Q.2 a. Compare open loop and closed loop control system.

2a

Comparison

	Open Loop Control Systems	*Close Loop Control Systems*	
(1)	Accuracy depends on the calibration of Input	Accuracy depends upon feed back so they operate more accurately.	
(2)	Simple Construction and Ease of Maintenance	Complicated to construct	2 mark
(3)	Feedback element is absent	Feedback element is present	
(4)	Systems are cheap	Expensive	
(5)	No Stability Problem	Becomes unstable under some condition	2 mark
(6)	No change in Gain.	Feed-back Improves transient response	
(7)	Disturbances and change in Calibration causes Error.	Operates better than Open Loop Control System in disturbances.	2 mark
(8)	Band width is small	Band width is large	

1.4 EXAMPLES OF CONTROL SYSTEMS

b. Derive transfer function of the circuit shown in Fig.4. **(6)**

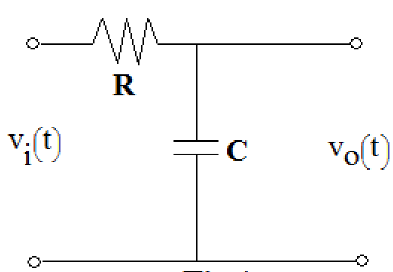
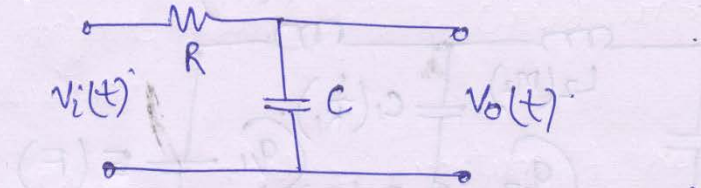
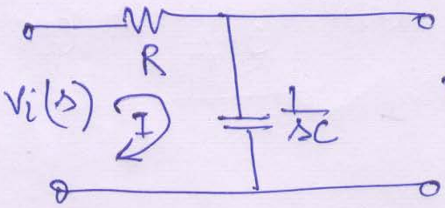


Fig.4

2b



By Laplace transform circuit is



1 mark.

here $V_o(s) = I\left(\frac{1}{sC}\right)$.

& $V_i(s) = I\left(R + \frac{1}{sC}\right)$.

2 marks.

$$T.F = \frac{V_o(s)}{V_i(s)}$$

$$= \frac{I\left(R+\frac{1}{sC}\right)}{I\left(\frac{1}{sC}\right)}$$

2 marks.

$$= \frac{R+\frac{1}{sC}}{\frac{1}{sC}}$$

$$\underline{T.F = sCR+1}$$

1 marks.

c. Draw electrical analogous circuit for mechanical system shown in Fig.5 based on Force-Voltage analogy. (4)

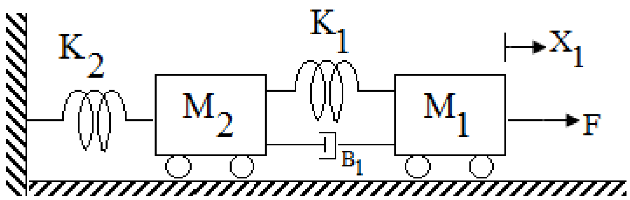
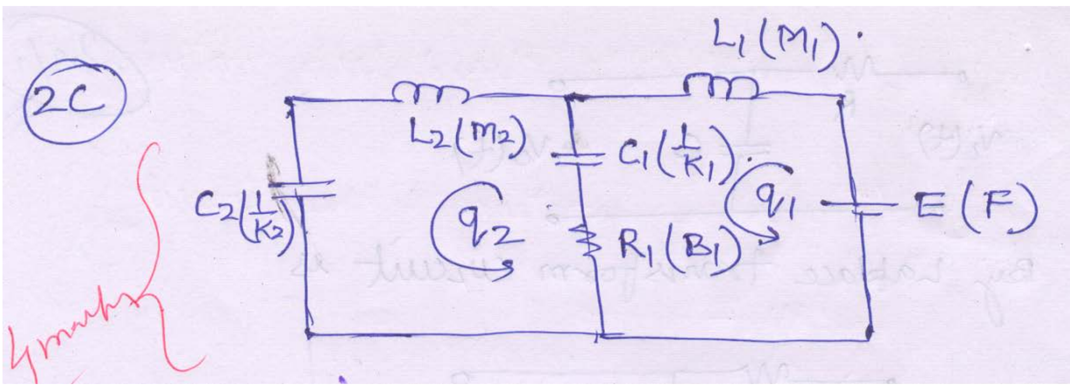


Fig.5



Q.3a. **Find transfer function $\left(C/R\right)$ for the system represented in Fig.6 using block diagram reduction method.** **)**

