Q.2a. Determine Vx in the circuit shown in Fig.5

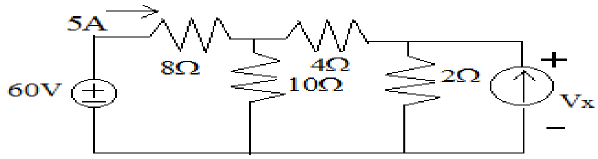
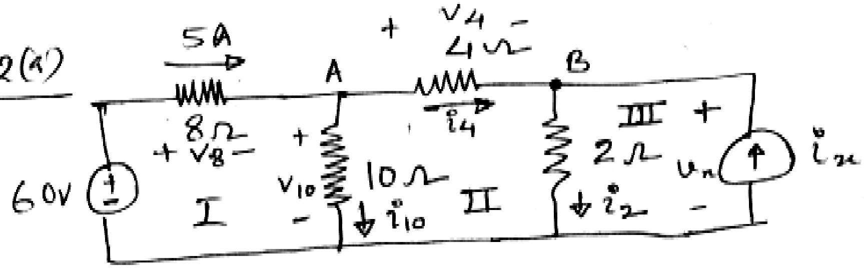


Fig. 5

Answer 2(a)



At node B

$$i_2 = i_4 + i_x$$

writing KVL for I

$$-60 + V_8 + V_{10} = 0 \qquad —(1)$$

and

$$-V_{10} + V_4 + V_x = 0 \qquad —(2)$$

where

$$V_8 = 40 \text{ V}$$

$$\therefore \; V_{10} = 60 - 40 = 20 \text{ V}.$$

$$\boxed{v_x = 20 - v_4}$$

$$\because \; i_4 = 5 - i_{10} = 5 - \frac{v_{10}}{10}$$

$$= 5 - \frac{20}{10} = 3 \text{ A}.$$

So $v_4 = 4 \times 3 = 12 \text{ V}$

HENCE $\Rightarrow$ $v_x = 20 - 12 = 8 \text{ V}.$

b.For the circuit shown in Fig.6, obtain the value of current through 2Ω resistor.

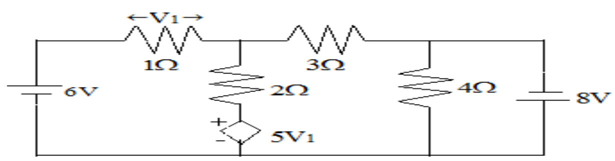


Fig.6

Soln $V_B = -8V$

Voltage accross 1Ω

$V_1 = 6 - V_A$

Applying KCL to node A, we have

$I_1 + I_2 = I_3$

$$\frac{6-V_A}{1} + \frac{5v_1 - V_A}{2} = \frac{V_A - V_B}{3}.$$

substituting $v_1 = 6 - V_A$ & $V_B = -8V$

$$(6-V_A) + \frac{5(6-V_A) - V_A}{2} = \frac{V_A + 8}{3}$$

$13V_A = 55$ or $V_A = 55/13$ V.

$\therefore$ $v_1 = 6 - V_A$

$= 23/13$ V.

and

current through 2Ω resistor $= I_2$

$$= \frac{5v_1 - V_A}{2}$$

$$= \frac{5 \times 23/13 - 55/13}{2} = \frac{30}{13} A.$$ Ans.

Q.3 a. In the network shown in Fig.7, if the switch k is opened at t = 0, then find the following quantities at t = 0^+ (i) v_1 & v_2 (ii) $\frac{dv_1}{dt}$ & $\frac{dv_2}{dt}$. **(8)**

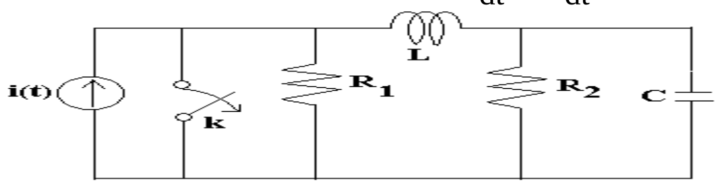


Fig.7

SOLN.

