ROLL NO. ___________________

Code: AC09/AT09 Subject: NUMERICAL COMPUTING

AMIETE – CS/IT (OLD SCHEME)

Time: 3 Hours **OCTOBER 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2 × 10)

a. Compute the middle value of the numbers a=4.568 and b=6.762 using the four digit arithmetic.

(A) 0.5660×10^1 **(B)** 0.5665×10^1
(C) 0.0566 **(D)** 0.6650

b. The iterative method $x_{k+1} = 2x_k - N^3 x_k^2$, where N is a positive number, is being used to evaluate a certain quantity. If the iteration converges, the method is used for finding

(A) $N^{3/2}$ **(B)** $1/N^3$
(C) $N^{1/3}$ **(D)** $N^{2/3}$

c. In bisection method, if the permissible error is $\in$, then the approximate number of iterations required may be determined from relation. [It is assumed that a root of $f(x) = 0$ lies in the interval (a_0, b_0) and n denotes the number of iterations]

(A) $\dfrac{b_0 - a_0}{2^n} \le \in$ **(B)** $\dfrac{b_0 - a_0}{2^n} \ge \in$
(C) $\dfrac{b_0 - a_0}{n \log 2} \le \in$ **(D)** $\dfrac{b_0 - a_0}{n \log 2} \ge \in$

d. LU decomposition method requires.

(A) forward substitution
(B) backward substitution
(C) both forward and backward substitution
(D) none of these

e. Consider the following statements:

(i) the largest eigenvalue in modulus of a square matrix A cannot exceed the largest sum of the moduli of the elements along any row or column

(ii) the eigenvalues of the matrix A are given by the diagonal elements when it has any one of the forms, diagonal, upper triangular or lower triangular

Which of the above statements are correct?

(A) (i) only **(B)** (ii) only
(C) (i) and (ii) both **(D)** none of the these

f. Let f(x) = e^{ax}. Then, $\Delta^2 f(x)$, where Δ is forward difference operator, with step size h, is given by.

(A) $(e^{ah} - 1)e^{ax}$ **(B)** $e^{ah} - 1$
(C) $(e^{ah} - 1)^2 e^{ax}$ **(D)** $e^{ax} - 1$

g. The least squares linear polynomial approximation for the following data is:

x	-2	-1	0	1	2
f(x)	15	1	1	3	19

(A) -1.5 + 5.6 x **(B)** -1.5 + x
(C) 5.4 + x **(D)** 7.4 + x

h. If λ is an eigen value of a Matrix A, then $\frac{1}{\lambda}$ is the eigen value of

(A) A^2 **(B)** A^T
(C) A^{-1} **(D)** None of the above

i. The order of Convergence of Newton-Raphson Method is

(A) 1 **(B)** 2
(C) 1.67 **(D)** 1.42

j. Taylor series method of order 1 is also known as

(A) Euler's method **(B)** Milne's method
(C) Runge-Kutta method **(D)** None of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Given the following equation, $x\text{-}e^{-x}=0$, determine the initial approximations for finding the smallest positive root. Use these to find the root correct to three decimal places using Secant method. **(8)**

b. Find the iterative method based on Newton-Raphson method for finding $N^{1/k}$, where N is a positive real number. Apply the method to N=18, K=2 to obtain the results correct to two decimal places. **(8)**

Q.3 a. Find the inverse of the matrix.

$$\begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

by Cholesky method **(8)**

b. Set up the Gauss-Jacobi scheme in matrix form to solve the system of equations

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 5 & 2 \\ 5 & 4 & 10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$$

and obtain three iterates starting with initial vector $(0,0,0)^T$, hence find the rate of convergence of the method. **(8)**

Q.4 a. Transform the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$ to tridiagonal form using Givens method. Using Sturm's sequence, obtain exact eigenvalues of matrix A. **(8)**

b. Show that the matrix

$\begin{bmatrix} 12 & 4 & -1 \\ 4 & 7 & 1 \\ -1 & 1 & 6 \end{bmatrix}$ is positive definite. **(8)**

Q.5 a. Using repeated Richardson extrapolation formula, given by

$f''(x) = \dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2}$, compute $f''(0.3)$ from the following table of values: **(8)**

x	0.1	0.2	0.3	0.4	0.5
f(x)	17.60519	17.68164	17.75128	17.81342	17.86742

b. Obtain an approximation using principle of least squares in the form of a polynomial of the degree 2 to the function $\dfrac{1}{(1+x^2)}$ in the range $-1 \le x \le 1$. **(8)**

Q.6 a. Using the data sin (0.1) = 0.09983 and sin (0.2) = 0.19867, find an approximate value of sin (0.15) by Lagrange interpolation. Obtain a bound on the truncation error. **(8)**

b. Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ by subdividing the interval $[0, 1]$ into two equal parts and then applying the Gauss-Legendre three point formula. **(8)**

Q.7 a. Compute the Integral $I = \int_0^1 \frac{dx}{x^3+10}$, using Simpson's rule with the number of points 3,5 and 9. Improve the results using Romberg integration. **(8)**

b. Consider the four point formula

$$f'(x_2) = \frac{1}{6h}\left[-2f(x_1) - 3f(x_2) + 6f(x_3) - f(x_4)\right] + TE + RE$$

Where $x_j = x_0 + jh, j = 1,2,3,4$ and TE, RE are respectively the truncation error and round off error.

(i) Determine the form of TE and RE

(ii) Obtain the optimum step length h satisfying the criterion $|TE| = |RE|$. **(8)**

Q.8 a. Given $\frac{dy}{dx} = \frac{1}{x+y}$, where y(0)=1, find y(0.5) and y(1.0), using Runge-Kutta fourth order method (take h=0.5). **(8)**

b. Given the initial value problem $u' = t^2 + u^2, u(0) = 0$, determine the first three non-zero terms in the Taylor series for u(t) and hence obtain the value for u(1). Also determine t when the error in u(t) obtained from the first two non-zero terms is to be less than 10^{-6} after rounding. **(8)**

Q.9 a. Show that

(i) $\delta = \nabla(1-\nabla)^{-1/2}$ (ii) $\mu = \left[1 + \frac{\delta^2}{4}\right]^{1/2}$ **(6)**

b. Determine an appropriate step size to use, in the construction of a table of $f(x) = (1+x)^6$ on [0,1]. The truncation error for linear interpolation is to be bounded by 5×10^{-5}. **(6)**

c. The matrix $A = \begin{bmatrix} 1+S & -S \\ S & 1-S \end{bmatrix}$ is given. Calculate p and q such that $A^n = pA + qI$.

(4)