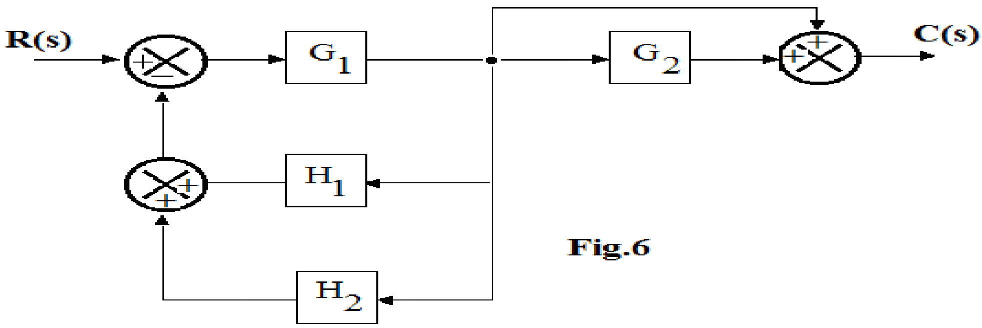


Fig.6

3a

~~Example 2.6~~ Derive the Transfer Function of the System shown in Fig. 2.17 using Block Reduction Method.

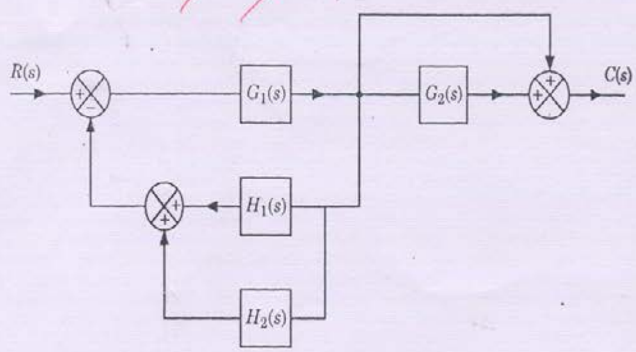


Fig. 2.17

Solution. **Rule 2:** *Solving Blocks in Parallel:* By Rule 2, the Block diagram of Fig. 2.17 has been reduced to Fig. 2.18.

2 marks

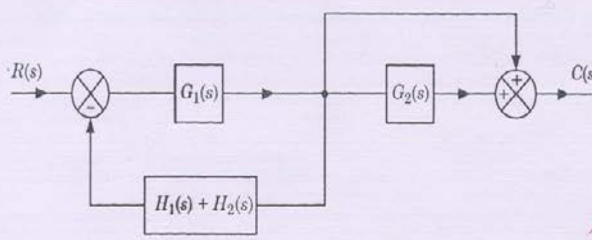


Fig. 2.18

Rule 7: *Eliminating a Feed-back Loop:* By Rule 7, we have reduced it to Fig. 2.19.

2 marks

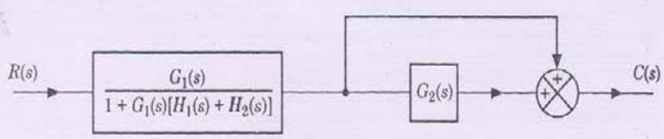


Fig. 2.19

Rule 2: *Solving Blocks in Parallel:* By Rule 2, we have reduced it to Fig. 2.20.

— 2 marks

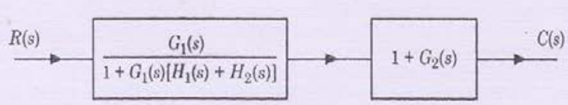


Fig. 2.20

REPRESENTATION OF SYSTEMS 25

Rule 1: *Solving Blocks in Cascade:* By Rule 1. We have reduced it to Fig. 2.21.

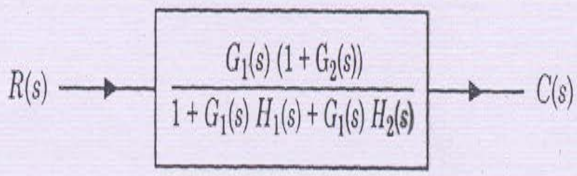


Fig. 2.21

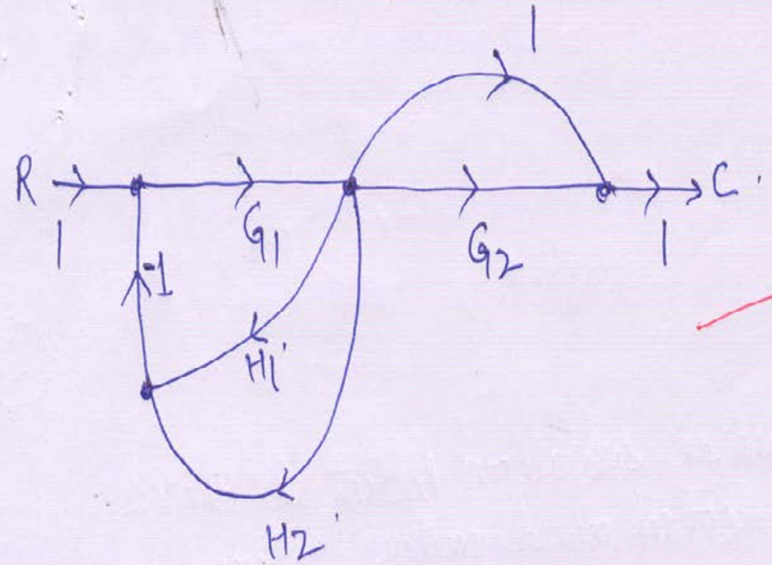
The Transfer Function of System is given by

$$\frac{C(s)}{R(s)} = \frac{G_1(s)(1+G_2(s))}{1+G_1(s)H_1(s)+G_1(s)H_2(s)}$$

2 marks

b. Find transfer function $\frac{C}{R}$ using Maison's gain formula of block diagram shown in Fig. 6.

