\frac{dx}{-(x-q)}=\frac{dy}{-(y-p)}=\frac{dz}{-p(x-q)-q(y-p)}=\frac{dp}{p}=\frac{dq}{q}=\frac{df}{0}$$

We have to choose the simplest integral involving p and q.

or $\frac{dp}{p}=\frac{dq}{q}$ or $\log p=\log q+\log a \Rightarrow p=aq$ 1

Putting for p in the given equation (I), we get

$q(ax+y)=aq^2$, $\therefore q=\frac{y+ax}{a}$ 1

$\therefore p=aq=y+ax$

Now $dz=pdx+qdy$ — (II)

Putting for p and q in (II), we get

$dz=(y+ax)dx+\frac{y+ax}{a}dy$ 1

$adz=a(y+ax)dx+(y+ax)dy$

$adz=(y+ax)(adx+dy)$

Integrating $az=\frac{(y+ax)^2}{2}+b$ 1 Ans.

b. Using the method of separation of variable, solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$**, where** $u(x,0)=6e^{-3x}$

b. Soln: $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ ——— (i)

Let $u = X(x) \cdot T(t)$ ① ——— (ii)

where X is a function of x only and T is a function of t only.

Putting the value of u in (i), we get

$$\frac{\partial (X \cdot T)}{\partial x} = 2\frac{\partial}{\partial t}(X \cdot T) + X \cdot T$$

$$T \cdot \frac{dX}{dx} = 2X\frac{dT}{dt} + X \cdot T$$

or $T \cdot X' = 2X \cdot T' + X \cdot T$ or $\frac{X'}{X} = 2\frac{T'}{T} + 1 = C$ (say) ①

(a) $\frac{X'}{X} = c$ or $\frac{1}{X}\frac{dX}{dx} = c$ or $\frac{dX}{X} = c\,dx$

On integrating $\log X = cx + \log a$ (1)

or $\log\frac{X}{a} = cx$ or $\frac{X}{a} = e^{cx}$ or $X = ae^{cx}$ (1)

(b) $\frac{2T'}{T} + 1 = c$

or $\frac{T'}{T} = \frac{1}{2}(c-1)$ or $\frac{1}{T}\frac{dT}{dt} = \frac{1}{2}(c-1)$ or $\frac{dT}{T} = \frac{1}{2}(c-1)dt$

On integrating, $\log T = \frac{1}{2}(c-1)t + \log b$ (1)

or $\log\frac{T}{b} = \frac{1}{2}(c-1)t$

or $\frac{T}{b} = e^{\frac{1}{2}(c-1)t}$

or $T = be^{\frac{1}{2}(c-1)t}$ (1)

Putting the value of X and T in (ii), we get

$u = ae^{cx} \cdot be^{\frac{1}{2}(c-1)t}$ —— (iii)

or $u = abe^{cx + \frac{1}{2}(c-1)t}$

or $u(x,0) = abe^{cx}$

But $u(x,0) = 6e^{-3x}$

i.e. $abe^{cx} = 6e^{-3x}$ or $ab = 6$ and $c = -3$ (1)

Putting the value of ab and c in (iii), we get

$u = 6e^{-3x + \frac{1}{2}(-3-1)t}$

$u = 6e^{-3x-2t}$ which is the reqd. soln. Ans (1)

Q.8 a. An urn 'A' contains 2 white nd 4 black balls. Another urn 'B' contains 5 white and 7 black balls. A ball is transferred from the urn 'A' to the urn 'B', then a ball is drawn from urn 'B'. Find the probability that it is white.

Q.8 a. Soln:

Urn A contains 2 white and 4 black balls.
Urn B contains 5 white and 7 black balls.
Now there are two cases of transferring a ball from A to B.

Case-I when a white ball is transferred from A to B:

Probability (Transfer of a white ball) $= \frac{2}{2+4} = \frac{1}{3}$ (1)

After transfer of a white ball, urn B contains 6 white balls and 7 black balls.

Probability (Drawing a white ball from urn B after transfer) = P (transfer of a white ball) × P (Drawing of a white ball) (1)

$= \left(\frac{1}{3}\right)\left(\frac{6}{6+7}\right) = \frac{1}{3} \times \frac{6}{13} = \frac{2}{13}$ (1)

Case-II. when a black ball is transferred from A to B:

P (Transfer of a black ball) $= \frac{4}{2+4} = \frac{2}{3}$ (1)

After transfer of a black ball, urn B contain 5 white and 8 black balls.

