ROLL NO. ____________________

Code: AE56/AC56/AT56/AE107/AC107/AT107
Subject: ENGINEERING MATHEMATICS - II

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours | **JUNE 2017** | **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If $u(x,y)= 2x-x^2+a^2y$ is to be a harmonic function, then 'a' should be

(A) 2 **(B)** 3
(C) 0 **(D)** 1

b. The value of $\int_C \frac{3z^2+7z+1}{z+1}dz$, where C is the circle $|z|=\frac{1}{2}$ is

(A) $2\pi i$ **(B)** 0
(C) $\pi i/2$ **(D)** $3\pi i$

c. A unit normal vector to the surface $x^3+y^3+3xyz=3$ at the point $(1,2,-1)$ is

(A) $(-\mathrm{I}+3\mathrm{J}+2\mathrm{K})/\sqrt{14}$ **(B)** $(\mathrm{I}-3\mathrm{J}+2\mathrm{K})/\sqrt{14}$
(C) $(\mathrm{I}+3\mathrm{J}+2\mathrm{K})/\sqrt{14}$ **(D)** $(-\mathrm{I}-3\mathrm{J}+2\mathrm{K})/\sqrt{14}$

d. Which of the following relations is not correct :

(A) $\nabla\times\nabla f=0$ **(B)** $\nabla\bullet\nabla\times F=0$
(C) $\nabla\times(\nabla\times F)=\nabla(\nabla\bullet F)+\nabla^2F$ **(D)** $\nabla\times(\nabla\times F)=\nabla(\nabla\bullet F)-\nabla^2F$

e. The values of $a, b\ and\ c$ such that the vector $(x+y+az)I+(bx+2y-z)J+(-x+cy+2z)K$ is irrotational, are:

(A) $1,1,1$ **(B)** $-1,1,1$
(C) $1,-1,-1$ **(D)** $-1,\ 1,-1$

f. If the interval of differencing is 1, then the value of $\Delta^6\left[(1-x)(1-2x^2)(1-3x^3)\right]$ is

(A) -4320 **(B)** -720
(C) 4320 **(D)** 720

g. Eliminating c from $z=(c+x)^2+y$, we obtain the partial differential equation

(A) $z=\left(\frac{\partial z}{\partial x}\right)^2+y$ **(B)** $z=\left(\frac{\partial z}{\partial y}\right)^2+y$

(C) $z=\frac{1}{4}\left(\frac{\partial z}{\partial x}\right)^2+y$ **(D)** $z=\frac{1}{4}\left(\frac{\partial z}{\partial y}\right)^2+y$

h. If P(A) = 0.5, P(B) =0.7, P(A U B) = 0.75, Then $P(A\cap B)=$

(A) 0.35 **(B)** 0.45

(C) 0.25 **(D)** 0

i. If in binomial distribution the probability of success is 0.75, and the number of trials is 9, then probability of more than one success is

(A) 0.5997 **(B)** 0.7998

(C) 0.6997 **(D)** 0.4997

j. If two dice are rolled then the probability that the sum is 8 and there is an even number on first dice, is

(A) $8/9$ **(B)** $7/36$

(C) $13/18$ **(D)** $1/12$

Answer any FIVE questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. Find the analytic function, whose real part is $\dfrac{\sin 2x}{\cosh 2y - cos\, 2x}$. **(8)**

b. Find the bilinear transformation which maps the point z = 1, i, –1 on to the points w = i, 0, -i and hence find the image of $|z|$ < 1. **(8)**

Q.3 a. Show that the function $f(z)=\begin{cases}\dfrac{x\,y^2\,(x+iy)}{(x^2+y^4)}, & z\neq 0\\ 0, & otherwise\end{cases}$ is not analytic at $z=0$, although Cauchy-Riemann equations are satisfied at $z=0$. **(8)**

b. Find the residue of $f(z)=\dfrac{z^3}{(z-1)(z-2)^2\,(z-3)}$ at its poles and hence evaluate $\oint_C f(z)dz$ where C is the circle $|z|=2.5$. **(8)**

Q.4 a. A particle moves along the curve x = t^3 + 1, y = t^2 z = 2t + 3, where t is the time. Find the components of its velocity and acceleration at t=1 in the diretion I + J + 3k. **(8)**

b. Evaluate $div\,F$ and $curl\,F$ at the point $(1,2,3)$ for $F=x^2yzI+xy^2zJ+xyz^2K$. **(8)**

Q.5 a. Using Green's theorem, evaluate $\int_C \left[(y - \sin x)\,dx + \cos x\,dy \right]$ where C is the triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$. **(8)**

b. Verify Stoke's theorem for $F = (x^2 + y^2) I - 2xy\, J$ taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. **(8)**

Q.6 a. Solve the partial differential equation $(y+z)p - (z+x)q = x - y$. **(8)**

b. Solve the partial differential equation $2xz - px^2 - 2qxy + pq = 0$ by Charpit's method. **(8)**

Q.7 a. Use Newton's divided difference formula to find method to find $f(8)$ from the following data: **(8)**

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

b. The following data gives the velocity of a particle for 20 seconds at an interval of 5 sec. Find the initial acceleration using the entire data. **(8)**

Time *t* (sec)	0	5	10	15	20
Velocity *v* (m/sec)	0	3	14	69	228

Q.8 a. A closet contains 8 pairs of shoes. If 4 shoes are randomly selected, what is the probability that there will be (i) no complete pair and (ii) exactly 1 complete pair? **(8)**

b. A continuous random variable X has the PDF given as

$$f(x) = \begin{cases} a + bx & 0 < x < 4 \\ 0 & \text{otherwise.} \end{cases}$$

If the mean of the distribution is 1/3. Find the values of *a*, *b*. **(8)**

Q.9 a. The marks obtained by students in Mathematics exam has a mean of 62 and variance 20. If the marks obtained has a normal distribution. Find the probability that a student would obtain marks
(i) Greater than 82 (ii) between 48 and 72, (iii) less than 52. **(8)**

b. On an average, 1 computer in 800 crashes during a severe thunderstorm. A certain company had 4,000 working computers when the area was hit by a severe thunderstorm.
(i) Compute the expected value and variance of the number of crashed computers.
(ii) Compute the probability that less than 10 computers crashed.
(iii) Compute the probability that exactly 10 computers crashed. **(8)**