(3b) from the block diagram, we get signal flow graph:



2 mark

Forward Path gains

$P_1 = G_1 G_2 \qquad \Delta_1 = 1$

$P_2 = G_1. \qquad \Delta_2 = 1.$

Loops:- $L_1 = -G_1 H_1$

$L_2 = -G_1 H_2$

2 mark.

There is no nontouching loops.

So $\Delta = 1 - [L_1 + L_2] +$ — 2 marks.

$$= 1 + G_1H_1 + G_1H_2$$

$$T \cdot F = \frac{P_K \Delta_K}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

2 marks.

$$= \frac{G_1G_2 + G_1}{1 + G_1H_1 + G_1H_2}$$

Q.4 a. Explain effect of feedback on sensitivity of open loop and closed loop control systems.

where S_K is ...

4a

Let $T(s)$ be the over all Transfer Function of a Control System and $G(s)$ is the Forward Gain. Then the Sensitivity of overall Transfer Function $T(s)$ with respect to variation in Forward Gain $G(s)$ is given by

$$S_G^T = \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)} = \frac{\partial T(s)}{\partial G(s)} \times \frac{G(s)}{T(s)} \qquad \ldots(4.6)$$

1 mark

Let us discuss Sensitivity of Open-Loop Control System and Close Loop Control System.

(a) *Sensitivity of Open-Loop Control System:* In accordance to Fig. 4.1 Transfer Function of overall Transfer Function of Open-Loop Control System is

$$T(s) = G(s)$$

So Sensitivity of the Open Loop Control System is

$$S_G^T = \frac{\partial G(s)}{\partial G(s)} * \frac{G(s)}{G(s)} = 1 \qquad \ldots(4.7)$$

Then Sensitivity of Open-Loop Transfer Function $T(s)$ with respect to $G(s)$ is unity.

2 mark

(b) *Sensitivity of Close-Loop Control System:* As shown in Fig. 4.2. The Transfer Function of Close-Loop System is

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)}$$

$$\therefore \quad \frac{\partial T(s)}{\partial G(s)} = \frac{1+G(s)\,H(s)-G(s)\,H(s)}{[1+G(s)\,H(s)]^2}$$

$$= \frac{1}{[1+G(s)\,H(s)]^2}$$

$\therefore$ Sensitivity of the Close-Loop System is

$$S_G^T = \frac{\partial T(s)}{\partial G(s)} \times \frac{G(s)}{T(s)}$$

2 mark

$$= \frac{1}{[1+G(s)H(s)]^2} \times \frac{G(s)}{[G(s)/1+G(s)H(s)]}$$

1 mark

$$= \frac{1}{1+G(s)H(s)} \qquad ...(4.8)$$

From eq. 4.7 and 4.8, it is observed that due to presence of Feed-back Sensitivity is reduced by the factor $[1 + G(s) H(s)]$. It mean that Sensitivity of a Close-Loop System with respect to variation in $G(s)$ is Less as compared to Open-Loop System.

2 marks

b.Find transfer function and write features of AC servomotor.

4b

~~5.3.1~~ A.C. Servomotors

Most of the smaller motors used in low power servomechanism are a.c. servomotors. It is basically two-phase Induction Motor. It is divided into two main parts i.e. Stator and Rotor as shown in Fig. 5.8. Stator contains two Stator windings displaced 90° in space. One winding is called Main winding. It is also known as Fixed or Reference winding, which is excited by a.c. Voltage(V_r). The other winding is called control winding which is excited by variable control voltage (V_c). This voltage (V_c) is obtained from Servo amplifier which should be 90° out of phase w.r.t. V_r. This is the necessary condition to obtain rotating Magnetic Field. The rotating Magnetic field interacts with currents producing a torque on the rotor in the direction of field rotation. The direction depends upon the phase relationship of V_c and V_r. It is clear that

2 marks

Torque developed by motor is function of motor-shaft angular speed $\dot{\theta}_m$ **and** Control Voltage (V_c), the equation is

$$T_m = -K_a \dot{\theta}_m + K_b V_c \qquad \text{...(5.12)}$$

where K_a and K_b are constants.

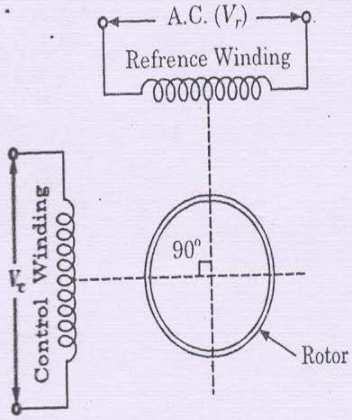


Fig. 5.8

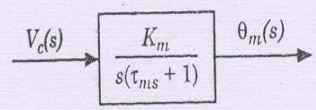


Fig. 5.9

2mark

If load consists of Inertia J_m and friction B_m, we can write

$$T_m(s) = J_m s^2 \theta_m(s) + B_m s\, \theta_m(s) \qquad \text{...(5.13)}$$

Equating eq. 5.12 and eq. 5.13, we have:

$$(J_m s + B_m)\, s\, \theta_m(s) = -K_a s\, \theta_m(s) + K_b V_c(s)$$

$$\therefore \quad J_m s^2\, \theta_m(s) + (B_m + K_a)\, s\, \theta_m(s) = K_b V_c(s)$$