At $t = 0^+$, the inductor acts as open circuit; no current flows through it,.. so entire current i(t) flows through R_1.

$$\therefore v_2(0+) = 0$$

$$v_1(0+) = R_1 i(0+).$$

writing KCL at V_1

$$\frac{V_1}{R_1} + \frac{1}{L}\int_0^t (V_1 - V_2)dt = i(t) \quad —(1)$$

differentiating (1)

$$\frac{1}{R_1}\frac{dv_1}{dt} + \frac{V_1}{L} - \frac{V_2}{L} = \frac{di(t)}{dt}$$

$$\therefore \frac{dv_1}{dt}(0+) = R_1\frac{di(0+)}{dt} - R_1\frac{V_1(0+)}{L} + R_1\frac{V_2(0+)}{L}$$

$$= R_1\frac{di(0+)}{dt} - \frac{R_1^2 i(0+)}{L} + 0$$

writing KCL at V_2

$$\frac{1}{L}\int_0^t (v_2 - v_1)dt + \frac{v_2}{R_2} + C\frac{dv_2}{dt} = 0$$

Since at $t = 0+$ $v_2(0+) = 0$ and current through inductor is zero

$$\therefore \frac{dv_2(0+)}{dt} = 0. \quad \text{Ans.}$$

b.For the circuit shown in Fig.8, find the voltage labelled v at t = 200 μ sec.

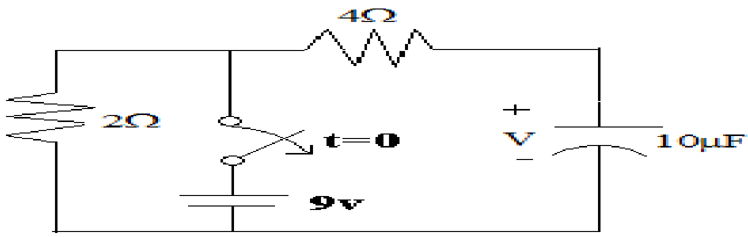
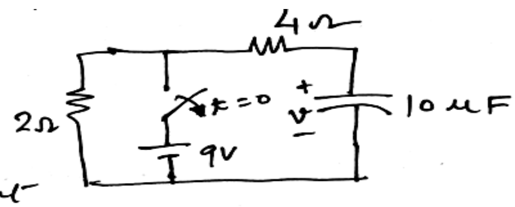


Fig.8

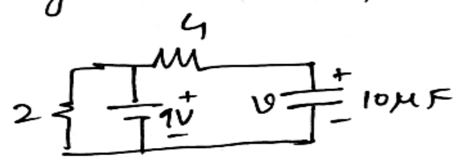
SOLN

To obtain the initial capacitor voltage: we assume any transients in that circuit died out long ago.



∴ with no current through either the capacitor or the 4Ω resistor.

$$v(0) = 9V$$

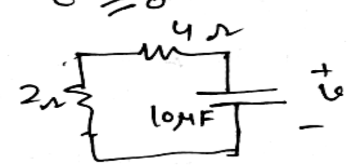


for $t \leq 0$

The time constant of the circuit is

$$T = RC = (2+4)(10 \times 10^{-6}) = 60 \times 10^{-6}\,s$$

Thus for $t \geq 0$



and

$$v(t) = v(0)\,e^{-t/RC} = v(0)\,e^{-t/60 \times 10^{-6}}$$

The capacitor voltage must be same in both circuits at $t = 0$:

$$\therefore \quad v(t) = 9e^{-t/60 \times 10^{-6}}\ V$$

and at $t = 200\,\mu s$

$$v(200 \times 10^{-6}) = 321.1\ mV. \quad \text{Ans.}$$

Q.4 a. Obtain the Laplace transform of
(i) The delayed step, function k[u(t-a)].
(ii) The ramp function k t u(t).

