ROLL NO. ____________________

Code: AE35/AC35/AT35 **Subject: MATHEMATICS-II**

AMIETE – ET/CS/IT (OLD SCHEME)

Time: 3 Hours **OCTOBER 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The value of $\lim\limits_{z\to 0}\dfrac{z}{|z|}$

(A) Exist **(B)** Does not exist
(C) 1 **(D)** z

b. The value of the integral $\oint\limits_c \dfrac{dz}{2-\bar{z}}, C: |z| = 1$ is

(A) $\dfrac{\pi i}{2}$ **(B)** $-\dfrac{\pi i}{2}$

(C) $\pm\dfrac{\pi i}{2}$ **(D)** 0

c. The poles of the function $f(z) = \dfrac{z^2}{(z^2+1)^2}$ are of

(A) Order two at z= $\pm i$ **(B)** Order three at z= $\pm i$
(C) Simple poles at z= $\pm i$ **(D)** No poles at z= $\pm i$

d. The angle between the surfaces $x^2 \log z = y^2 - 1$ and $x^2y=2-z$ at point (1,1,1) is

(A) $\theta = 0^\circ$ **(B)** $\theta = \sin 90^\circ$

(C) $\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{30}}\right)$ **(D)** $\theta = 180^\circ$

e. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then the value of $\operatorname{div}\left(\dfrac{\vec{r}}{r^3}\right)$ is

(A) –1 **(B)** 1

(C) $\dfrac{1}{2}$ **(D)** 0

f. The line integral of $\vec{A} = x^2 i - 2yj + z^2 k$ over the straight line path from (–1,2,3) to (2,3,5) is

(A) $\frac{50}{2}$ **(B)** $\frac{92}{3}$

(C) $\frac{35}{2}$ **(D)** $\frac{61}{3}$

g. Two players 'L' and 'M' play tennis. Their chance of winning a game are in the ratio 3:2 respectively then the chance of at winning L's at least two games out of four games are

(A) $\frac{513}{625}$ **(B)** $\frac{413}{625}$

(C) $\frac{213}{625}$ **(D)** $\frac{100}{625}$

h. A die is tossed thrice. A success is getting 1 or 6 on a toss, then the variance of the number of successes is:-

(A) 1 **(B)** –1

(C) $\frac{2}{3}$ **(D)** $-\frac{2}{3}$

i. The Cauchy integral theorem states that if f(i) is analytic in a simply connected domain D, then $\int_c f(z)dz = 0$ on every simple closed path C in D. The condition of analyticity in this theorem is

(A) Necessary **(B)** Sufficient

(C) Both **(A)** and **(B)** **(D)** None of these

j. Inverse transformation $w = \frac{1}{z}$ transforms the straight line ax+by=0 into

(A) Straight line **(B)** Circle

(C) Straight line through the origin **(D)** None of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find the first four terms of the Taylor series expansion of the complex variable function f(z)= $\frac{z+1}{(z-3)(z-4)}$ about $z = 2$. **(8)**

b. If the variance of the Poisson distribution is 2, find the probabilities for r=1,2,3,4 from the recurrence relation of the Poisson distribution. **(8)**

Q.3 a. Evaluate $\oint_c \dfrac{\cos \pi z^2}{(z-1)(z-2)}dz$, where C is the circle $|z|=3$. **(8)**

b. If f(z) is an analytic function of z, prove that $\left(\dfrac{d^2}{dx^2}+\dfrac{d}{dy^2}\right)\log|f'(z)|=0$. **(8)**

Q.4 a. Use Green's theorem to evaluate $\int_c (x^2+xy)dx+(x^2+y^2)dy$, where C is the square formed by the lines x= ± 1, y= ± 1. **(8)**

b. Evaluate $\int_s \vec{A}.\vec{ds}$ by using divergence theorem, where

$\vec{A}=x^3i+y^3j+z^3k$ and s is the surface of the sphere $x^2+y^2+z^2=a^2$. **(8)**

Q.5 a. Use contour integration techniques, evaluate the real integral $\int_0^\infty \dfrac{dx}{1+x^6}$. **(8)**

b. Solve the wave equation $\dfrac{d^2u}{dt^2}=a^2\dfrac{d^2u}{dx^2}$ under the conditions:

(i) u(0,t)=u(x, π)=0
(ii) u(x,0)=x, $0<x<\pi$
(iii) $\dfrac{du}{dt}=0$, when $t=0$. **(8)**

Q.6 a. A six-faced die is so biased that, when thrown, it is twice as likely to show an even number than an odd number. If it is thrown twice, what is the probability that the sum of two numbers thrown is odd. **(8)**

b. Use the method of separation of variables to solve equation $\dfrac{d^2u}{dx^2}=\dfrac{dv}{dt}$ under the conditions:

(i) v=0 at x=0 & x=l
(ii) v=0 when $t\to\infty$. **(8)**

Q.7 a. If $r^2=x^2+y^2+z^2$ and $\vec{R}=xi+yj+zk$ then prove that $\nabla^2 f(r)=f''(r)+\dfrac{2}{r}f'(r)$. **(8)**

b. Find the analytic function $f(z)=u+iv$ where $v(r,0)=\left(r-\dfrac{1}{r}\right)\sin\theta,\quad r\neq 0$. **(8)**

Q.8 a. Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the paths (i) $y = x$ and (ii) $y = x^2$. **(8)**

b. Find the image in the ω plane, of the disk $|z-1| \le 1$ under the mapping $\omega = \frac{1}{z}$. **(8)**

Q.9 a. Evaluate $\iint_s \vec{F}.\hat{n}\, d\vec{A}$, where $\vec{F} = z^2 i + xyj - y^2 k$ and s is the portion of the surface of the cylinder $x^2+y^2=36,\ 0 \le z \le 4$ included in the first octant. **(8)**

b. If f(z) is an analytic function with constant modulus, show that f(z) is constant. **(8)**