The Transfer Function of System $\dfrac{\theta_m(s)}{V_c(s)}$ is

$$\frac{K_b}{J_m s^2 + (B_m + K_a)\, s} = \frac{K_m}{s(\tau_m s + 1)}$$

2mark

where $\quad K_m = \dfrac{K_b}{B_m + K_a}$ = Motor Gain Constant

$$\tau_m = \frac{J_m}{B_m + K_a} = \text{Time Gain Constant}$$

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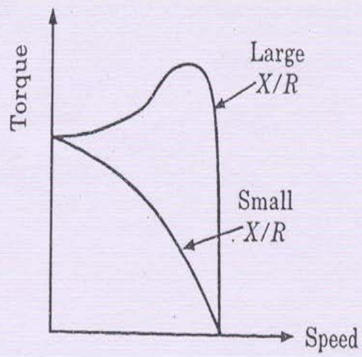


Fig. 5.10

1 mark

1. Small X to R ratio.
2. Large Torque to weight ratio.
3. Light in weight because it eliminates the use of brushes and commutator as in D.C. Servomotor.
4. Reliable for low power application.
5. Noise free and stable operation.

Q.5 a. For a unit step response of second order system, determine damping ratio for overshoot of 37% and ω_n if response takes 10 second to reach within 5% of final value.)

5a) $M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = 0.37$ — 2 mark

by solving this we get

$\xi = 0.3$.

settling time for 5% error — 2 mark

$t_s = \frac{3}{\xi \omega_n} = 10$

so $\omega_n = \frac{t_s \times \xi}{3}$ — 1 mark

$= 1$ rad/sec.

b. **Unity feedback system has open loop T.F.** $G(s) = \frac{k(1+2s)}{s(1+s)(1+4s)^2}$. **Find value of k to limit steady state error to 10% when input is unit ramp.**

(b) $$\left.\begin{aligned} G(s)H(s) &= \frac{K(1+2s)}{s(1+s)(1+4s)^2} \\ r(t) = t \quad \text{so} \quad R(s) &= \frac{1}{s^2} \end{aligned}\right\} \text{1 mark}$$

$$\left.\begin{aligned} e_{ss} &= \lim_{s \to 0} \frac{s R(s)}{1 + G(s)H(s)} \\ &= \lim_{s \to 0} \frac{s \times \frac{1}{s^2}}{\frac{K(1+2s)}{s(1+s)(1+4s)^2}} \end{aligned}\right\} \text{2 mark}$$

$$\left.\begin{aligned} &= \lim_{s \to 0} \frac{1}{s + \frac{K(1+2s)}{(1+s)(1+4s)^2}} = \frac{1}{0+K} \\ &= \frac{1}{K} \\ K &= \frac{1}{e_{ss}} = \frac{1}{0.1} = 10 \end{aligned}\right\} \text{2 mark}$$

c. Determine value of K for stable system shown in Fig.7. **)**

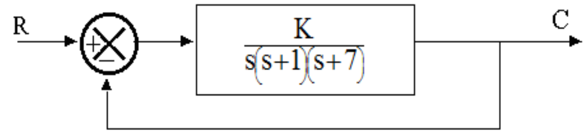


Fig.7

(c) We know that ch. eqn is

$$1+G(s)H(s)=0$$

$$1+\frac{K}{s(s+1)(s+7)}=0$$

$$s^3+8s^2+7s+K=0$$

2 marks

By Routh's array

s^3	1	7
s^2	8	K
s	$\frac{56-K}{8}$	0
s^0	K	0

2 marks

for stable system all elements in Ist column in routh's array must be +ve.

so

$$K>0 \quad \& \quad \frac{56-K}{8}>0$$

$$\text{or } K<56$$

so system is stable when

$$\boxed{0<K<56}$$

2 marks

Q.6 **Draw Root locus for the system having** $G(s)H(s) = \dfrac{k}{s(s+3)(s^2+3s+4.5)}$**. Also comment on its stability.** **)**

Fig. 8.27

Q6

~~Example 8.19~~ Discuss the stability of the System by using Root Locus for

$$G(s)\,H(s) = \frac{K}{s\,(s+3)\,(s^2+3s+4.5)}$$

Solution.

Step 1: There are four poles ($N = 4$) at $s = 0, -3, -1.5 \pm j1.5$. Since there is no zero so all four branches will approach to Infinity.

Step 2: Root Locus on Real axis between $s = 0$ and $s = -3$.