SOLN

(i) $f(t) = k\,u(t-a)$

$$F(s) = \int_0^{\infty} k\,U(t-a)\,e^{-st}\,dt$$

By definition of $U(t-a)$, we have

$$= \int_a^{\infty} k e^{-st}\,dt$$

$$= k\,\frac{e^{-st}}{s}\Big|_a^{\infty} = k\,\frac{e^{-as}}{s}.$$

(ii) $f(t) = k\,t\,u(t)$

$$F(s) = \int_0^{\infty} k\,t\,u(t)\,e^{-st}\,dt$$

$$= \int_0^{\infty} k\,t\,e^{-st}\,dt$$

$$= k\left[t\cdot\frac{e^{-st}}{-s}\Big|_0^{\infty} - \int_0^{\infty} 1\cdot\frac{e^{-st}}{-s}\,dt\right]$$

$$= k\,[0-0] + \frac{1}{s}\int_0^{\infty} e^{-st}\,dt$$

$$= -\frac{1}{s^2}\,e^{-st}\Big|_0^{\infty}$$

$$= \frac{1}{s^2}$$ Ans.

b. Consider the R-L circuit with R = 4Ω and L = 1H excited by a 48V dc source as shown in Fig.9. Assume the initial current through the inductor is 3A. Using Laplace transform method, determine the current i(t); at t ≥ 0. Also draw the s-Domain representation of the circuit.

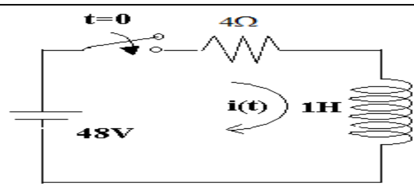


Fig.9

SOLN Applying KVL, $R\,i(t) + L\dfrac{di(t)}{dt} = 48$

taking Laplace transform

$$R\,I(s) + L(s\,I(s) - i_L(0^+)] = 48/s$$

$$i_L(0^+) = 3\text{ A} \quad \therefore \quad I(s) = \frac{3s+48}{s(s+4)}$$

Applying partial fraction expansion, we get.

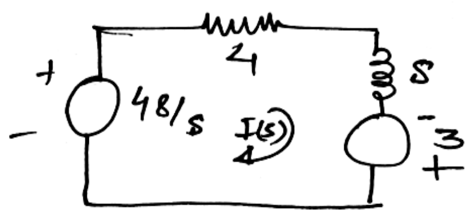
$$I(s) = \frac{12}{s} - \frac{9}{s+4}$$

taking inverse laplace transform.

$$i(t) = L^{-1}[I(s)]$$

$$= 12 - 9e^{-4t}\text{ A}.$$

s-domain representation of circuit



Ans

Q.5 a. Obtain the Thevenin's equivalent circuit across the terminal A & B of ckt shown in Fig.10.

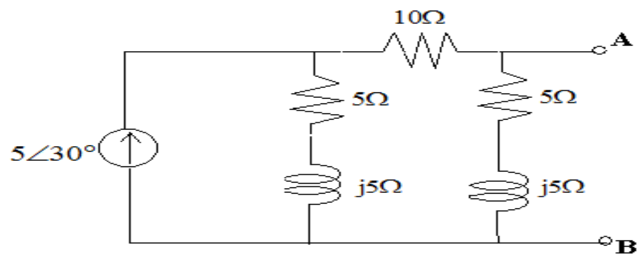


Fig.10

SOLN

For V_{th}.

$$V_{th} = (5+j5) \times \left\{ 5\angle 30^\circ \cdot \frac{(5+j5)}{(10+j10+10)} \right\}$$

$$= \frac{(5\sqrt{2}\angle 45^\circ)^2 (5\angle 30)}{22.36\angle 26.56} = 11.18\angle 93.44 \text{ V.}$$

As the current source is open ckt.