P (Drawing of a white ball from urn B after transfer)

= P (Transfer of black ball) × P (Drawing of a white ball). (1)

$= \frac{2}{3}\left(\frac{5}{5+8}\right) = \frac{10}{39}$ (1)

∴ Required Probability $= \frac{2}{13} + \frac{10}{39} = \frac{16}{39}$ (2) Ans.

b. Assuming half the population of a town consumes chocolates and that 100 investigators each take 10 individuals to see whether they are consumers, how many investigators would you expect to report that three people or less were consumers?

Q.8.b. Soln:

The chance for an individual to be consumer is $p = \frac{1}{2}$. 1

The chance of not being a consumer $= q = 1 - \frac{1}{2} = \frac{1}{2}$

Here, we have to find the probability of 0, 1, 2 and 3 successes. Formula for binomial distribution 1

$$P(r \le 3) = P(0) + P(1) + P(2) + P(3)$$

$$= q^{10} + {}^{10}C_1 q^9 p^1 + {}^{10}C_2 q^8 p^2 + {}^{10}C_3 q^7 p^3$$

$$= \left(\frac{1}{2}\right)^{10} + 10\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right) + 45\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + 120\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3$$

$$= \left(\frac{1}{2}\right)^{10}\left[1 + 10 + 45 + 120\right] = \frac{176}{1024} \quad 1$$

The number of investigators to report that three or less people were consumers of chocolates is given by

$$\frac{176}{1024} \times 100 = 17.2 \quad 1$$

Hence, 17 investigators would report that 3 or less people are consumers.

Ans

Q.9 a. If the variance of the Poisson distribution is 2, find the probabilities for $r = 1,2,3,4$ from the recurrence relation of the Poisson distribution. Also find $P(r \ge 4)$.

Q.9.a. Soln.

Variance = m = 2; Mean = 2 1

$P(r+1) = \frac{m}{r+1} P(r)$ 1 [Recurrence relation]

Now, $P(r+1) = \frac{2}{r+1} P(r)$ $\left[P(r) = \frac{e^{-m} \cdot m^r}{\lfloor r} \right.$

$P(0) = e^{-m} = e^{-2} = 0.1353$ 1

If $r=0$, $P(1) = \frac{2}{0+1} P(0) = \frac{2}{0+1}(0.1353) = 0.2706$

If $r=1$, $P(2) = \frac{2}{1+1} P(1) = \frac{2}{2}(0.2706) = 0.2706$ 1

If $r=2$, $P(3) = \frac{2}{2+1} P(2) = \frac{2}{3}(0.2706) = 0.1804$ 1

If $r=3$, $P(4) = \frac{2}{3+1} P(3) = \frac{1}{2}(0.1804) = 0.0902$ 1

$P(r \geq 4) = P(4) + P(5) + P(6) + \ldots\ldots$

$= 1 - [P(0) + P(1) + P(2) + P(3)]$ 1

$= 1 - [0.1353 + 0.2706 + 0.2706 + 0.1804]$

$= 1 - 0.8569$

$= 0.1431$ 1 Ans.

b. The life of army shoes is 'normally' distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months? [Given that $P(z \geq 2) = 0.0228$ **and** $z = \frac{x - \mu}{\sigma}$ **]**

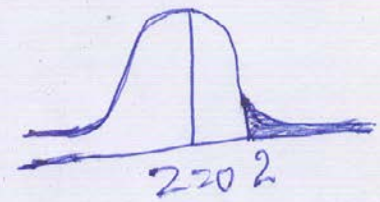
Q.9.b. Soln:

Mean $(\mu) = 8$

Standard deviation $(\sigma) = 2$

Number of pairs of shoes $= 5000$

Total months $(x) = 12$



when $z = \dfrac{x-\mu}{\sigma}$ (1) $= \dfrac{12-8}{2} = 2$ (1)

Area $(z \geq 2) = 0.0228$ (2)

Number of pairs whose life is more than 12 months $(z > 2)$ $= 5000 \times 0.0228 = 114$ (2)

Replacement after 12 months $= 5000 - 114$

$= 4886$ pairs of shoes. (2)

Ans

Text books

1. Higher Engineering Mathematics –Dr. B.S.Grewal, 40th Edition 2007, Khanna Publishers, Delhi

2. A Text book of engineering Mathematics – N.P. Bali and Manish Goyal , 7th Edition 2007, Laxmi Publication(P) Ltd