Step 3: Angle of Asymptotes

$$\theta = \frac{180°\,(2q+1)}{N-M} \text{ where } q = 0, 1, 2, 3$$

$$\therefore \qquad \theta = 45°,\ 135°,\ 225°,\ 315°$$

2 mark

Step 4: Centroid is given by

$$\sigma_A = \frac{0-3-1.5-1.5}{4} = -1.5$$

2 mark

Step 5: The characteristic equation is

$$1 + \frac{K}{s(s+3)(s^2+3s+4.5)} = 0$$

$$\therefore \quad s^4 + 6s^3 + 13.5s^2 + 13.5s + K = 0$$

also $$K = -s^4 - 6s^3 - 13.5s^2 - 13.5s$$

$$\frac{dK}{ds} = -4s^3 - 18s^2 - 27s - 13.5$$

2 marks

For Breakaway Point, $\frac{dK}{ds} = 0$

$$\therefore \quad 4s^3 + 18s^2 + 27s + 13.5 = 0$$

$$(s + 1.5)(4s^2 + 12s + 9) = 0$$

The Roots are $s = -1.5, -1.5, -1.5$

So $s = -1.5$ is valid breakaway point.

2 marks

Step 6: Intersection with Imaginary axis,

Routh's Array:

s^4	1	13.5	K
s^3	6	13.5	
s^2	11.25	K	
s	$\frac{151.8-6K}{11.25}$		
s^0	K		

$$\therefore \quad K_{\text{mar.}} = \frac{151.8}{6} = 25.31$$

2 marks

Auxiliary equation is

$$11.25s^2 + 25.31 = 0$$

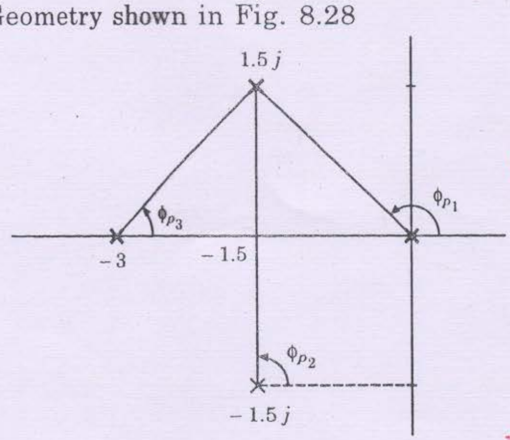
$$s^2 = -\frac{25.31}{11.25} = -2.25$$

$$s = \pm j\,1.5$$

1 marks

Step 7: Angle of Departure for Pole $s = -1.5 + j1.5$

From the Geometry shown in Fig. 8.28



2 marks

Fig. 8.28

we have calculated

$$\phi_{P1} = 135^\circ \quad \phi_{P2} = 90^\circ \quad \phi_{P3} = 45^\circ$$

$$\therefore \qquad \Sigma\phi_P = 135^\circ + 90^\circ + 45^\circ = 270^\circ$$

$$\Sigma\phi_z = 0^\circ$$

$\therefore$ Angle of Departure is

$$\phi_d = 180 - [\Sigma\phi_P - \Sigma\phi_z] = 180^\circ - 270$$
$$= -90^\circ$$

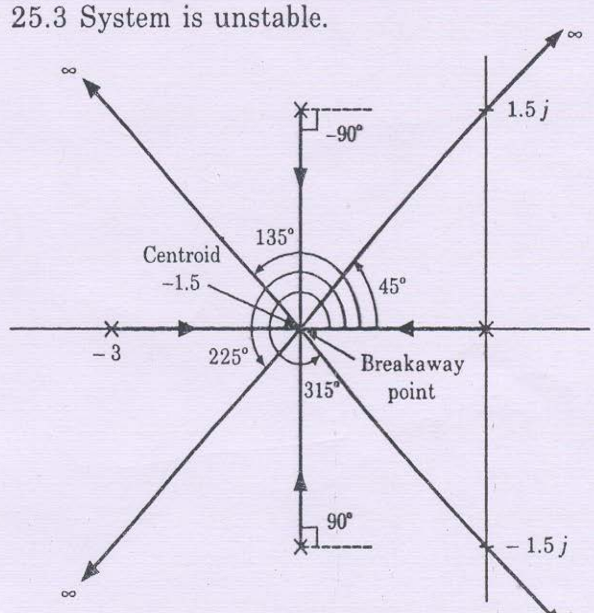
1 mark

Similarly at $s = -1.5 - j1.5 \qquad \phi_d = +90^\circ$

Step 8: The complete Root Locus is shown in the Fig. 8.29. For 25.3 > K > 0. System is absolutely Stable.

At $K = 25.31$. System is marginally Stable with Frequency 1.5 rad/sec.

For $K > 25.3$ System is unstable.



2 mark.