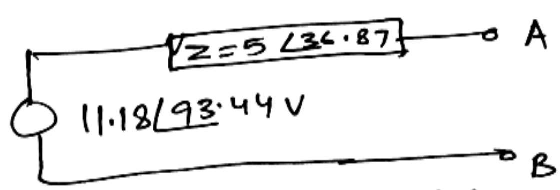
$$Z_{th} = \{(5+j5)+10\} \,//\, (5+j5)$$

$$= \frac{(15.81\angle 18.43)(5\sqrt{2}\angle 45^\circ)}{20+j10}$$

$$= \frac{111.80\angle 63.43}{22.36\angle 26.56}$$

$$= 5\angle 36.87^\circ \ \Omega$$

Ans

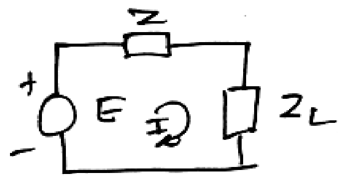


Thevenin's equivalent ckt.

b. Derive the condition for maximum power transfer to take place at a load impedance $Z_L = R_L + jX_L$, when the source is an ac source having an internal impedance of $Z_{in} = R + jX$.

SOLN

$$I = \frac{E}{Z+Z_L}$$



Power dissipated at load

$$P = I^2 R_L$$

where $Z = R+jX$ and $Z_L = R_L + X_L$

So $$P = \frac{E^2}{(R+R_L)^2 + (X+X_L)^2} \cdot R_L$$

For P to be maximum $\frac{dP}{dX_L} = 0$

$$\frac{dP}{dX_L} = \frac{0 - E^2 . R_L . 2(X+X_L)}{[(R+R_L)^2 + (X+X_L)^2]^2} = 0$$

$$E^2 . R_L . 2(X+X_L) = 0$$

$$X + X_L = 0$$

$$\boxed{X_L = -X.}$$

Putting $X_L = -X$., For P to be maximum $\frac{dP}{dR_L} = 0$

$$\frac{dP}{dR_L} = \frac{(R+R_L)^2 E^2 - E^2 R_L \cdot 2.(R+R_L)}{[(R+R_L)^2]^2} = 0$$

$$E^2 (R+R_L) - 2E^2 R_L = 0$$

$$\Rightarrow \boxed{R_L = R.}$$

Thus, the load impedance must be the complex conjugate of internal impedance of the circuit.

Q.6 a. Find the transfer impedance function $Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$ of the network shown in Fig.11 **(8)**

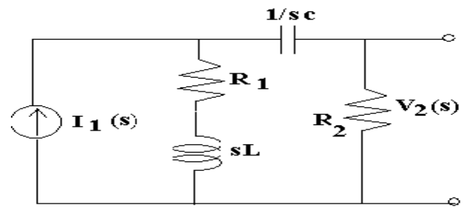


Fig.11

SOLN

Applying current division rule,

Current in the impedance $\left(R_2 + \frac{1}{sC}\right)$ is given by

$$I'(s) = I_1(s)\frac{R_1 + sL}{R_1 + sL + R_2 + \frac{1}{sC}}$$

$$= \frac{I_1(s)\ (R_1 + sL) \cdot sC}{sC\,(R_1 + R_2 + sL) + 1}$$

Then $V_2(s) = I_1'(s) \cdot R_2$

$$V_2(s) = I_1(s) \cdot \frac{(R_1 + sL)\,sC \cdot R_2}{sC\,(R_1 + R_2 + sL) + 1}$$

Therefore,

$$\boxed{Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = \frac{R_2\,(s^2LC + sR_1C)}{s^2LC + sC(R_1 + R_2) + 1}}$$

b.Check whether the given polynomial P(s) is Hurwitz or not.

$P(s) = s^4 + s^3 + 2s^2 + 4s + 1$

Ans

Since all a_i are +ve, the condition 1 is satisfied.

