Code: AE51/AC51/AT51/ AE101/AC101/AT101
Subject: ENGINEERING MATHEMATICS-I

ROLL NO. ___________________

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours | **June 2017** | **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Q2 TO Q8 CAN BE ATTEMPTED BY BOTH CURRENT AND NEW SCHEME STUDENTS.**
- **Q9 HAS BEEN GIVEN INTERNAL OPTIONS FOR CURRENT SCHEME (CODE AE51/AC51/AT51) AND NEW SCHEME (CODE AE101/AC101/AT101) STUDENTS.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $z = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to

(A) sin z **(B)** cos z
(C) tan z **(D)** cot z

b. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 3 & 4 \\ 1 & 2 & 4 & 4 \\ 1 & 2 & 3 & 5 \end{vmatrix}$ is

(A) 0 **(B)** 1
(C) 2 **(D)** 3

c. If in a determinant the corresponding elements of two rows (or columns) are identical to each other, then the value of the determinant is

(A) unity **(B)** zero
(C) infinity **(D)** None of these

d. The value of the double integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\,dy\,dx$ is

(A) $\frac{a^3}{4}$ **(B)** a^3
(C) $\frac{a^3}{2}$ **(D)** $\frac{a^3}{3}$

e. The value of K for which equations 3x+y–Kz=0, 4x–2y–3z=0 and 2Kx+4y+Kz=0 are consistent, is

(A) 4 **(B)** 3
(C) 2 **(D)** 1

f. Taylor's series solution of $\frac{dy}{dx} = -xy, y(0) = 1,$ upto x^2 is

(A) $y(x) = 1 + x + \frac{x^2}{2} + ...$ **(B)** $y(x) = 1 + x - \frac{x^2}{2} + ...$
(C) $y(x) = 1 + \frac{x^2}{2} + ...$ **(D)** $y(x) = 1 - \frac{x^2}{2} + ...$

g. Using Newton-Raphson method to root of the equation f(x)=0 fails if

(A) f(x) is an exponential function **(B)** $f'(x)$ is zero
(C) $|f'(x)| = 1$ **(D)** None of these

h. A family of straight lines passing through the origin is represented by

(A) ydx + xdy = 0 **(B)** ydx – xdy = 0
(C) xdx + ydy = 0 **(D)** xdx – ydy = 0

i. The orthogonal trajectories of the family of curves $y = a\,x^2$ is

(A) $x^2 + y^2 = c^2$ **(B)** $x^2 - y^2 = c^2$
(C) $x^2 + 2y^2 = c^2$ **(D)** $xy = c^2$

Where c is an arbitrary constant

j. A matrix 'A' is said to be idempotent matrix if

(A) $A^T A = I$
(B) $A^2 = A$
(C) $A^K = A,$ K is any positive integer value
(D) $A = A^T$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 **a.** If $x^x y^y z^z = c$, show that at x=y=z, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ **(8)**

b. If $x+y=2e^{\theta}\cos\phi$ and $x-y=2ie^{\theta}\sin\phi$, show that

$$\frac{\partial^2 u}{\partial\theta^2}+\frac{\partial^2 u}{\partial\phi^2}=4xy\frac{\partial^2 u}{\partial x\partial y}$$ **(8)**

Q.3 a. Change the order of integration and then evaluate $\int_0^{\infty}\int_0^{x} xe^{-\frac{x^2}{y}}\,dydx$ **(8)**

b. Compute $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ if the region of integration is bounded by the coordinate planes and the plane x+y+z=1 **(8)**

Q.4 a. Find the eigen values and eigen vectors of the matrix. **(8)**

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

b. Find the values of k for which the system of equations

$$(3k-8)x+3y+3z=0$$
$$3x+(3k-8)y+3z=0$$
$$3x+3y+(3k-8)z=0$$

has a non-trivial solution. **(8)**

Q.5 a. Develop Newton – Raphson formula for finding $\sqrt{N}$, where N is a real number. Use it to find $\sqrt{41}$. Correct to 3 decimal places. **(8)**

b. Find by Runge-Kutta method of order four, an approximate value of y at x = 0.2 for the equation $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$, y(0) = 1. Take h = 0.2. **(8)**

Q.6 a. Use method of variation of parameters to solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=\frac{e^x}{x}$. **(8)**

b. Find the general solution of the equation $y''-6y'+13y=8e^{3x}\sin 4x$. **(8)**

Q.7 a. Obtain the series solution of $\frac{d^2y}{dx^2} + x^2 y = 0$ **(8)**

b. Show that $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = 2\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta = 4\int_0^{\infty}\frac{x^2 dx}{1+x^4} = \pi\sqrt{2}$ **(8)**

Q.8 a. Prove that $\int_{-1}^{1} P_m(n)P_n(n)dx = \begin{cases} 0, & m \neq n \\ \frac{2}{2n+1}, & m = n \end{cases}$ **(8)**

b. Prove that $\left|\overline{m}\right. \left|\overline{\left(m+\frac{1}{2}\right)}\right. = \frac{\sqrt{\pi}}{2^{2m-1}} \left|\overline{2m}\right.$ and hence evaluate **(8)**

$\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, m)$

Q.9 **(For Current Scheme students i.e. AE51/AC51/AT51)**

a. Prove that $J_n(x) = \frac{x}{2n}\left[J_{n-1}(x) + J_{n+1}(x)\right]$ **(8)**

b. Show that the family of parabolas $x^2 = ua\,(y + a)$ is self orthogonal. **(8)**

Q.9 **(For New Scheme students i.e. AE101/AC101/AT101)**

a. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ **(8)**

b. Find a Fourier Series to represent x^2 in the interval $(-\ell, \ell)$ **(8)**