Fig 8.29

Example 8.20 Calculate the value of K at point P for the Root Locus

Q.7 a. Determine M_r & W_r for unity feedback system with $G(s)=\dfrac{10}{s^2+2s+10}$)

7a) here Ch eqn is

$s^2+2s+10=0$ — 1 mark

Compare it with

$s^2+2\xi W_n s+W_n^2=0$ — 1 mark

we get $W_n=3.16$

& $\xi=0.316$

$M_r=\dfrac{1}{2\xi\sqrt{1-\xi^2}}=1.66$ — 1 mark

& $W_r=W_n\sqrt{1-2\xi^2}$ — 1 mark

$=2.826$ rad/sec

b.Draw the complete Nyquist Plot and discuss stability of a system $G(s)H(s)=\dfrac{k}{s(s-a)}$ here a > 0)

7b

~~**Example 9.12**~~ Draw the complete Nyquist Plot and discuss stability for

$$G(s)\,H(s) = \frac{K}{s(s-a)} \qquad (a > 0)$$

Solution: Step 1: As $G(s)H(s) = \dfrac{K}{s(s-a)}$

Replace s by $j\omega$

$$\therefore \qquad G(j\omega)H(j\omega) = \frac{K}{j\omega\,(j\omega - a)}$$

Step 2: As there is one pole at origin. So in this case, Nyquist path will encircle the whole Right Half at S-plane (with Radius infinity), but avoid the pole at origin by a small circle with Radius ε approach to zero as shown in fig. 9.46.

2 marks

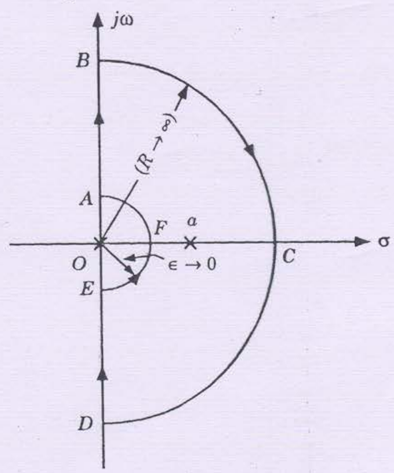


Fig. 9.46

Step 3: The different section form the Nyquist Contour are AB, BCD, DE and EFA as shown in fig. 9.46.

(i) Section AB: For this section, $s = j\omega$

$$\therefore \qquad G(j\omega)H(j\omega) = \frac{K}{j\omega\,(j\omega - a)}$$

$$|\,G(j\omega)H(j\omega)\,| = \frac{K}{\omega\sqrt{\omega^2 + a^2}} \quad \text{and} \quad \phi = -90^\circ - 180^\circ \tan^{-1}\frac{\omega}{a}$$

2 mark

At $\omega = 0$, $\quad |\,G(j\omega)H(j\omega)\,| = \infty$, $\quad \phi = -270^\circ$

$\omega = \infty$, $\quad |\,G(j\omega)H(j\omega)\,| = 0$, $\quad \phi = -180^\circ$

This portion is mapped into the curve $A'B'$ as shown in fig. 9.47.

(ii) Section BCD: For section BCD, we have $S = Re^{j\theta}$ where R approaches to infinity and θ varies from $(+90^\circ$ to -90° through $0^\circ)$.

$$\therefore \quad G(s)H(s) = \frac{K}{Re^{j\theta}(Re^{j\theta} - a)} = \frac{K}{R^2 e^{j2\theta}} \text{ (as } R \text{ is high)}$$
$$= 0\, e^{-j2\theta}$$

So corresponding to section BCD, we have locus $B'C'D'$ with Radius 0 from $-180°$ to $+180°$ through $0°$.

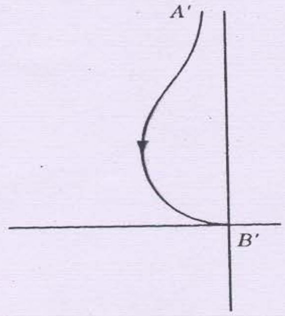


Fig. 9.47

(iii) **Section DE:** For this section, $s = -j\omega$. So this is mirror image of Plot $A'B'$.

(iv) **Section *EFA*:** For section EFA, we have $s = \varepsilon e^{j\theta}$ where ε approaches to zero ($\varepsilon \to 0$) and θ varies from $-90°$ to $+90°$ through $0°$.

2 mark

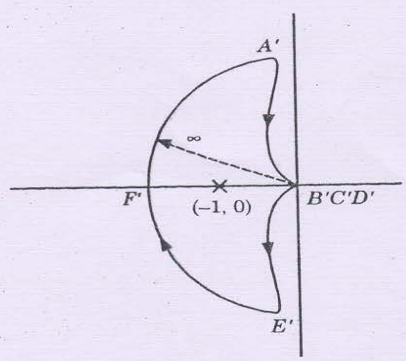
$$\therefore \quad G(s)\,H(s) = \frac{K}{\varepsilon e^{j\theta}(\varepsilon e^{j\theta} - a)} = -\frac{K}{a\varepsilon e^{j\theta}} \text{ (as } \varepsilon \text{ is less)}$$

The effect of negative sign is to add (or subtract) 180°. Hence

$$G(s)\,H(s) = \infty e^{-j(180\,+\,\theta)}$$

So corresponding to section EFA, we have E′F′A′ with radius equal to infinity from $-90°$ to $-270°$ through $-180°$ as shown in fig. 9.48.

Step 4: The complete Nyquist plot is shown in fig. 9.48.