Even and odd part of P(s) are

$$M(s) = s^4 + 2s^2 + 1$$

$$N(s) = s^3 + 4s.$$

So continued fraction expansion $\psi(s) = \dfrac{M(s)}{N(s)}$ is given as

$$
\begin{array}{l}
s^3+4s \,\big)\, s^4+2s^2+1 \,\big(\, s \\
\quad\quad\quad\;\; \underline{s^4+4s^2} \\
\quad\quad\quad\quad -2s^2+1 \,\big)\, s^3+4s \,\big(\, -\tfrac{1}{2}s \\
\quad\quad\quad\quad\quad\quad\quad\quad\;\; \underline{s^3-\tfrac{s}{2}} \\
\quad\quad\quad\quad\quad\quad\quad\quad\;\; \tfrac{9}{2}s \,\big)\, -2s^2+1 \,\big(\, -\tfrac{2}{9}\cdot 2s \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\;\; \underline{-2s^2} \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\;\; 1 \,\big)\, \tfrac{9}{2}s \,\big(\, \tfrac{9}{2}s \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\;\; \underline{\tfrac{9}{2}s} \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \times
\end{array}
$$

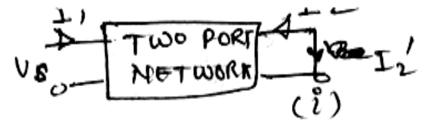
The given polynomial is not Hurwitz because of the presence of the negative quotient terms in the continued fraction expansion.

Q.7 a. Obtain the condition for reciprocity and symmetry in terms of h-parameters.

SOLN

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$



Let $V_1 = V_S,\ I_1 = I,\ V_2 = 0,\ I_2 = -I_2'$ (Fig i)

$$I_2' = -V_s \frac{h_{21}}{h_{11}}$$

Now interchanging the position of excitation to response.

$V_2 = V_S,\ I_2 = I_2,\ V_1 = 0$

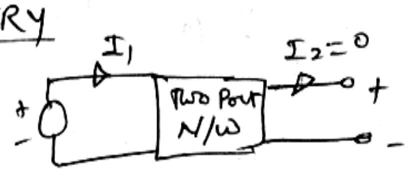
& $I_1 = -I_1'$

$$I_1' = V_s \frac{h_{12}}{h_{11}}$$

Thus for reciprocity $I_2' = I_1'$

$$\therefore \boxed{h_{21} = -h_{21}}$$

CONDITION FOR SYMMETRY



$V_1 = V_S,\ I_1 = I_1, I_2 = 0$

$V_2 = V_2$

$$\frac{V_S}{I_1} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}$$

For n/w-2

$V_2 = V_S,\ I_2 = I_2$

$I_1 = 0$ & $V_1 = V_1$

$V_1 = h_{12} V_S$ and $I_2 = h_{22} V_S$

$$\frac{V_S}{I_2} = \frac{1}{h_{22}}$$

For symmetry $\left.\frac{V_S}{I_1}\right|_{I_2 = 0} = \left.\frac{V_S}{I_2}\right|_{I_1 = 0}$

$$\therefore \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$$

Ans.

b. Calculate the Z – Parameters of the network shown in Fig.12. Determine whether the network is symmetrical or not?

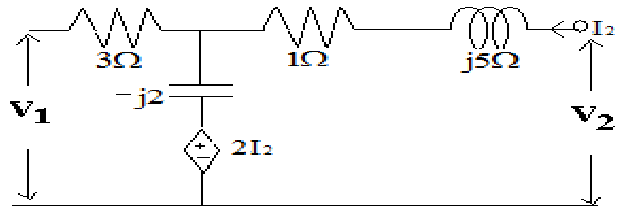


Fig.12

SOLN

The Loop equation can be written as

$V_1 = 3I_1 - j2(I_1+I_2) + 2I_2$

$V_1 = (3-j2)I_1 + (2-j2)I_2 \quad —(1)$

$V_2 = (1+j5)I_2 - j2(I_1+I_2) + 2I_2$

$V_2 = -j2 \cdot I_1 + (3+j3)I_2 \quad —(2)$

Therefore from (1) & (2)

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = (3-j2)\,\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = (-j2)\,\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = (2-j2)\,\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = (3+j3)\,\Omega$$

Since $Z_{11} \neq Z_{22}$, the network is not symmetrical.

Q.8 An impedance function given by $Z(s) = \frac{s(s+2)(s+5)}{(s+1)(s+4)}$ find the R-L representation of **(16)**

(i) FOSTER I and II forms. (ii) CAUER I and II forms.