1 mark

Fig. 9.48

c.Draw polar plot of $G(s)H(s) = \dfrac{k}{s(1+s)}$**.**

7C

1 mark

$$G(s)\,H(s) = \frac{K}{s(1+s)}$$

$$G(j\omega)\,H(j\omega) = \frac{K}{j\omega(1+j\omega)}$$

1 mark

$$M = \frac{K}{\omega\sqrt{1+\omega^2}}$$

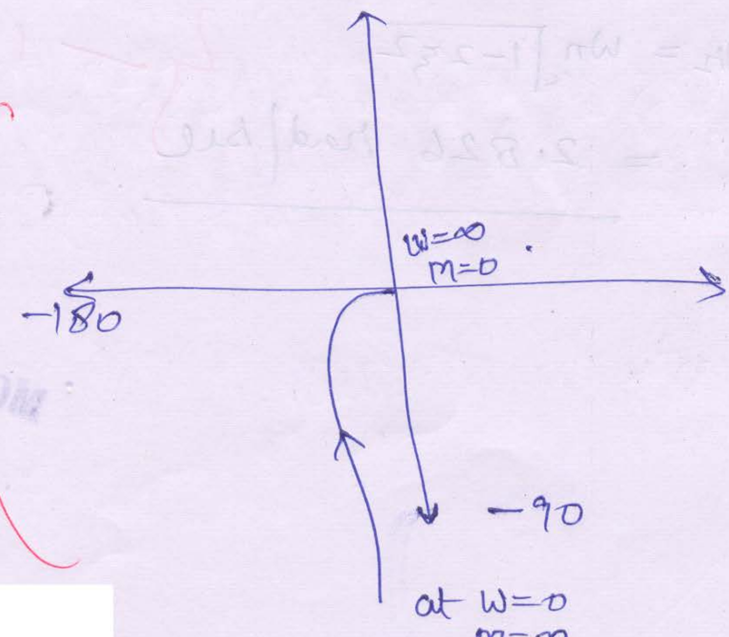
$$\phi = -90 - \tan^{-1}\omega$$

1 mark

at $\omega = 0$ $\quad M = \infty \quad \phi = \angle -90^\circ$

$\omega = \infty \quad M = 0 \quad \phi = -180^\circ$

1 mark



Q.8 **Consider unity feedback control system with the open loop transfer function** $G(s) = \dfrac{k}{s^2(0.2s+1)}$**. Design compensator using Bode plot to produce the following specifications** $K_a = 10$ **and phase margin =** 35°**.**

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Q8

For Ka=10 the value of K is

$$K_a = \lim_{s \to o} s^2 G(s) \qquad \therefore \lim_{s \to 0} \frac{s^2 K}{s^2(0.2s+1)} = 10 \quad \text{so } K = 10$$

2 mark

Step 2: Draw the Bode plot of the system $G(s) = \dfrac{10}{s^2(0.2s+1)}$ as shown in Fig. 10.29. The uncompensated system has PM of $-30°$ at gain cross-over frequency 2.94 rad/sec.

2 mark

2 marks

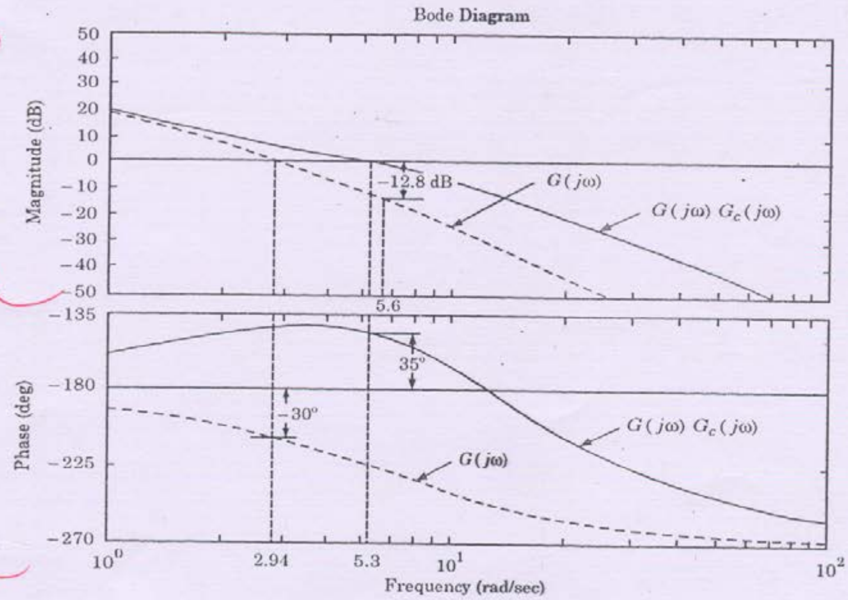

Fig. 10.29

2 mark

Step 3: The additional lead required is

$$\phi_m = \phi_S - \phi_P + \epsilon$$

We have $\quad \phi_S = 35° \quad \phi_P = -30° \quad$ and $\quad \epsilon = 10°$

Note: *ϵ has been selected as large value because of slope of $-$ 60 dB/dec.*

$$\therefore \qquad \phi_m = 35 + 30 + 10 = 75°$$

Since 75° > 60° which can not meet using single lead compensator. So we have to use double lead compensator which has phase lead of $\phi_L = 75/2 = 37\tfrac{1}{2}°$.