SOLN

FOSTER-I : Since we know that the residues of poles of $Z_{R-L}(s)$ are real & negative. So we determine the residues of $\frac{Z(s)}{s}$ as

$$\frac{Z(s)}{s} = \frac{(s+2)(s+5)}{(s+1)(s+4)}$$

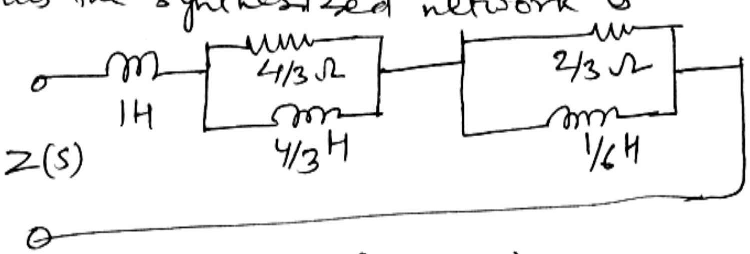
$$s^2+5s+4 \overline{) s^2+7s+10} (1$$
$$\underline{s^2+5s+4}$$
$$2s+6$$

$$\therefore \frac{Z(s)}{s} = 1 + \frac{2s+6}{(s+1)(s+4)}$$

$$= 1 + \frac{4/3}{s+1} + \frac{2/3}{s+4}$$

$$Z(s) = s + \frac{4/3\, s}{s+1} + \frac{2/3\, s}{s+4}$$

Thus the synthesized network is

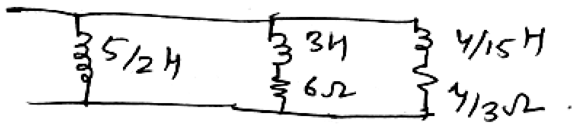


FOSTER-II : $Y(s) = \frac{(s+1)(s+4)}{s(s+2)(s+5)}$

Using Partial fraction expansion

$$Y(s) = \frac{\frac{2}{5}}{s} + \frac{1/3}{s+2} + \frac{4/15}{s+5}.$$

Therefore the synthesized network is as below

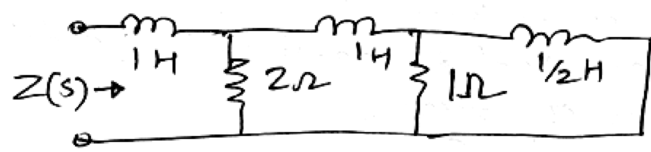


CAUER-I :

$$Z(s) = \frac{s^3+7s^2+10s}{s^2+5s+4}$$

$\therefore Z_1 = s, \quad Y_2 = \frac{1}{2}$

$Z_3 = s \quad Y_4 = 1 \quad Z_5 = \frac{1}{2}s.$



CAUER-II :

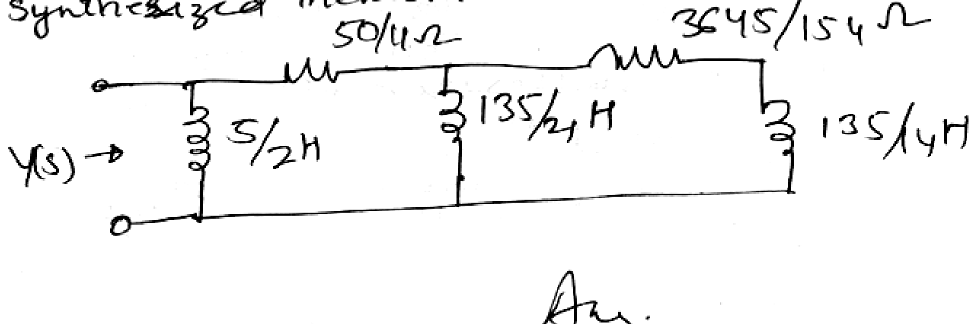
$$\cancel{Z(s) = s^3} \quad Y(s) = \frac{4+5s+s^2}{10s+7s^2+s^3}$$

$\therefore Y_1 = \frac{2}{5s}, \quad Z_2 = \frac{50}{11}$

$Y_3 = \frac{121}{135s} \quad Z_4 = \frac{3645}{154}$

$Y_5 = \frac{14}{135s}.$

Synthesized network is as below



Ans.