1 mark

Step 4: Attenuation factor α is given as

$$\alpha = \frac{1 - \sin\phi_L}{1 + \sin\phi_L}$$

1 mark

$$\alpha = \frac{1-\sin 37.5°}{1+\sin 37.5°} \simeq 0.239$$

Step 5: The uncompensated system will have gain of $-20 \log \frac{1}{\sqrt{\alpha}}$

$$= -10 \log \frac{1}{\alpha} = -12.8 \text{ db}$$

The frequency corresponding to -12.8 db is $\omega'_g = 5.6$ rad/sec.

1 mark

Step 6: Set $\omega'_g = \omega_m$ $\quad \therefore \omega_m = 5.6$ rad/sec.

Now we find out T parameter of lead compensator as

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \quad \therefore \frac{1}{T} = 2.6$$

2 mark

Step 7: Two corner frequencies are

$$\omega_{C1} = \frac{1}{T} \quad \therefore \frac{1}{T} = 2.6 \quad \text{or } T = 0.373$$

$$\omega_{C2} = \frac{1}{T\alpha} \quad \therefore \frac{1}{T\alpha} = 11.8 \quad \text{or } T\alpha = 0.08$$

$\therefore$ The lead compensator $G_C(s) = \frac{(1+0.373s)^2}{(1+0.08s)^2}$ and compensated open loop transfer function $G(s)\, G_C(s)$ is

$$= \frac{10\,(1+0.373s)^2}{s^2\,(1+0.2s)\,(1+0.08s)^2}$$

Sketch the Bode Plot of compensated system as shown in Fig. 10.29. From this plot it is clear that phase margin is increased from $-30°$ to $35°$. Thus the designed transfer function satisfies all above specifications.

3 mark

Q.9 a. Define the following:

(i) State variables **(ii) State vector**

(iii) State space **(iv) State space equations**

Ans 9a

t_0 be zero.

The concept of State is applicable to different type of Systems like physical systems, Biological systems, Economic systems and others.

State Variables: The State variables of a dynamic System are the smallest set of variables which determines the State of dynamic System. If $x_1, x_2 \ldots x_n$ are the variables choosen to describe the behaviour of dynamic System, then the set of these variables are a set of State variables. 1 mark

An important point to be remembered that State variables need not be physically measurable or observable. A variable which does not represent any physical quantity and neither measurable nor observable can be choosen as State variable. 1 mark

State Vector: The n State variables can be considered the n components of a vector $X(t)$ such a vector is called State vector. A State vector is thus a vector that determines the behaviour of System (State) $X(t)$ for any time $t \geq t_0$, once the State at $t = t_0$ is given. 2 marks

State Space: The n-dimensional space whose coordinate axis consists of the x_1 axis, x_2 axis ... x_n axis is called a State Space. 2 marks.

State Space Equation: If n elements of the vector are a set of State variables, then the vector-Matrix differential equation is called State Space Equation. 1 mark

A dynamic System involve elements that memorize the value of Input $t \geq t_1$. Since Integrator in a Continuous-time Control System acts as Memory device. The output of such Integrator can be considered as the variable that define the Internal State of the System. Thus the Output of Integrator serve as State variables so total number of integrators used in the System is equal to State variables. 1 mark

~~Assume that MIMO System involve n-Integrators. Hence State variables are $X_1(t)$, $X_2(t)$ ... $X_n(t)$. Assume also r Inputs as~~

b. Obtain transfer function of system given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -7 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$\& \; Y = [1 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

9b

~~Example 11.3~~ Obtain the transfer function of the System given as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -7 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_1\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$Y = [1 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Solution. From the State Space Equation, we have

$$A = \text{State Matrix } (2 \times 2) = \begin{bmatrix}0 & 1\\ -7 & -2\end{bmatrix}$$ — 1 mark

$$B = \text{Input Matrix } (2 \times 1) = \begin{bmatrix}0\\ 1\end{bmatrix}$$ — 1 mark.

$$C = \text{Output Matrix } [1 \times 2] = [1 \quad 0]$$ — 1 mark

The Transfer Function is given by

$$G(s) = C(sI - A)^{-1} B + D$$ — 1 mark.

where $D = 0$

so
$$(sI - A)^{-1} = \frac{\text{adjoint}\begin{bmatrix}s & -1\\ 7 & s+2\end{bmatrix}}{\begin{vmatrix}s & -1\\ 7 & s+2\end{vmatrix}} = \frac{\begin{bmatrix}s+2 & 1\\ -7 & s\end{bmatrix}}{s^2+2s+7}$$

$$= \frac{1}{s^2+2s+7}\begin{bmatrix}s+2 & 1\\ -7 & s\end{bmatrix}$$

2 mark

$$\therefore \quad G(s) = [1 \quad 0]\begin{bmatrix}\dfrac{s+2}{s^2+2s+7} & \dfrac{1}{s^2+2s+7}\\ \dfrac{-7}{s^2+2s+7} & \dfrac{s}{s^2+2s+7}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$= \begin{bmatrix}\dfrac{s+2}{s^2+2s+7} & \dfrac{1}{s^2+2s+7}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$= \frac{1}{s^2+2s+7}$$

2 mark

Text book

1.Control Systems Engineering, Fifth Edition, Reprint 2011, I.J. Nagrath and M. Gopal, New Age International Pvt. Ltd.