Q9 a. Find the driving point impedance of the network shown in Fig.13. Find the poles and zeros of the network and locate. them

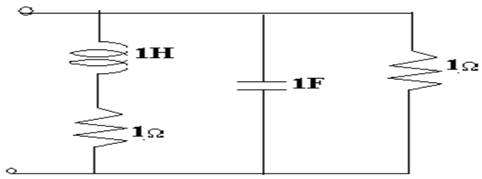


Fig.13

Soln

The impedances can be transformed into s-domain.
There are 3-parallel branches having impedances

$Z_1(s) = s+1$; $Z_2(s) = \frac{1}{s}$; $Z_3 = 1$

So, $Y(s) = Y_1(s) + Y_2(s) + Y_3(s)$

$$= \frac{s^2+2s+2}{s+1}$$

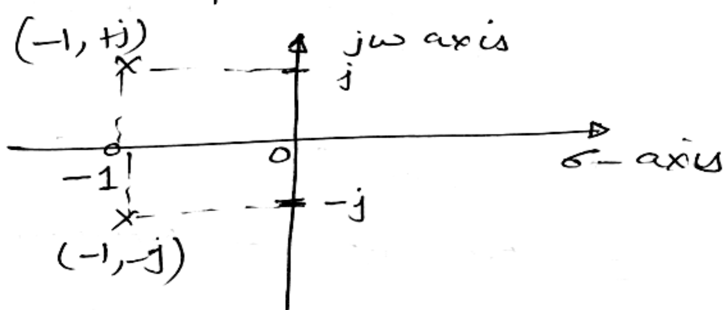
Therefore

$$Z(s) = \frac{1}{Y(s)} = \frac{s+1}{s^2+2s+2}$$

$$Z(s) = \frac{s+1}{(s+1)^2 - (j1)^2}$$

$$= \frac{s+1}{(s+1+j)(s+1-j1)}$$

Thus pole zero plot can be drawn as



pole zero location of Z(s).

b .If a T-section of a constant k- low pass filter has series inductance 85 mH and shunt capacitance of 0.025μF, calculate its cut off frequency and the nominal design impedance R_o. Design an equivalent π-section too.

SOLN The cut-off frequency fc is given by

$$f_c = \frac{1}{\pi\sqrt{LC}} = \frac{1}{3.14 \times \sqrt{85\times10^{-3}\times 0.025\times10^{-6}}}$$

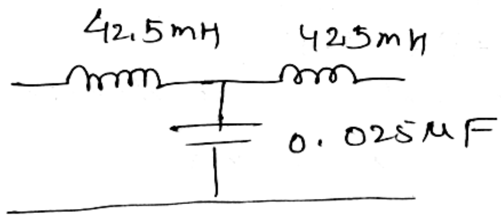
$$= 6.9\ \text{kHz}$$

The nominal design impedance Ro is given by

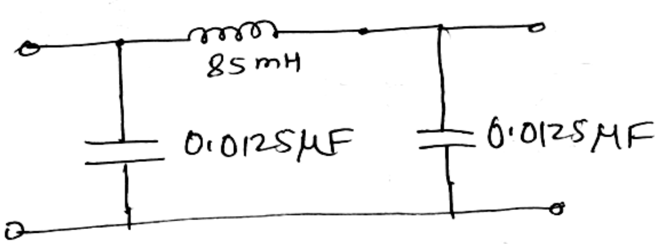
$$R_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{85\times10^{-3}}{0.025\times10^{-6}}}$$

$$= 1.844\ \text{k}\Omega.$$

The required T & π section are



T- section



π-section

Textbook

1. Network Analysis, M.E.Van Valkenberg, 3rd Edition, Prentice-Hall India, EEE 2011

2.Network Analysis and Synthesis, Franklin F Kuo, 2nd Edition, Wiley India